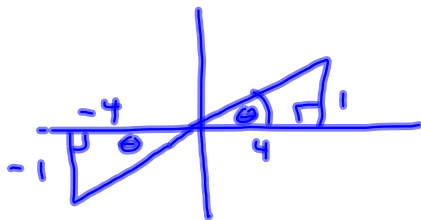
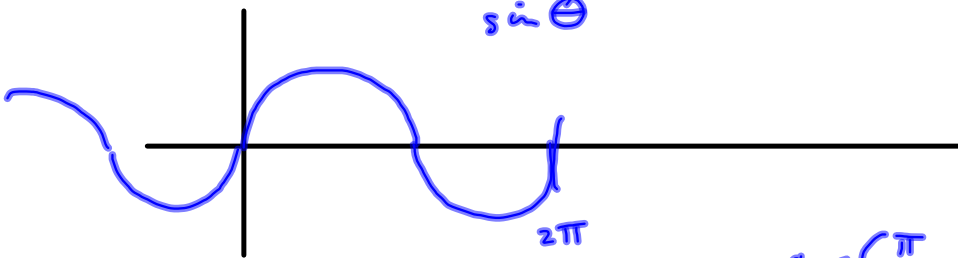


$\cot \theta = 4$  is ambiguous.

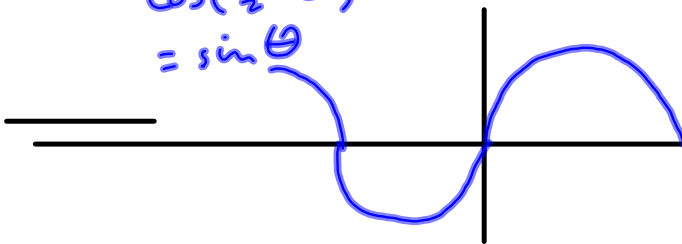


$$\cos\left(\frac{\pi}{2} - \theta\right) = \frac{\sin \theta}{\sin \theta}$$

cofunction identity

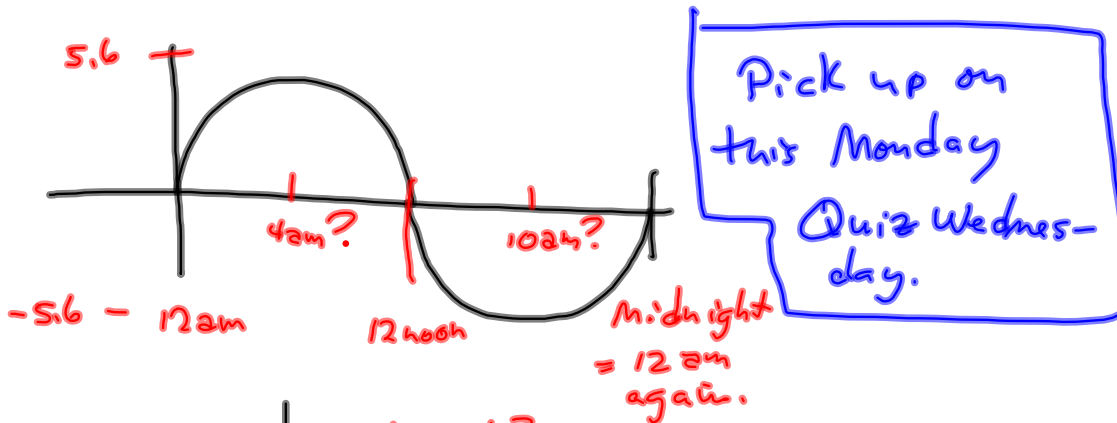


$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

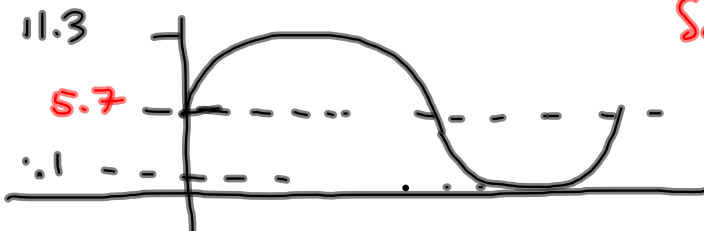


$$\begin{aligned} \cos\left(\frac{\pi}{2} - \theta\right) &= \cos(-\theta + \frac{\pi}{2}) \\ &= \cos(-(\theta - \frac{\pi}{2})) \\ &\textcircled{1} \cos(-\theta) \\ &\textcircled{2} \cos(-(\theta - \frac{\pi}{2})) \end{aligned}$$

$5.6 \sin\left(\frac{\pi}{12}x\right)$  has these 2 features.



We want  $\text{High} = 11.3$   
 $\text{Low} = 0.1$



So now, we're here:

$$5.6 \sin\left(\frac{\pi}{12}x\right) + 5.7$$

5.6 is the amplitude.  
what's the middle height?

$$\frac{11.3 + 0.1}{2} = \frac{11.4}{2} = 5.7$$

Example 7

