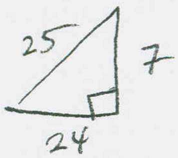
4pts each. Throw out #5 36 poss

MAT 122 Homework 4 Name KEY

Due Wednesday, February 29th 2.1 – 2.4

Do your work on separate paper, organize it, and then show your work, here, but *organized* !!!

1. **2.1** Suppose $\csc(x)=\dfrac{25}{7}$ and $\tan(x)=\dfrac{7}{24}$. Find the values of the other four trigonometric functions.




$\sin x = \dfrac{7}{25}$

$\cos x = \dfrac{24}{25}$ $\sec x = \dfrac{25}{24}$

$\cot x = \dfrac{24}{7}$

Discuss

2. **2.1** Multiply and simplify $(3\sin x-3)(3\sin x+3)$

$= 9\sin^2 x - 9 = 9(\sin^2 x - 1) = 9(-\cos^2 x) = \boxed{-9\cos^2 x}$

3. **2.1** Let $x = 3\sec\theta$ and write $\sqrt{x^2-9}$ as a trigonometric function of $\theta$. Assume $0 \le \theta < 2\pi$.

$\sqrt{(3\sec\theta)^2 - 9}$

$= \sqrt{9\sec^2\theta - 9}$

$= \sqrt{9(\sec^2\theta - 1)}$

$= 3\sqrt{\sec^2\theta - 1}$

$= 3\sqrt{\tan^2\theta}$ Discuss

$= \boxed{3\,|\tan\theta|}$

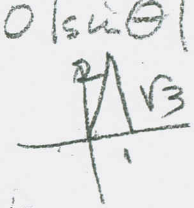
= as far as we can go.

4. **2.1** Assume $-\dfrac{\pi}{2} \le \theta < \dfrac{\pi}{2}$ and make the substitution $x = 10\cos\theta$ in the equation $5\sqrt{3} = \sqrt{100-x^2}$. Solve for $\sin\theta$ and $\cos\theta$.

Discuss in class

$\sqrt{100 - 100\cos^2\theta} = \sqrt{100(1-\cos^2\theta)} = 10\sqrt{\sin^2\theta} = 10|\sin\theta|$

$= 10\sin\theta = \pm 5\sqrt{3} \Rightarrow \boxed{\sin\theta = \pm\dfrac{\sqrt{3}}{2}} \Rightarrow$



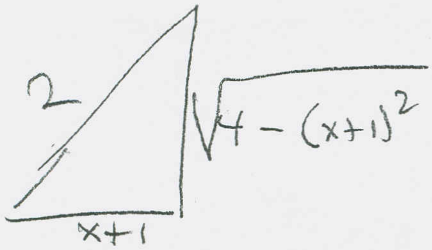

$\boxed{\cos\theta = \dfrac{1}{2}}$ ok

5. **2.2** Verify the identity $\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}} = \dfrac{1-\cos\theta}{|\cos\theta|}$ $\rightarrow |\sin\theta|$

Bad Prob.

$\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}\cdot\dfrac{1-\cos\theta}{1-\cos\theta}} = \sqrt{\dfrac{(1-\cos\theta)^2}{1-\cos^2\theta}} = \dfrac{|1-\cos\theta|}{\sqrt{\sin^2\theta}} = \dfrac{1-\cos\theta}{|\sin\theta|}$

6. **2.2** Use a drawing to verify the identity $\tan\left(\cos^{-1}\left(\frac{x+1}{2}\right)\right)=\frac{\sqrt{4-(x+1)^2}}{x+1}$

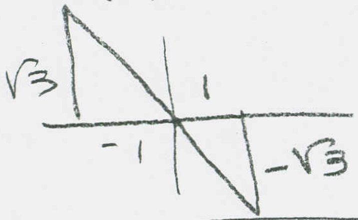



$\Rightarrow \tan(\text{the mess}) = \frac{\sqrt{4-(x+1)^2}}{x+1}$

7. **2.3** Solve the equations:

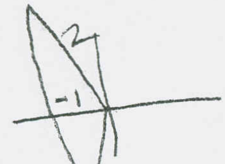
a. $\tan\theta+\sqrt{3}=0$

$\tan\theta = -\sqrt{3}$




$\boxed{\theta = \frac{2\pi}{3} + n\pi,\ n\in\mathbb{Z}}$

b. $\cos(2x)(2\cos(x)+1)=0$




$\cos(2x)=0$

$2x = \frac{\pi}{2}+n\pi$

$x = \frac{\pi}{4}+n\frac{\pi}{2},\ n\in\mathbb{Z}$

$2\cos x + 1 = 0$

$2\cos x = -1$

$\cos x = -\frac{1}{2}$

$\frac{2\pi}{3}+2n\pi,\ n\in\mathbb{Z}$

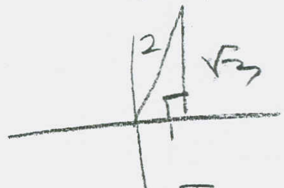
$\frac{4\pi}{3}+2n\pi,\ n\in\mathbb{Z}$

8. **2.3** Find all solutions of $2\cos^2 x-7\cos x+3=0$ in the interval $[0,2\pi)$.

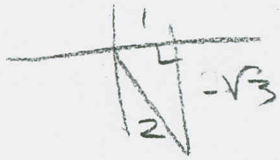
$2\cos^2 x - 7\cos x + 3 = 0$

$(2\cos x - 1)(\cos x - 3)$

$\cos x = \frac{1}{2}$ $\quad$ $\cancel{\cos x = 3}$




$x = \frac{\pi}{3}$




$x = \frac{5\pi}{3}$

$\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$

x =

9. **2.4** Find the exact values of sine, cosine, and tangent of $\theta=\frac{5\pi}{12} = \frac{2\pi+3\pi}{12} = \frac{\pi}{6}+\frac{\pi}{4}$

$\sin\frac{5\pi}{12} = \sin\frac{\pi}{6}\cos\frac{\pi}{4}+\sin\frac{\pi}{4}\cos\frac{\pi}{6} = \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)+\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) = \boxed{\frac{1+\sqrt{3}}{2\sqrt{2}}}$

$\cos\frac{5\pi}{12} = \cos\frac{\pi}{6}\cos\frac{\pi}{4}-\sin\frac{\pi}{6}\sin\frac{\pi}{4} = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)-\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) = \boxed{\frac{\sqrt{3}-1}{2\sqrt{2}}}$

$\tan\left(\frac{5\pi}{12}\right) = \left(\frac{1+\sqrt{3}}{2\sqrt{2}}\right)\left(\frac{2\sqrt{2}}{\sqrt{3}-1}\right) = \boxed{\frac{\sqrt{3}+1}{\sqrt{3}-1}} = \frac{\sin\frac{5\pi}{12}}{\cos\frac{5\pi}{12}}$