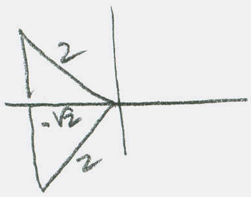
MAT 122 Homework 3 Name KEY

Due February 20th 1.4 – 1.7

Do your work on separate paper, organize it, and then show your work, here, but *organized* !!!

1. Find two solutions for θ. Give both solutions in degrees *and* radians (which makes for *four* answers). Assume $0^0 \le \theta < 360^0$ and $0 \le \theta < 2\pi$ for the answers in radians.

   a. $\cos\theta = -\frac{\sqrt{2}}{2}$

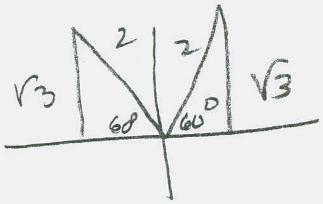


$\theta = \frac{3\pi}{4}, \frac{5\pi}{4}$

$\theta = 135^0, 225^0$

$180-45=$
$180+45=$

   b. $\sin\theta = \frac{\sqrt{3}}{2}$

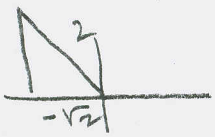


$\theta = \frac{\pi}{3}, \frac{2\pi}{3}$

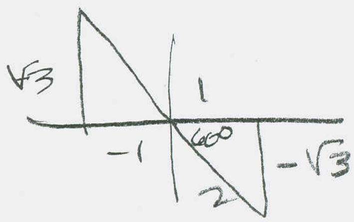
OR $60^0, 120^0$

2. Evaluate the following:

   a. $\arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$



   b. $\operatorname{arccot}(-\sqrt{3})$



$\boxed{= -\frac{\pi}{3}}$

3. Construct a cosine function, $f(x)$, that models daily temperatures in Gunnison Colorado, in midwinter, with a high of 25^0 at 6 p.m. (a bit of a stretch on time of day for peak temperature, I realize...), and a low of -10^0 at 6 a.m. One day represents one period.

$MAX = 25,\ MIN = -10 \Rightarrow MID = \frac{15}{2},\ AMP = \frac{35}{2}$

$T = 24 = \frac{2\pi}{b} \Rightarrow b = \frac{\pi}{12}$ up $\frac{15}{2}$

Max @ $t = 12$

$\frac{35}{2}\cos\left(\frac{\pi}{12}(x-18)\right) + \frac{15}{2}$

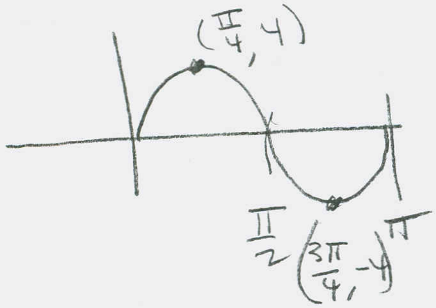
$x=0$ corresponds to 12 midnite

$x=24$ corresponds to 12 midnite, next day.

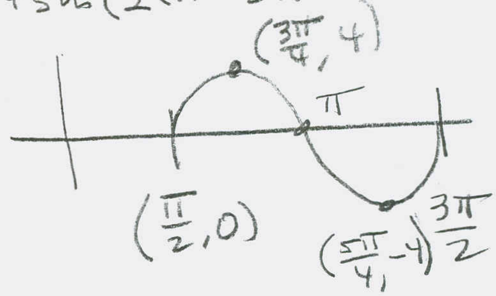
4. Sketch the graph of $g(x) = 4\sin(2x - \pi) + 7$ by transforming the function $f(x) = \sin(x)$.