High @ 4am }  
 Low @ 10am } 6 hrs from high to low.

Let 12am correspond to  $t=0$  (and  $t=24$ )

Period of the tides is 24 hours.

$\sin(\frac{\pi}{12}x)$  has period 24.  $\frac{2\pi}{b} = 24$   
 $\sin(bx)$

High point : 11.3 feet 4am

Low point : .1 foot 10am

$\frac{2\pi}{24} = b = \frac{\pi}{12}$

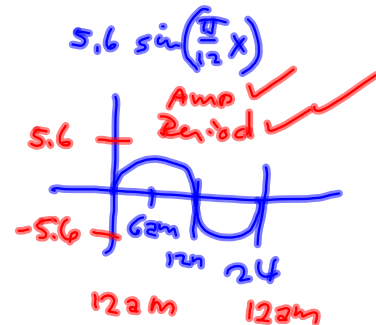
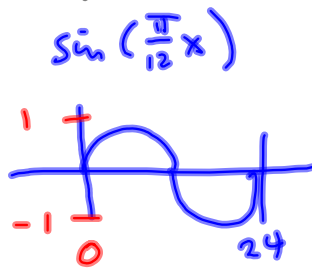
When does it reach mid-height?

$\frac{10+4}{2} = 7 \text{ am}$

Want midpoint @ 7am.

When's the NEXT midpoint? 7pm (12 hrs later)

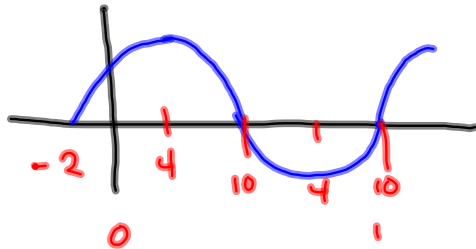
Amplitude =  $\frac{\text{High} - \text{Low}}{2} = \frac{11.3 - .1}{2} = \frac{11.2}{2} = 5.6 \text{ ft}$



Want high to be @ 4am

The way we built it, its high is @ 6am

Need to shift the function  
2 units left to get  
high @ 4am:



$$5.6 \sin\left(\frac{\pi}{12}(x+2)\right)$$

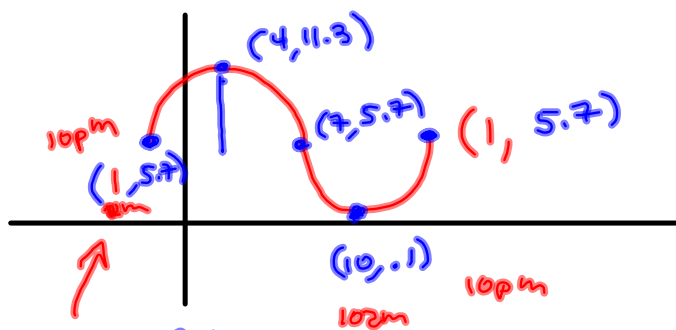
Now a couple ways to view the last step.

