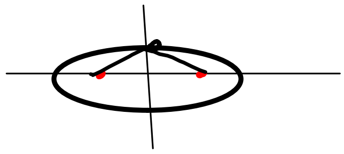
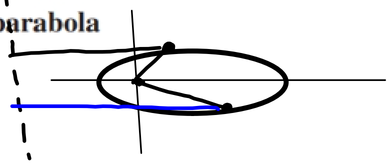
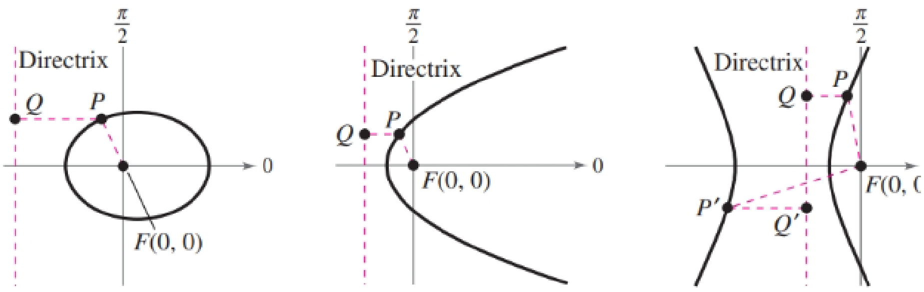
Final is Comprehensive. Due Midnight, December 9th.



Alternative Definition of a Conic

The locus of a point in the plane that moves such that its distance from a fixed point (focus) is in a constant ratio to its distance from a fixed line (directrix) is a **conic.** The constant ratio is the *eccentricity* of the conic and is denoted by e. Moreover, the conic is an **ellipse** when $0 < e < 1$, a **parabola** when $e = 1$, and a **hyperbola** when $e > 1$. (See the figures below.)





Ellipse: $0 < e < 1$

$\frac{PF}{PQ} < 1$

Parabola: $e = 1$

$\frac{PF}{PQ} = 1$

Hyperbola: $e > 1$

$\frac{PF}{PQ} = \frac{P'F}{P'Q'} > 1$

Polar Equations of Conics

The graph of a polar equation of the form

1. $r = \dfrac{ep}{1 \pm e\cos\theta}$ or **2.** $r = \dfrac{ep}{1 \pm e\sin\theta}$

Do right & go up to heaven
This is a positive thing

is a conic, where $e > 0$ is the eccentricity and $|p|$ is the distance between the focus (pole) and the directrix.

$r = \dfrac{ep}{1+e\sin\theta}$ Horizontal Directrix above

$r = \dfrac{ep}{1-e\sin\theta}$ " " BELOW

$r = \dfrac{ep}{1+e\cos\theta}$ vertical Dir. to right

$r = \dfrac{ep}{1-e\cos\theta}$ " " " left.

$$r = \frac{ep}{1 \pm e\cos\theta}$$ Vertical directrix

$$r = \frac{ep}{1 \pm e\sin\theta}$$ Horizontal directrix

1. Horizontal directrix above the pole: $r = \dfrac{ep}{1 + e\sin\theta}$
2. Horizontal directrix below the pole: $r = \dfrac{ep}{1 - e\sin\theta}$
3. Vertical directrix to the right of the pole: $r = \dfrac{ep}{1 + e\cos\theta}$
4. Vertical directrix to the left of the pole: $r = \dfrac{ep}{1 - e\cos\theta}$

we're treating this as axiomatic. (Take on faith.)

Identifying a Conic In Exercises 5–8, write the polar equation of the conic for each value of e. Identify the type of conic represented by each equation. Verify your answers with a graphing utility.

(a) $e = 1$ (b) $e = 0.5$ (c) $e = 1.5$

5. $r = \dfrac{2e}{1 + e\cos\theta}$

6. $r = \dfrac{2e}{1 - e\cos\theta}$

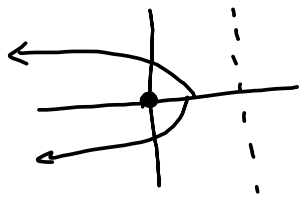
7. $r = \dfrac{2e}{1 - e\sin\theta}$

8. $r = \dfrac{2e}{1 + e\sin\theta}$

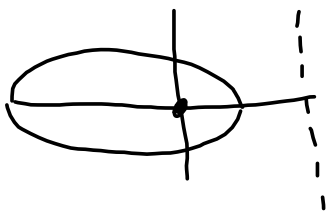
$e<1$ ellipse
$e=1$ parabola
$e>1$ hyperbola

5 a parabola
vert. dir. to right.