$= 4\sin\left(2\left(x - \frac{\pi}{2}\right)\right) + 7$

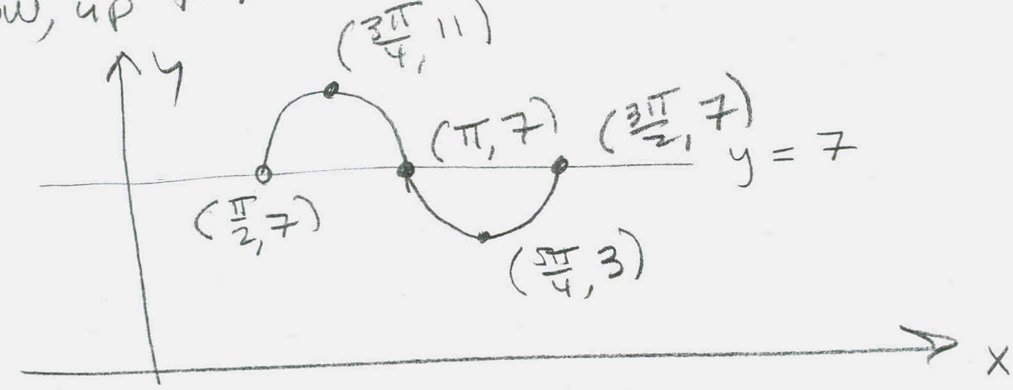
$4\sin(2x)$



$4\sin\left(2\left(x - \frac{\pi}{2}\right)\right)$

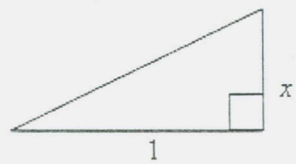


Now, up 7:

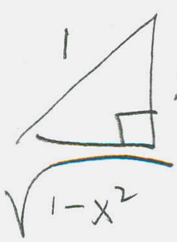


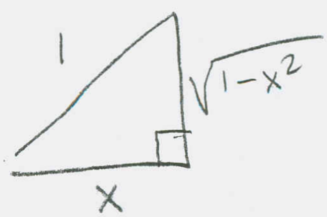
$4\sin(2x - \pi) + 7$

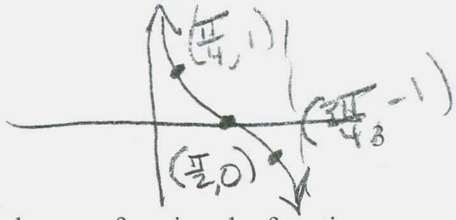
5. In Calculus II, *trigonometric substitution* is a technique for finding the area under the graph of a function involving Pythagorean-type expressions, such as $\sqrt{1 - x^2}$ or $\sqrt{x^2 + 3^2}$. The technique involves evaluating expressions such as $\sin(\arctan(x))$, by constructing an appropriate right triangle, in this case, one with an angle whose tangent is x. From the triangle, and a little Pythagorean action, we see that $\sin(\arctan(x)) = \frac{x}{\sqrt{x^2 + 1}}$. In this spirit, evaluate the following:



a. $\cot(\arctan(x)) = \frac{1}{x}$ b. $\tan(\arcsin(x)) = \frac{x}{\sqrt{1-x^2}}$ c. $\sin(\arccos(x)) = \sqrt{1-x^2}$





