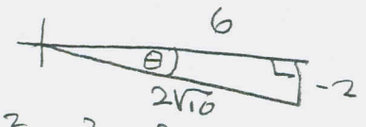
122 HOMEWORK #2

(1a)



$(-2)^2 + 6^2 = c^2$

$c^2 = 4 + 36 = 40$

$c = \sqrt{40}$ (Take it positive)

$= 2\sqrt{10}$

$\sin\theta = \frac{-2}{2\sqrt{10}} = -\frac{1}{\sqrt{10}} \approx -.3162$

$\cos\theta = \frac{6}{2\sqrt{10}} = \frac{3}{\sqrt{10}} \approx .9487$

$\tan\theta = \frac{-2}{6} = -\frac{1}{3} \approx -.3333$

(1b) $\csc\theta = \frac{5}{2}$ is poorly posed, because there are two possible situations:

(I)

(II)

Both satisfy $\csc\theta = \frac{5}{2}$, but $\tan\theta$ & $\cos\theta$ are different.

For situation (I):

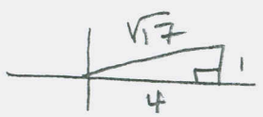
$\sin\theta = \frac{2}{5} = .4$

$\cos\theta = \frac{\sqrt{21}}{5} \approx .9165$

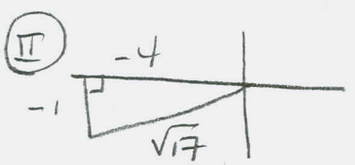
$\tan\theta = \frac{2}{\sqrt{21}} \approx .4364$

(2) $\cot\theta = 4$. This one is ALSO poorly posed. No restriction on θ gives two possibilities.

(I)



(II)



Both satisfy $\cot\theta = 4$

For situation (I):

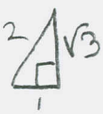
(a) $\tan\theta = \frac{1}{4} = .25$

(b) $\sin\theta = \frac{1}{\sqrt{17}} \approx .2425$

(c) $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta = \frac{1}{\sqrt{17}} \approx .2425$

(3)

is similar to



$\Rightarrow \frac{x}{32} = \cot 60^\circ$

$\Rightarrow x = 32\cot 60^\circ = 32 \cdot \frac{1}{\sqrt{3}}$

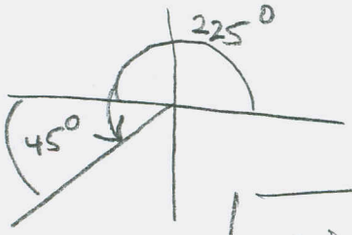
$x = \frac{32}{\sqrt{3}} \approx 18.4752$

122 HOMEWORK #2

(4) (a) $\theta = 225^\circ$

Scratch

$225 - 180 = 45^\circ = \theta'$



$\sin\theta = -\frac{1}{\sqrt{2}} \approx -.7071$

$\cos\theta = -\frac{1}{\sqrt{2}} \approx -.7071$

$\tan\theta = 1$

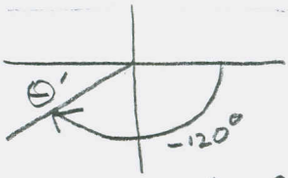
(b) $\theta = -840^\circ$ Scratch: (& thought process) START HERE

$= -120^\circ \pmod{360^\circ}$ $\frac{840}{360} = 2.\overline{3}$, so its twice around plus some. We want the "plus some." We get it by subtracting off the $(2)(360) = 720$:

$840 - 720 = 120$

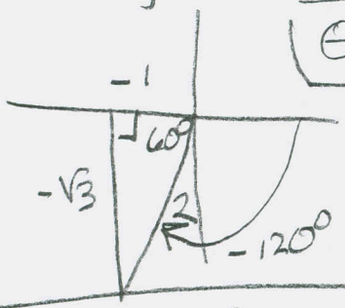
Now, we're working with -120°, since it was -840° to start.

Now go to top left of this problem.



$180^\circ - 120^\circ = 60^\circ$, so we're looking at this picture:

$\theta' = 60^\circ$



$\sin\theta = -\frac{\sqrt{3}}{2}$

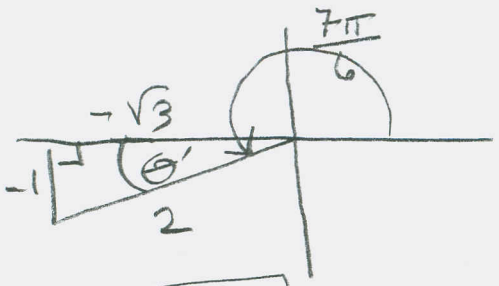
$\cos\theta = -\frac{1}{2}$

$\tan\theta = -\frac{\sqrt{3}}{-1} = \sqrt{3}$

(4) (c) $\theta = \frac{7\pi}{6}$ I "see" this as $\frac{\pi}{6}$ past $\pi = \frac{6\pi}{6}$

π is my "touchstone," here.

Since $\frac{7\pi}{6}$ is between 0 & 2π, I don't need to do the "division thing."



$\theta' = \frac{\pi}{6}$ radians $= 30^\circ$

$\sin\theta = -\frac{1}{2} = -.5$

$\cos\theta = -\frac{\sqrt{3}}{2} \approx -.8660$

$\tan\theta = -\frac{1}{-\sqrt{3}} = \frac{1}{\sqrt{3}} \approx .5774$

(4d) Similar to 4b, only with radians.