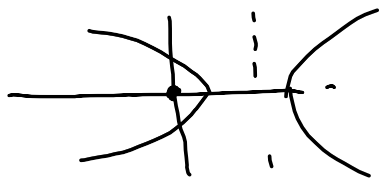
$r = \dfrac{2e}{1+e\cos\theta}$



(b) $e = 0.5$ ellipse

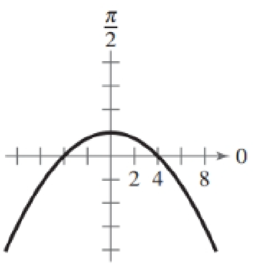


(c) $e = 1.5$ hyperbola

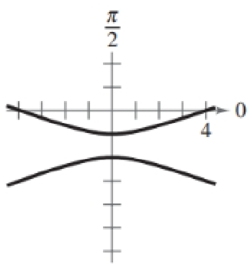


Matching **In Exercises 9–12, match the polar equation with its graph. [The graphs are labeled (a), (b), (c), and (d).]**

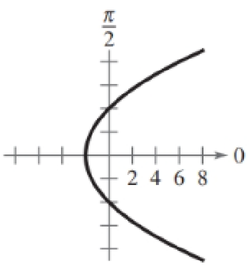
(a)



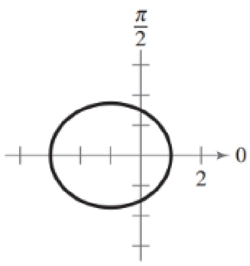
(b)



(c)



(d)



9. $r = \dfrac{4}{1 - \cos\theta}$ c

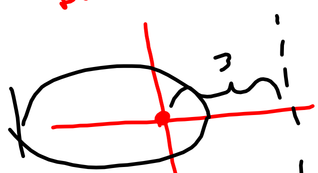
10. $r = \dfrac{3}{2 + \cos\theta}$ $= \dfrac{3}{2\left(1 + \frac{1}{2}\cos\theta\right)} = \dfrac{\frac{3}{2}}{1 + \frac{1}{2}\cos\theta}$ d

$= \dfrac{ep}{1 + e\cos\theta}$

$p = 3$ = distance from pole to directrix

11. $r = \dfrac{4}{1 + \sin\theta}$

12. $r = \dfrac{4}{1 - 3\sin\theta}$



$a = \dfrac{4}{2} = \dfrac{\text{length of major axis}}{2} = 2$

Sketching a Conic In Exercises 13–24, identify the conic represented by the equation and sketch its graph.

13. $r = \dfrac{3}{1 - \cos\theta}$

14. $r = \dfrac{7}{1 + \sin\theta}$

15. $r = \dfrac{5}{1 - \sin\theta}$

16. $r = \dfrac{6}{1 + \cos\theta}$

17. $r = \dfrac{2}{2 - \cos\theta}$

18. $r = \dfrac{4}{4 + \sin\theta}$

19. $r = \dfrac{6}{2 + \sin\theta}$

20. $r = \dfrac{6}{3 - 2\sin\theta}$

21. $r = \dfrac{3}{2 + 4\sin\theta}$

22. $r = \dfrac{5}{-1 + 2\cos\theta}$

23. $r = \dfrac{3}{2 - 6\cos\theta}$

24. $r = \dfrac{3}{2 + 6\sin\theta}$

Graphing a Rotated Conic In Exercises 33–36, use a graphing utility to graph the rotated conic.

33. $r = \dfrac{3}{1 - \cos[\theta - (\pi/4)]}$ (See Exercise 13.)

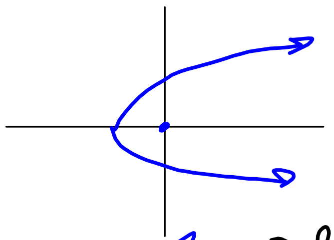
34. $r = \dfrac{4}{4 + \sin[\theta - (\pi/3)]}$ (See Exercise 18.)

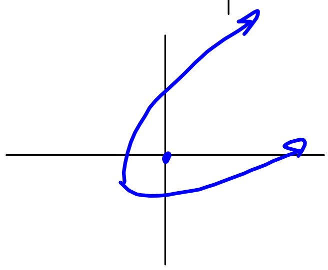
35. $r = \dfrac{6}{2 + \sin[\theta + (\pi/6)]}$ (See Exercise 19.)

