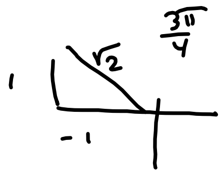
De Moivre

Multiplication of Complex #s.

Trig Form

$$\boxed{\begin{aligned} z_1 &= r_1(\cos\theta_1 + i\sin\theta_1) \\ z_2 &= r_2(\cos\theta_2 + i\sin\theta_2)\end{aligned}}$$

Length of 1. $\sqrt{\cos^2\theta + \sin^2\theta} = 1$



$$z_1 z_2 = \underline{r_1 r_2}\left(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\right)$$

$$z_1 = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = 2\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = \sqrt{3} + i$$

$$z_2 = 3\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) = 3\left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \left(-\frac{3}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} + \frac{3}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}i\right)$$

$$= -\frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i$$

$$\Rightarrow z_1 z_2 = 6\left(\cos\left(\frac{11\pi}{12}\right) + i\sin\left(\frac{11\pi}{12}\right)\right)$$

$$\frac{3\pi}{4}\cdot\frac{3}{3} + \frac{\pi}{6}\cdot\frac{2}{2} = \frac{11\pi}{12}$$

$$(\sqrt{3} + i)\left(-\frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i\right)$$

$$= -\frac{3\sqrt{6}}{2} + \frac{3\sqrt{6}}{2}i - \frac{3\sqrt{2}}{2}i + \frac{3\sqrt{2}}{2}i^2$$

$$= -\frac{3\sqrt{6}}{2} - \frac{3\sqrt{2}}{2} + \left(\frac{3\sqrt{6}}{2} - \frac{3\sqrt{2}}{2}\right)i$$

Fernando

$$\frac{11\pi}{12} = \frac{9\pi}{12} + \frac{2\pi}{12} = \frac{3\pi}{4} + \frac{\pi}{6}$$

$$11 = 10 + 1 = 9 + 2$$

$$\cos(u+v) = \cos u\cos v - \sin u\sin v$$

$$\sin(u+v) = \sin u\cos v + \sin v\cos u$$

$$\Rightarrow z_1 z_2 = 6\left(\cos\left(\frac{3\pi}{4} + \frac{\pi}{6}\right) + i\sin\left(\frac{3\pi}{4} + \frac{\pi}{6}\right)\right)$$

$$= 6\left[\left(\cos\frac{3\pi}{4}\cos\frac{\pi}{6} - \sin\frac{3\pi}{4}\sin\frac{\pi}{6}\right) + i\left(\sin\frac{3\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{6}\cos\frac{3\pi}{4}\right)\right]$$

$$= 6\left[\left(-\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \frac{1}{\sqrt{2}}\cdot\frac{1}{2}\right) + i\left(\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} + \frac{1}{2}\cdot\left(-\frac{1}{\sqrt{2}}\right)\right)\right]$$

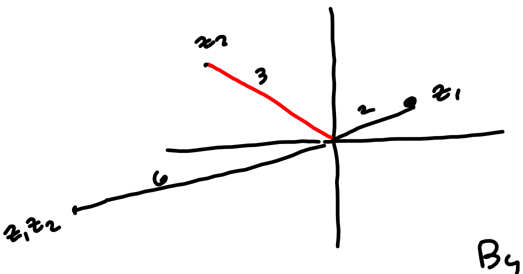
$$= 6\left[\left(\frac{\sqrt{3}}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} - \frac{1}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}\right) + i\left(\frac{\sqrt{3}}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} - \frac{1}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}\right)\right]$$

$$= 6\left[\left(-\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}\right) + i\left(\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}\right)\right]$$

$$= \left(\frac{-3\sqrt{6}}{2} - \frac{3\sqrt{2}}{2}\right) + i\left(\frac{3\sqrt{6}}{2} - \frac{3\sqrt{2}}{2}\right)$$

$$-\frac{3\sqrt{6}}{2} - \frac{3\sqrt{2}}{2} + \left(\frac{3\sqrt{6}}{2} - \frac{3\sqrt{2}}{2}\right)i$$

See? Same!
De Moivre's Made it FAST



Implications for Powers & Roots.

$$z^n = r^n\left(\cos(n\theta) + i\sin(n\theta)\right)$$

By De Moivre:

$$z^n = r\cdot r\cdot r \cdots r\left(\cos(\underbrace{\theta+\theta+\theta+\cdots+\theta}_{n \text{ of 'em}}) + i\sin(\theta+\theta+\cdots+\theta)\right)$$

$$= \boxed{r^n\left(\cos(n\theta) + i\sin(n\theta)\right) = z^n}$$

$$z_1 = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

$$z_2 = 3\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

$$z^{\frac{1}{n}} = r^{\frac{1}{n}}\left(\cos\left(\frac{\theta + 2\pi k}{n}\right) + i\sin\left(\frac{\theta+2\pi k}{n}\right)\right),\ k = 0, 1, \ldots, n-1$$

6

Write the 3 3rd roots of $z_2 = 3\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

$\sqrt[3]{z_2} = \boxed{\sqrt[3]{3}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)}$ = PRINCIPLE 3rd ROOT OF z_2.