$\theta = -\frac{23\pi}{4}$ First we get it between 0 & 2π:

$\frac{\frac{-23\pi}{4}}{2\pi} = \frac{-23}{8} = -2.875$, So it's twice around the circle, plus another little bit.

This work says:

$\frac{\frac{23\pi}{4}}{2\pi} = 2 + .875$, i.e.

$\frac{23\pi}{4} = 2\cdot 2\pi + .875\cdot 2\pi$

$2\cdot 2\pi$: Twice around

$.875\cdot 2\pi$: plus a little bit

One way to get that little bit is to convert $(.875)(2\pi)$ to something we can see.

→ The other way is to subtract off the "twice around"

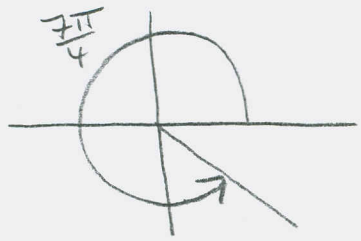
$2\cdot 2\pi = 4\pi$,

$\frac{23\pi}{4} - 4\pi = \frac{23\pi}{4} - \frac{16\pi}{4}$

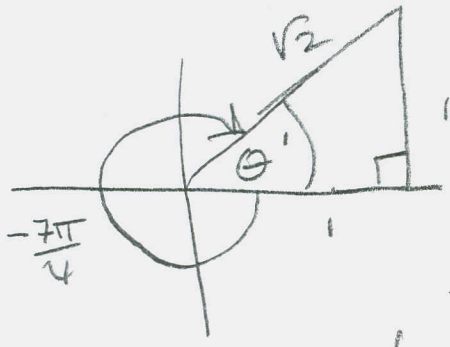
$= \frac{7\pi}{4}$. This says that

$\underbrace{\frac{23\pi}{4} = \frac{7\pi}{4}}_{\text{coterminal, in other words.}} \pmod{2\pi}$

(4d) cnt'd. So, we now know that we're working with an angle coterminal to $\Theta = \frac{23\pi}{4}$. That angle is $\frac{7\pi}{4}$, which (observe!) is $\frac{\pi}{4}$ less than 2π.



But WAIT! I forgot that we started with $-\frac{23\pi}{4}$! That means we need to go CLOCKWISE to get $-\frac{7\pi}{4}$



See $\boxed{\Theta' = \frac{\pi}{4}}$?

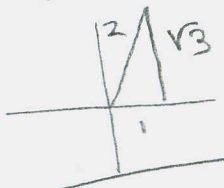
The rest is simple

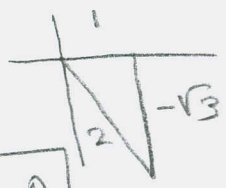
$$\sin\left(\frac{23\pi}{4}\right) = \frac{1}{\sqrt{2}} \approx .7071$$

$$\cos\left(\frac{23\pi}{4}\right) = \frac{1}{\sqrt{2}} \approx .7071$$

$$\tan\left(\frac{23\pi}{4}\right) = 1$$

(5) (a) $\cos\Theta = \frac{1}{2}$

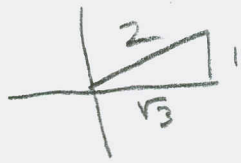




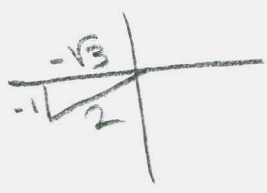
$\Theta = \frac{\pi}{3}$ OR 60°

$\Theta = 360^\circ - 60^\circ = 300^\circ$ OR $2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

(b) $\tan\Theta = \frac{1}{\sqrt{3}}$



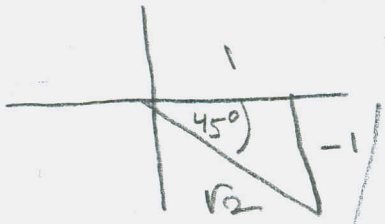
$\Theta = \frac{\pi}{6}$ OR 30°



$\Theta = \frac{7\pi}{6}$ OR 210°

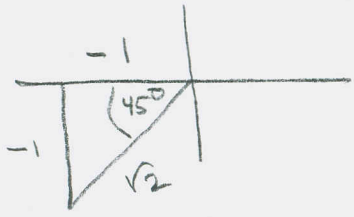
122 HOMEWORK #2

5a $\sin\theta = -\frac{1}{\sqrt{2}}$



$\theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

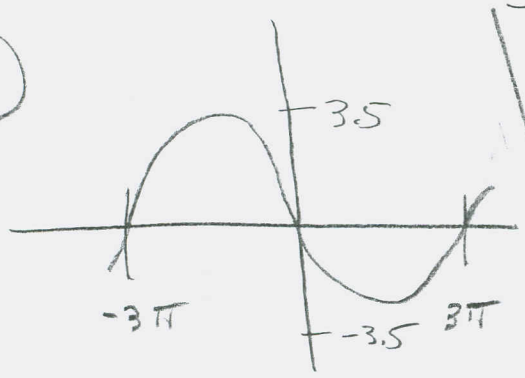
OR $\theta = 360^\circ - 45^\circ = 315^\circ$



$\theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

OR $\theta = 180^\circ + 45^\circ = 225^\circ$

6



Amplitude ≈ 3.5

Period = 6π