$11.3 - 5.6 = 5.7$  Adding 5.7 to this function,  
makes its high point the 11.3 we want.

Midheight:  $\frac{11.3 + 1}{2} = 5.7$  add 5.7 to move  
midheight to the 5.7 we want.

Check: Want Low @ 0.1

$$\text{Low was } -5.6 \text{ \& } -5.6 + 5.7 = .1$$



"(-2, 5.7)  
but clock  
time is periodic, too.

$$5.6 \sin\left(\frac{\pi}{12}(x+2)\right) + 5.7$$

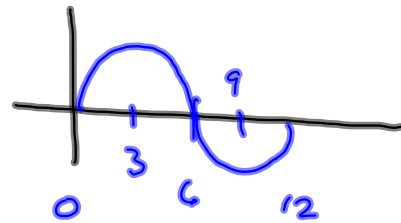
Dummy!  
The period is  
12 hours.

$$5.6 \sin(\quad) + 5.7$$

Period is 12 hrs !!!

$$\frac{2\pi}{b} = 12$$

$$\frac{\pi}{6} = \frac{2\pi}{12} = b$$



$$5.6 \sin\left(\frac{\pi}{6}x\right)$$

high is 4 @  $x=3$  want the high @ 4am.

So delay it by 1 hour!

$$5.6 \sin\left(\frac{\pi}{6}(x-1)\right) \quad \text{Right shift of 1}$$

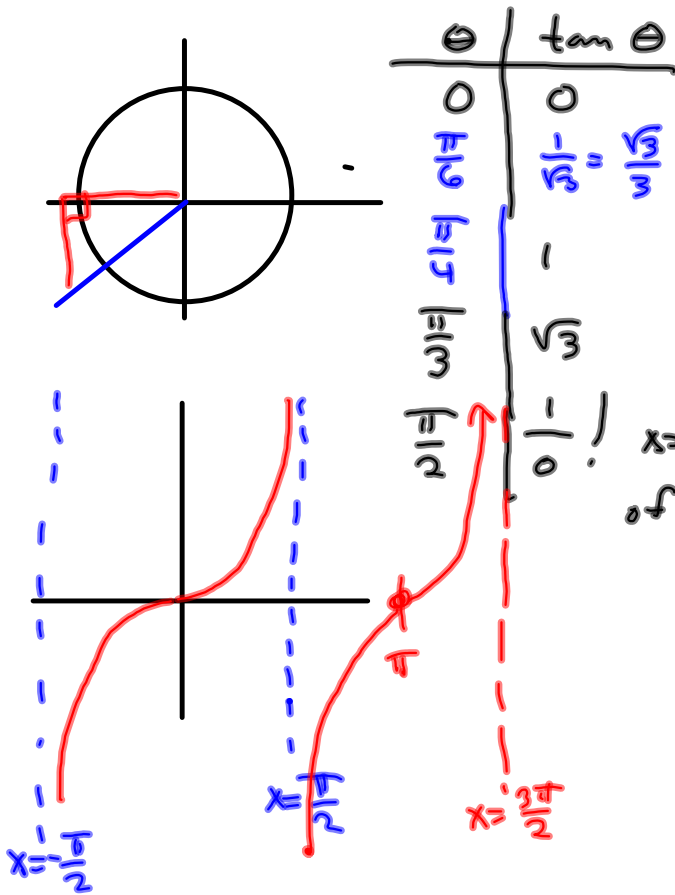
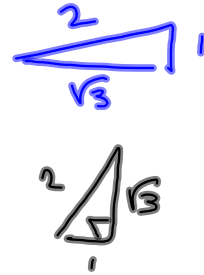
$$5.6 \sin\left(\frac{\pi}{6}(x-1)\right) + 5.7 \quad \text{does it!}$$

Graphing  
confirms  
this



tangent:  

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{y}{x}$$



$x = \frac{\pi}{2}$  is the equation of its vertical asymptote.