36. $r = \dfrac{3}{2 + 6\sin[\theta + (2\pi/3)]}$ (See Exercise 24.)

#33

$\cos(\theta - \frac{\pi}{4})$ is a right shift by $\frac{\pi}{4}$ in rectangular



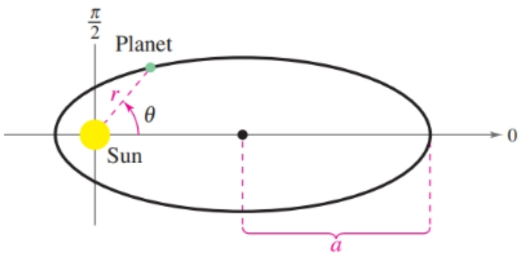


Delayed things by $\frac{\pi}{4}$

Astronomy The planets travel in elliptical orbits with the sun at one focus. Assume that the focus is at the pole, the major axis lies on the polar axis, and the length of the major axis is $2a$ (see figure). Show that the polar equation of the orbit is $r = a(1 - e^2)/(1 - e\cos\theta)$, where e is the eccentricity.

$$r = \frac{ep}{1-e\cos\theta} \overset{\text{want}}{=} \frac{a(1-e^2)}{1-e\cos\theta}$$

It amounts to showing $ep = a(1-e^2)$



Alternative Definition of a Conic

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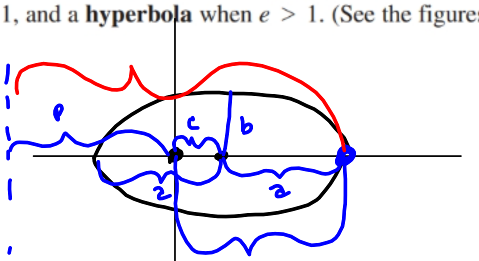
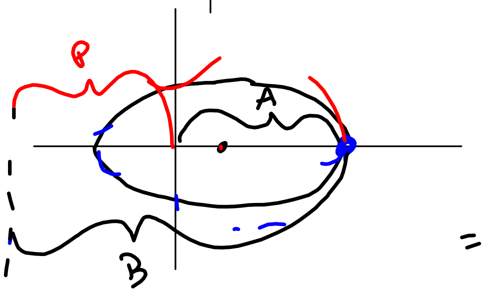


This 'p' is different from the 'p' in earlier section $(x-h)^2 = 4p(y-k)$ Not this p



$$\frac{A}{B} = e = \text{eccentricity}$$

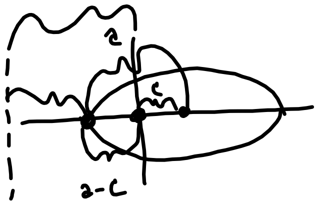
$$= \frac{a+c}{p+A} = \frac{a+c}{p+a+c} = e$$

$$\Rightarrow a+c = e(p+a+c) = ep+ea+ec$$

$$\Rightarrow ep = ea+ec-a-c$$

It's somewhere in here!

Try from the closer vertex.



FACT: $e = \frac{c}{a} \Rightarrow ea = c$

$= ea + ac - a - ea$

$ep = ac - a \cancel{= ac}$

$= a(c-1)$

$= a(ea - 1)$ WANT $= a(1-e^2)$

$$\frac{a-c}{p-(a-c)} = e = \frac{a-c}{p-a+c} = e \Rightarrow$$

$e = \frac{c}{a}$

$a = \frac{c}{e}$

$a - c = e(p - a + c) = ep - ea + ec$

$ep = a - c + ea - ec$

$= a - ea + ea - ec$

$= a - ec = a - e(ea) = a - e^2 a = a(1-e^2)$

So $ep = a(1-e^2)$ ¢

$$\frac{ep}{1-e\cos\theta} = \frac{a(1-e^2)}{1-e\cos\theta}$$

$\boxed{\text{FACT: } e = \frac{c}{a}}$

Conic in Rectangular Coordinates.

