Today: 6.6 'n' 6.7

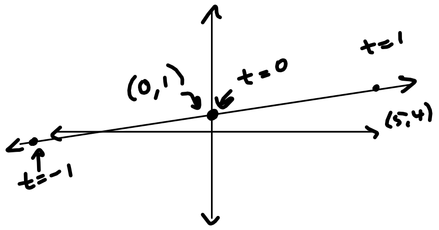
Somethin' screwy about Wednesday's recording.

Parametric Equations

$(x(t), y(t))$, where $x = x(t)$ & $y = y(t)$, is a plane curve.

$x = 5t$, $y = 3t + 1$ is a plane curve.

$\frac{x}{5} = t \Rightarrow y = 3\left(\frac{x}{5}\right) + 1 = \boxed{\frac{3}{5}x + 1 = y}$ → after eliminating the parameter.



t	x	y
-1	-5	-2
0	0	1
1	5	4

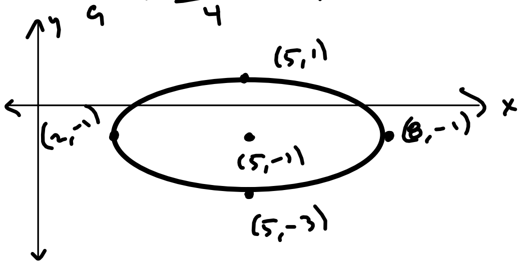
$y = ax^2 + bx + c$ parabola

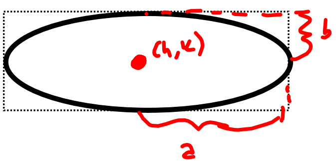
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ ellipse (h,k) = center

$2a$ = horizontal axis length

$2b$ = vertical axis length

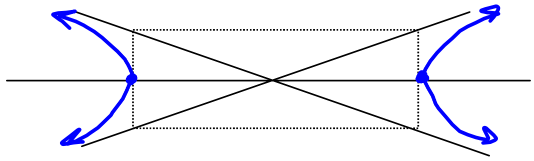
$\frac{(x-5)^2}{9} + \frac{(y+1)^2}{4} = 1$



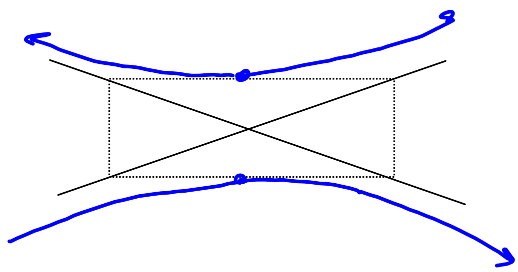


$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Hyperbola



$$\frac{(y-k)^2}{b^2} - \frac{(x-h)}{a^2} = 1$$



$x^2 - y^2 = 1 \quad x \neq 0 \quad (-y^2 = 1$
$y^2 = -1$
$y = \pm i$
Not Real)

) (

$y^2 - x^2 = 1 \quad y \neq 0$

What is it?

$x = e^{-t}$

$y = 2e^{-t} + 1$

$y = 2x + 1$ A line!

52. Circle: $x = h + r\cos\theta, \quad y = k + r\sin\theta$

53. Ellipse with horizontal major axis:

$x = h + a\cos\theta, \quad y = k + b\sin\theta$

54. Hyperbola with horizontal transverse axis:

$x = h + a\sec\theta, \quad y = k + b\tan\theta$

$x = h + r\cos\theta$ $\qquad$ $y = k + r\sin\theta$

$x - h = r\cos\theta$ $\qquad$ $y - k = r\sin\theta$

$\frac{x-h}{r} = \cos\theta$ $\qquad$ $\frac{y-k}{r} = \sin\theta$

$\cos^2\theta + \sin^2\theta = 1$ (by Pythagorus on unit circle)

$\frac{(x-h)^2}{r^2} + \frac{(y-k)^2}{r^2} = 1$ CIRCLE!

$(x-h)^2 + (y-k)^2 = r^2$

$x = h + a\sec\theta, \quad y = k + b\tan\theta$

$\frac{x-h}{a} = \sec\theta, \quad \frac{y-k}{b} = \tan\theta$

Pythagorus says! $\sec^2\theta = \tan^2\theta + 1$

$\left(\frac{x-h}{a}\right)^2 = \left(\frac{y-k}{b}\right)^2 + 1 \implies$

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ Hyperbola! $\quad) \; ($

$x^2 - y^2 = 1 \implies x \neq 0 \implies) \; ($

$(1-t)\langle 1,-4\rangle + t\langle 9,0\rangle$

FROM $(1,-4)$ TO $(9,0)$ LINE SEGMENT.

$(x(t), y(t)) = t(1,-4) + (1-t)(9,0)$ is Backwards.

$t \in [0,1]$

$t=0: \; (9,0)$

$t=1: \; (1,-4)$ so.

$(1-t)\langle 1,-4\rangle + t\langle 9,0\rangle$ is parametric equation of line.

