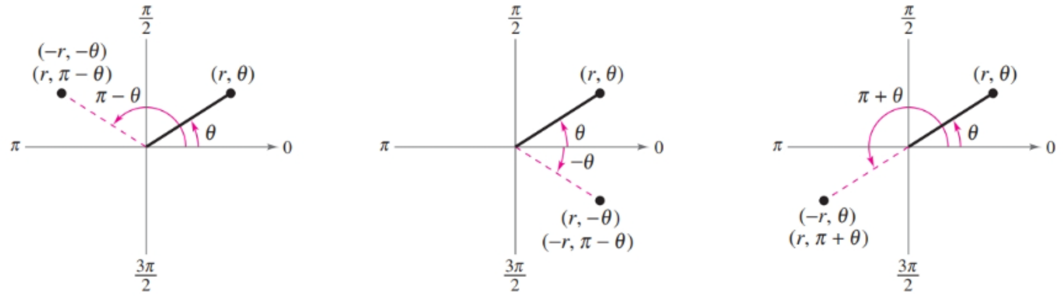
Today: Some symmetry tests, questions answered and then VACATION. (Working vacation?)



Symmetry with Respect to the Line $\theta = \frac{\pi}{2}$

Symmetry with Respect to the Polar Axis

Symmetry with Respect to the Pole

Quick Tests for Symmetry in Polar Coordinates

1. The graph of $r = f(\sin\theta)$ is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

2. The graph of $r = g(\cos\theta)$ is symmetric with respect to the polar axis.

Alex Bingham: who are you?

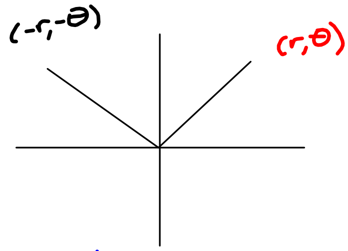
$r = 3\sin\theta + 4$ It's an $f(\sin\theta)$, so symmetric w.r.t. $\theta = \frac{\pi}{2}$

$(-r, -\theta)$:

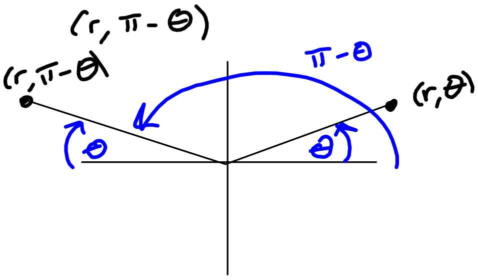
$-r = 3\sin(-\theta) + 4$

$-r = -3\sin\theta + 4 = -(3\sin\theta - 4) \rightarrow$

$r = 3\sin\theta - 4$ Not same.



$(r, \pi - \theta)$



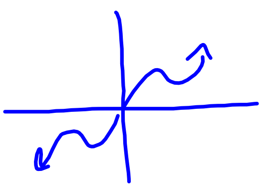
$r = 3\sin(\pi-\theta) + 4$

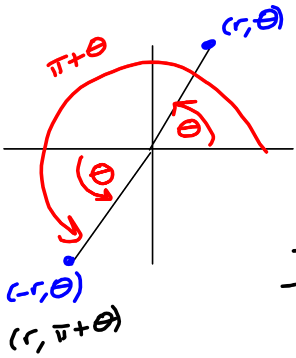
$= 3[\sin\pi\cos(-\theta) + \sin(-\theta)\cos\pi] + 4$

$= 0 + (-3\sin\theta)(-1) + 4$

$r = 3\sin\theta + 4$ SAME!

w.r.t. POLE





2 ways to reflect thru the pole!

$r = 3\sin\theta + 4$

$-r = 3\sin\theta + 4$ Not the same.

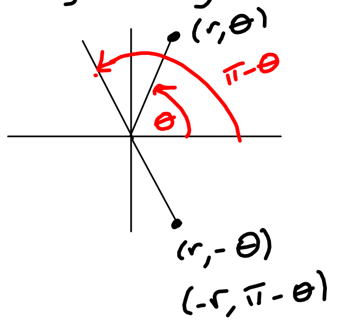
$r = 3\sin(\pi+\theta) + 4$

$= 3[\sin\pi\cos\theta + \sin\theta\cos\pi] + 4$

$= (3\sin\theta)(-1) + 4$

$= -3\sin\theta + 4$ Nope.