$$\frac{(x-7)^2}{36} + \frac{(y+8)^2}{81} = 1$$

81
36

$$81(x-7)^2 + 36(y+8)^2 = 2916$$

$$81(x^2 - 14x + 49) + 36(y^2 + 16y + 64) = 2916 \quad 81$$

$$81x^2 + 1134x + 3969 + 36y^2 + 576y + 2304 = 2916$$

START $81x^2 + 36y^2 + 1134x + 576y = -3357$

Give center & length of major & minor axes

$$\begin{array}{r} 2916 \\ -3969 \\ \hline -1053 \\ -2304 \\ \hline -3357 \end{array}$$

$$81x^2 + 1134x \qquad 36y^2 + 576y = -3357$$

$$81(x^2 + 14x + 7^2) + 36(y^2 + 16x + 8^2) = -1357 + (81)(49) + 36(64)$$

$\frac{14}{2} = 7 \to 7^2 = 49$ $\qquad$ $\frac{16}{2} = 8 \to 8^2$ $\qquad$ -3357

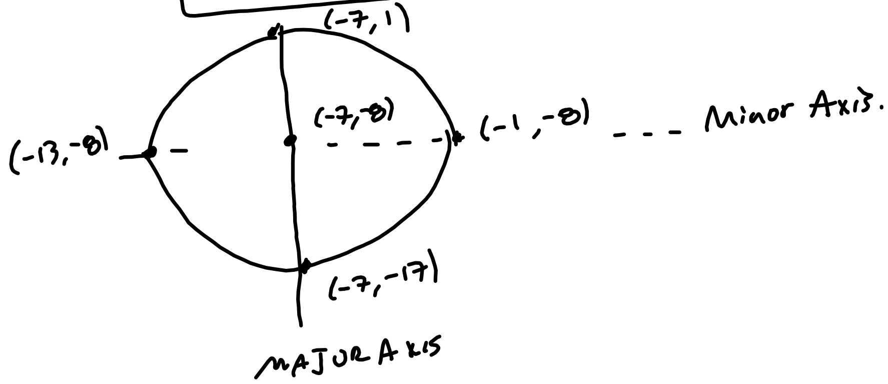
$$81(x+7)^2 + 36(y+8)^2 = 4916 \rightarrow 2$$

$$\frac{81(x+7)^2}{4916 \to 2916} + \frac{36(y+8)^2}{4916 \to 2916} = 1$$

$$\frac{(x+7)^2}{36} + \frac{(y+8)^2}{81} = 1$$

$(h,k) = (-7,-8) =$ center

$a = 6,\ b = 9$



20. 0/1 points

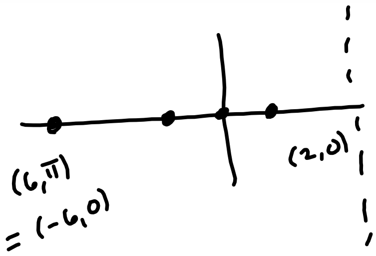
Find a polar equation of the conic in terms of *r* with its focus at the pole.

Conic	*Vertices*
Ellipse	(2, 0), (6, π)

$r = \dfrac{6}{\cos(\theta)+2}$

$2a = 8 \Rightarrow a = 4$

$2 - 4 = -2$



Directrix to the right

$r = \dfrac{ep}{1+e\cos\theta}$

$r(\pi) = 6 = \dfrac{ep}{1+e\cos\pi} = \dfrac{ep}{1-e} = 6$

$r(0) = 2 = \dfrac{ep}{1+e\cos(0)} = \dfrac{ep}{1+e} = 2$

So $2a = \dfrac{ep}{1-e} + \dfrac{ep}{1+e} = 8$

$ep(1+e) + ep(1-e) = 8(1-e)(1+e) = 8(1-e^2)$

$ep + e^2p + ep - e^2p = 8(1-e^2)$

$2ep = 8(1-e^2)$

So, Fernando says: solve for p!

$\dfrac{ep}{1-e} = 6 \Rightarrow ep = 6 - 6e \Rightarrow p = \dfrac{6-6e}{e}$

$\dfrac{ep}{1+e} = 2 \Rightarrow ep = 2 + 2e = \left(\dfrac{6-6e}{e}\right)e$

$2 + 2e = 6 - 6e \Rightarrow$

$8e = 4$

$\boxed{e = \frac{1}{2}}$

$\Rightarrow p = \dfrac{6-6e}{e} = \dfrac{6-3}{\frac{1}{2}} = 2(3) = 6$

$r = \dfrac{ep}{1+e\cos\theta} = \dfrac{3}{1+\frac{1}{2}\cos\theta}$

$= \dfrac{6}{2+\cos\theta}$

$r = \frac{-2}{1-\cos\theta}$ & $r = \frac{2}{1+\cos\theta}$ have same graph!

Just traced out differently