Build the line

$(x_1, y_1) = (1,-4)$

$(x_2, y_2) = (9,0)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4}{8} = \frac{1}{2} = m$

$y = m(x - x_1) + y_1$ POINT-SLOPE

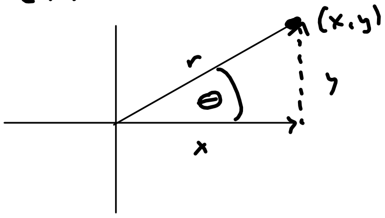
$y = \frac{1}{2}(x-1) - 4$ is perfect!

$y = \frac{1}{2}x - \frac{9}{2}$ is BOOK WAY

That's pretty much all (and more than) you need for 6.6

6.7 : Basics of polar coordinates.

$(x,y) \longmapsto (r, \theta)$



$\tan\theta = \frac{y}{x} \Rightarrow$

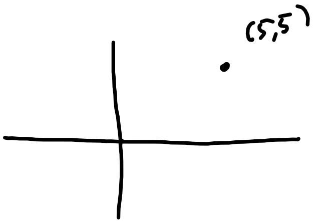
$\theta = \arctan\left(\frac{y}{x}\right)$?

Not quite.

Depends on what quadrant.

$\mathcal{R}(\text{arctangent}) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$x^2+y^2=r^2$

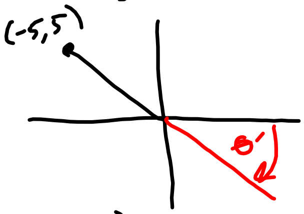


$\theta = \arctan\left(\frac{y}{x}\right) = \arctan(1) = 45^\circ = \frac{\pi}{4}$

$\sqrt{5^2+5^2} = \sqrt{2(5^2)} = 5\sqrt{2} = r,$

so $(r,\theta) = \left(5\sqrt{2}, \frac{\pi}{4}\right)$

" arctangent bad "

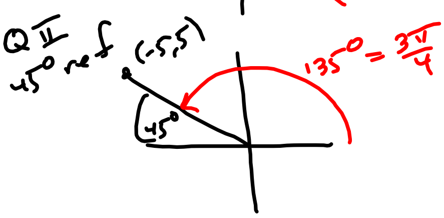


$\arctan\left(\frac{5}{-5}\right) = \arctan(-1) = -\frac{\pi}{4} = -45^\circ$?!

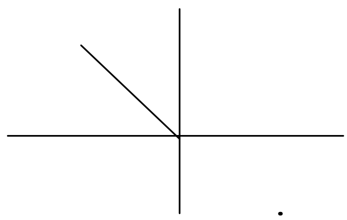
Wrong quadrant!

Reference angle is 45°.

You need to know this is QII



So, $(r,\theta) = \left(5\sqrt{2}, \frac{3\pi}{4}\right)$



$(5\sqrt{2}, \frac{3\pi}{4})$

$(-5\sqrt{2}, -\frac{\pi}{4})$! Same!

$(-5\sqrt{2}, \frac{7\pi}{4})$!

r negative
$r < 0$ } First time in trig.

Next Time:
Wipe Out 6.6 & 6.7, start 6.8

Next Monday, Finish 6.8

$x^2+y^2=9$

$r^2=9$

$r=\pm 3$

$r=3$ gives whole thing

$2xy=1$

$x = r\cos\theta$
$y = r\sin\theta$

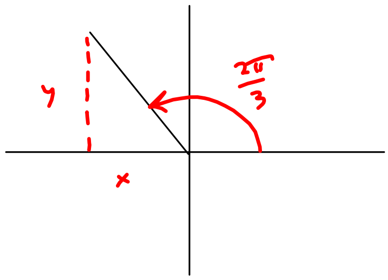
$2(r\cos\theta)(r\sin\theta) = 1$

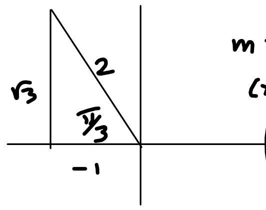
$2r^2\sin\theta\cos\theta = 1$

$r^2 = \frac{1}{2\sin\theta\cos\theta} = \frac{\csc\theta\sec\theta}{2}$

Convert to rectangular

$\theta = \frac{2\pi}{3}$





$m = \frac{\sqrt{3}}{-1} = -\sqrt{3}$

$(x_1, y_1) = (0,0)$

$\boxed{y = -\sqrt{3}\, x}$

$\frac{\pi}{3}$ ref angle in QII

$r = 4\sin\theta =$

$\sqrt{x^2+y^2} = 4\left(\frac{y}{r}\right)$

$r - 4\sin\theta = 0$

$r = 4\sin\theta$

$x^2 + y^2 = \frac{16y^2}{r^2} = \frac{16\,y^2}{x^2+y^2}$

$(x^2+y^2)^2 = 16y^2$ is AN eq'n in rectangular form.
(may be not the best.)