Today – §2.2 Verifying Identities

§2.3 Solving Trig Eq'ns

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§2.1 Backtrack

#s 93-108 come up a lot in Calc II

93 $\sqrt{9-x^2}$, $x = 3\cos\theta$

$\sqrt{ab} = \sqrt{a}\sqrt{b}$

$\sqrt{9-(3\cos\theta)^2} = \sqrt{9 - 3^2\cos^2\theta}$

$= \sqrt{9-9\cos^2\theta} = \sqrt{9(1-\cos^2\theta)}$

$= \sqrt{9}\sqrt{1-\cos^2\theta}$ $\sqrt{9}\sqrt{\sin^2\theta}$ $= 3\sqrt{\sin^2\theta}$

$= 3|\sin\theta|$

Assume $0 < \theta < \frac{\pi}{2}$

$= 3\sin\theta$

Principal Square Root. $\sqrt{}$

$\sqrt{x^2} = |x|$

$\sqrt{x}^2 = x$, and the hidden assumption is that $x \geq 0$ to start with.

Next Worksheet this afternoon.

Simplify:

$\sin x \cot(-x)$

$= -\sin x \cot x$

$= -\cancel{\sin x} \cdot \frac{\cos x}{\cancel{\sin x}}$

$= -\cos x$

$\frac{\cos(-x)}{\sin(-x)}$

$= \frac{\cos(x)}{-\sin(x)}$

$= -\cot x$

$\int \frac{1}{\sqrt{4-x^2}}dx$

$x = 2\cos\theta, \ldots$

Calc II

#s 37-58

$\frac{1}{\frac{1}{\cos^2 x}} = 1 \cdot \frac{\cos^2 x}{1}$

$\frac{1}{\tan^2 x + 1} = \frac{1}{\sec^2 x} = \cos^2 x$

$\sin\beta \tan\beta + \cos\beta$

$\sin\beta \cdot \frac{\sin\beta}{\cos\beta} + \cos\beta$

when in doubt, break it into sines & cosines.

$\frac{\sin^2\beta}{\cos\beta} + \cos\beta \cdot \frac{\cos\beta}{\cos\beta}$

$= \frac{\sin^2\beta}{\cos\beta} + \frac{\cos^2\beta}{\cos\beta}$

$= \frac{\sin^2\beta + \cos^2\beta}{\cos\beta}$

$= \frac{1}{\cos\beta} = \sec\beta$

#s 81-84

$$\frac{3}{\sec x - \tan x}$$

write w/o fracs.

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$= \frac{3}{\frac{1}{\cos x} - \frac{\sin x}{\cos x}} = \frac{3}{\frac{1-\sin x}{\cos x}}$$

$$\frac{1}{1-\sqrt{2}} = \left(\frac{1}{1-\sqrt{2}}\right)\left(\frac{1+\sqrt{2}}{1+\sqrt{2}}\right)$$

$$= \boxed{\frac{3\cos x}{1-\sin x}} \cdot \frac{1+\sin x}{1+\sin x}$$

A "conjugate" thing.

$$= \frac{1+\sqrt{2}}{1^2-\sqrt{2}^2} = \frac{1+\sqrt{2}}{1-2} = \frac{1+\sqrt{2}}{-1} = -1-\sqrt{2}$$

$$= \frac{(3\cos x)(1+\sin x)}{1^2 - \sin^2 x}$$

$$(a-b)(a+b) = a^2 - b^2$$

$$= \frac{3\cos x + 3\sin x \cos x}{\cos^2 x}$$

$$= \sec^2 x\,(3\cos x + 3\sin x \cos x)$$

$$= (\sec^2 x)(\cos x)(3 + 3\sin x)$$

$$= \left(\frac{1}{\cos^2 x} \cdot \cos x\right)(3)(1+\sin x)$$

$$\frac{1}{\cos x} \cdot \sin x = \frac{\sin x}{\cos x}$$

$$= \frac{1}{\cos x} \cdot 3 \cdot (1+\sin x)$$

$$= (3\sec x)(1+\sin x) = 3\sec x + 3\underline{\sec x \sin x}$$

$$= \boxed{3\sec x + 3\tan x}$$

$$3\sec x + 3\tan x = \frac{3\cos x}{1-\sin x}$$

This is a typical §2.2 problem.

$$3\sec x + 3\tan x = \frac{3\cos x}{1-\sin x}$$

These are sort of like busy-work, because you can't immediately see WHY.

Good thing is, you know where you want to end up, unlike some "Simplify" question.

§2.2: you may safely skip all the "$\frac{\pi}{2} - x$" cofunction question.

#s 33, 34, 43-46
Blech.

FACTOR THEOREM:

If $f(x)$ is a polynomial, and $f(c)=0$, then $x-c$ is a factor.

$$x^2-5x-6 = (x-6)(x+1) \overset{\text{SET}}{=} 0$$

$$\Longrightarrow x=6,\ x=-1$$

$$x^2-5x-6=0$$

$$a=1, b=-5, c=-6$$

$$\text{Discriminant} = b^2-4ac$$
$$= (-5)^2-4(1)(-6)$$
$$= 25+24$$
$$= 49$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{5\pm\sqrt{49}}{2(1)} = \frac{5\pm7}{2}$$

$$\frac{5+7}{2}=6$$

$$\frac{5-7}{2}=-1$$

$$x=6,\ x=-1$$

$$\Longrightarrow x^2-5x-6=(x-6)(x+1)$$

$x = -\frac{1}{2},\ 1$

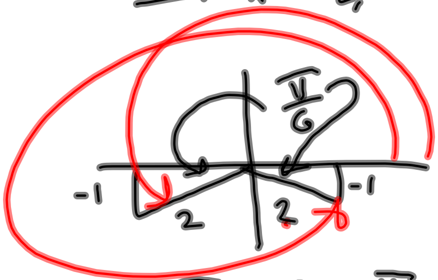
$(x+\frac{1}{2})(x-1)$

$\Longrightarrow 2(x+\frac{1}{2})(x-1)$

$(2x+1)(x-1)$

$= 2x^2 - x - 1 \overset{\text{set}}{=} 0$

$\Longrightarrow x = -\frac{1}{2},\ 1$



$2\pi - \frac{\pi}{6} = \frac{12\pi - \pi}{6} = \frac{11\pi}{6}$

$\pi + \frac{\pi}{6} = \frac{6\pi + \pi}{6} = \frac{7\pi}{6}$

§2.3

Solve

$2\sin^2\theta - \sin\theta - 1 = 0$

Let $u = \sin\theta$

$\Longrightarrow 2u^2 - u - 1 = 0$

OR

$\Longrightarrow u = -\frac{1}{2},\ u = 1$

$\sin\theta = -\frac{1}{2}$ OR $\sin\theta = 1$

Look for sol'ns in $[0, 2\pi]$

$\sin\theta = -\frac{1}{2}$

$\boxed{\theta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}}$

$\sin\theta = 1 \longrightarrow \theta = \frac{\pi}{2}$