Symmetry w.r.t. polar axis (horizontal axis)



$r = 3\sin\theta + 4$

$r = 3\sin(-\theta) + 4$?

$= -3\sin\theta + 4$ Not same

$-r = 3\sin(\pi - \theta) + 4$?

$= 3[\sin\pi \cos(-\theta) + \sin(-\theta)\cos\pi] + 4$

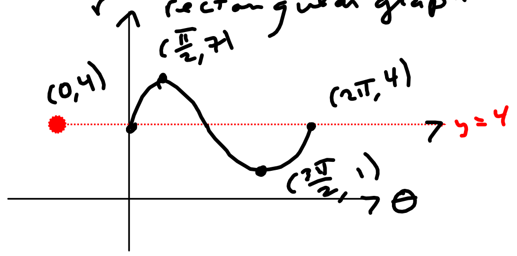
$= 3[(-\sin\theta)(-1)] + 4$

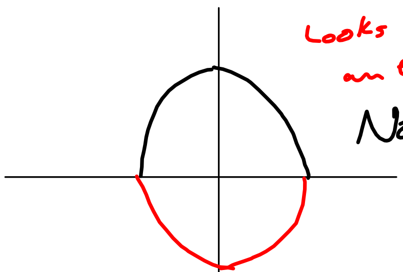
$-r = 3\sin\theta + 4$, so yes! → So No!

So, symmetric w.r.t. $\theta = \frac{\pi}{2}$ & Polar Axis.

$r = 3\sin\theta + 4$

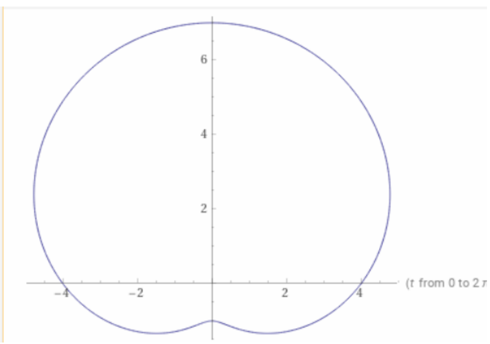
rectangular graph





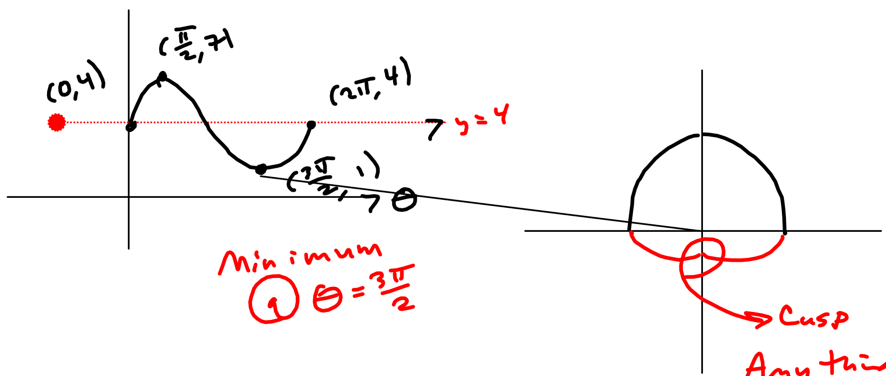
Looks Like an Ellipse!

Nooooooo!



But the graph on wolfram doesn't show symmetry with respect to the polar acis!!!

My symmetry test was wrong!



Minimum @ $\theta = \frac{3\pi}{2}$

Cusp

Anything that can give you a puncture wound.

$x = \sqrt{t} - 3$

$y = t^3$

Observe: $t \geq 0$, b/c $\sqrt{t}$

BASICS OF DOMAIN:

Even index $\sqrt[2n]{\text{stuff}} \Rightarrow \text{stuff} \geq 0.$

$x^3 + \ldots$

$\sqrt{x^3 - 4x}$ Need:

$x^3 - 4x \geq 0$

$x(x^2 - 4) = x(x-2)(x+2) = x(x-2)(x+2)$

N Y Y Y N Y Y

$- \quad + \quad - \quad +$

Want ≥ 0

$-2 \quad 0 \quad 2$

$=0 \quad =0 \quad =0$

$\mathcal{D}$ = Domain $= [-2, 0] \cup [2, \infty)$

Test-value method is O.K., but tedious and slower than general considerations. (General Analysis).