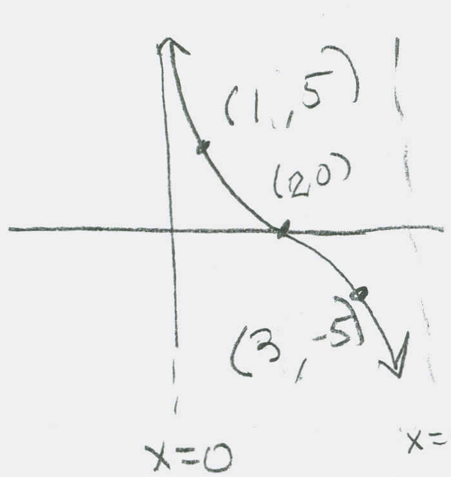


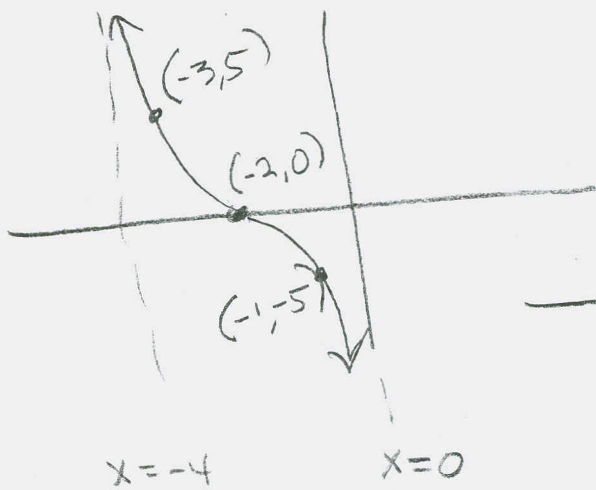
6. Sketch one period of the graph of $g(x) = 5\cot\left(\dfrac{\pi}{4}x + \pi\right) + 7$, by transforming the function $f(x) = \cot(x)$. I want to see the 3 key points corresponding to $x = \dfrac{\pi}{4}, \dfrac{\pi}{2}$, and $\dfrac{3\pi}{4}$, in the graph of $f(x)$. I want to see where these points are moved by each succeeding transformation you apply to $f(x)$, and where they show up in the final graph of $g(x)$.

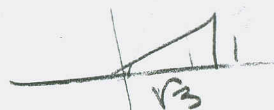
7. Now that you have the graph of g, sketch the graph of $y = 5\sqrt{3} + 7$ on the same coordinate axes, and show where it intersects the graph of g.

$g(x) = 5\cot\left(\frac{\pi}{4}(x+4)\right) + 7$ $\quad T = \dfrac{\pi}{\frac{\pi}{4}} = 4$

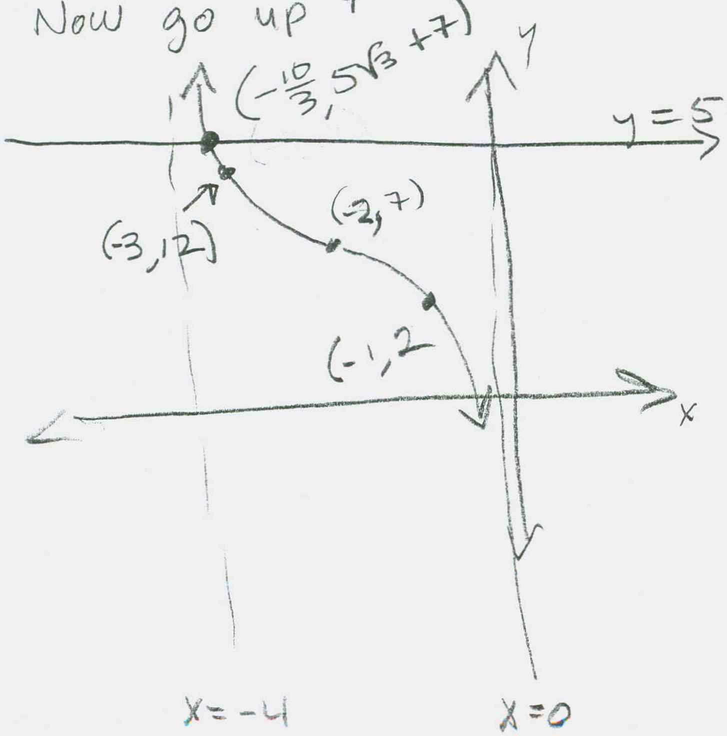
$5\cot\left(\frac{\pi}{4}x\right)$ $\quad 5\cot\left(\frac{\pi}{4}(x+4)\right)$







Now go up 7



Solve: $5\cot\left(\frac{\pi}{4}x+\pi\right)+7$
$= 7 + 5\sqrt{3}$

$\Rightarrow 5\cot\left(\frac{\pi}{4}x+\pi\right) = 5\sqrt{3}$

$\Rightarrow \cot\left(\frac{\pi}{4}x+\pi\right) = \sqrt{3}$

$\Rightarrow \frac{\pi}{4}x + \pi = \frac{\pi}{6}$

$\Rightarrow \frac{\pi}{4}x = -\frac{5\pi}{6}$

$\Rightarrow x = \left(-\frac{5\pi}{6}\right)\left(\frac{4}{\pi}\right) = -\frac{10}{3}$

$= -3.\overline{3}$