$\dfrac{\text{Anything}}{\text{stuff}}$ Need stuff $\neq 0$

solve "stuff = 0" & throw out the solution.

$\dfrac{\text{Something}}{x^3 - 4x}$ Need $x^3 - 4x \neq 0$

$x(x-2)(x+2) \neq 0$

$x \neq 0$, $x \neq 2$, and $x \neq -2$

$\mathcal{D} = \mathbb{R} \setminus \{-2, 0, 2\}$

Y N Y N Y N Y $\quad \neq 0$

$=$

$-2 \quad 0 \quad 2$

$=0 \quad =0 \quad =0$

$= (-\infty, -2) \cup (-2, 0) \cup (0, 2) \cup (2, \infty)$

2 more weeks after this week. Next week: Conic Sections in Polar Coordinates.

READ 6.9. If you have time, review the conic sections in RECTANGULAR coordinates before Monday.

The $c^2 = a^2 - b^2$ stuff → Ellipse

& $c^2 = a^2 + b^2$ → Hyperbola

$c =$ focal length

Skim §6.2 - 6.4

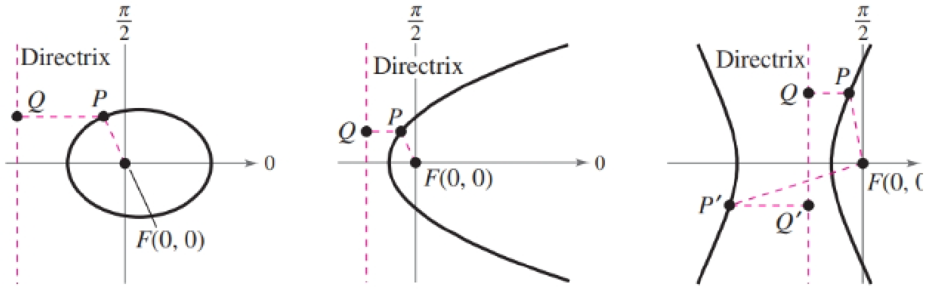
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ $(a > b)$ Ellipse

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ Hyperbola.

Final is Comprehensive. Due Midnight, December 12th.

Alternative Definition of a Conic

The locus of a point in the plane that moves such that its distance from a fixed point (focus) is in a constant ratio to its distance from a fixed line (directrix) is a **conic.** The constant ratio is the *eccentricity* of the conic and is denoted by e. Moreover, the conic is an **ellipse** when $0 < e < 1$, a **parabola** when $e = 1$, and a **hyperbola** when $e > 1$. (See the figures below.)



Ellipse: $0 < e < 1$ $\frac{PF}{PQ} < 1$

Parabola: $e = 1$ $\frac{PF}{PQ} = 1$

Hyperbola: $e > 1$ $\frac{PF}{PQ} = \frac{P'F}{P'Q'} > 1$

Polar Equations of Conics

The graph of a polar equation of the form

1. $r = \dfrac{ep}{1 \pm e\cos\theta}$ or **2.** $r = \dfrac{ep}{1 \pm e\sin\theta}$

is a conic, where $e > 0$ is the eccentricity and $|p|$ is the distance between the focus (pole) and the directrix.

$$r = \frac{ep}{1 \pm e\cos\theta}$$ Vertical directrix

$$r = \frac{ep}{1 \pm e\sin\theta}$$ Horizontal directrix

1. Horizontal directrix above the pole: $r = \dfrac{ep}{1 + e\sin\theta}$
2. Horizontal directrix below the pole: $r = \dfrac{ep}{1 - e\sin\theta}$
3. Vertical directrix to the right of the pole: $r = \dfrac{ep}{1 + e\cos\theta}$
4. Vertical directrix to the left of the pole: $r = \dfrac{ep}{1 - e\cos\theta}$