1

SAME!

$\frac{2\pi}{3}$ = Increment

Next one: $\frac{\pi}{4} + \frac{2\pi}{3} = \frac{3\pi + 8\pi}{12} = \frac{11\pi}{12}$ $k=1$

$\frac{11\pi}{12} + \frac{8\pi}{12} = \frac{19\pi}{12}$ $k=2$

$\frac{19\pi}{12} + \frac{8\pi}{12} = \frac{27\pi}{12} = \frac{9\pi}{4}$ $k=3$

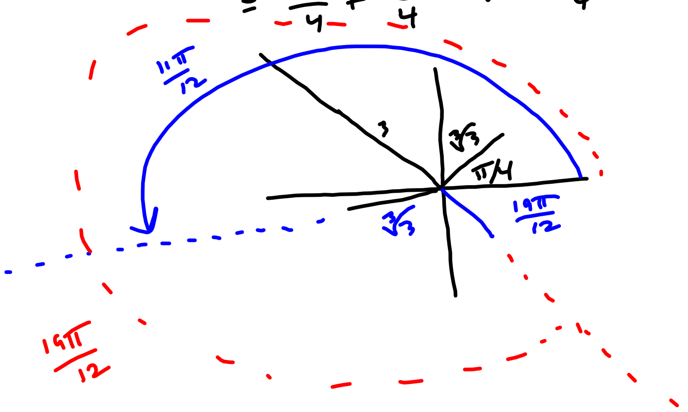
$= \frac{8\pi}{4} + \frac{1\pi}{4} = 2\pi + \frac{\pi}{4}$ is coterminal with $\sqrt[3]{z_2}$

$\sqrt[3]{3}\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right)$ 2

$\sqrt[3]{3}\left(\cos\frac{19\pi}{12} + i\sin\frac{19\pi}{12}\right)$ 3

$\boxed{\sqrt[3]{3}\left(\cos\frac{9\pi}{4} + i\sin\frac{9\pi}{4}\right)}$

$= \sqrt[3]{z_2}$



Discriminant

$5x^2 - x - 3$ HOW MANY REAL SOL'NS?

$a=5,\ b=-1,\ c=-3 \Rightarrow b^2-4ac = 1^2-4(5)(-3) = 1+60=61>0 \Rightarrow$ $\boxed{\text{2 real sol'ns.}}$

$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$ is real b/c radicand (which is the discriminant) is positive.

$\boxed{\text{2 real sol'ns}}$

12. Previous Answers LarTrig10 4.5.044. (3883181)

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Consider the following.

Fifth roots of $243\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

(a) Use the formula $z_k = \sqrt[n]{r}\left(\cos\frac{\theta + 2\pi k}{n} + i\sin\frac{\theta + 2\pi k}{n}\right)$ to find the indicated roots of the complex number. (Enter your answers in trigonometric form. Let $0 \le \theta < 2\pi$.)

5th root: $\frac{2\pi}{5}$ = increment $= \frac{12\pi}{30}$

$\sqrt[5]{243} = \sqrt[5]{3^5} = 3$

$3\,|\,243$, $3\,|\,81$, $3\,|\,27$, $3\,|\,9$, 3 — 5

$\dfrac{\frac{5\pi}{6}}{5} = \frac{5\pi}{6}\cdot\frac{1}{5} = \boxed{\frac{\pi}{6} \quad k=0}$

$k=1 \quad \frac{\pi}{6} + \frac{2\pi}{5} = \frac{5\pi}{30} + \frac{12\pi}{30} = \frac{17\pi}{30}$

$k=2 \quad \frac{17+12}{30}\pi = \frac{29}{30}\pi$

$k=3 \quad \frac{29+12}{30}\pi = \frac{41\pi}{30}$

$k=4 \quad \frac{41+12}{30}\pi = \frac{53\pi}{30}$

$k=5 = k=0$?

$\frac{53+12}{30}\pi = \frac{65}{30}\pi = \frac{13}{6}\pi = \frac{12\pi}{6} + \frac{\pi}{6}$

is coterminal w/ $\frac{\pi}{6}$ ✓

$3\left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right)$

$3\left(\cos\left(\frac{17\pi}{30}\right) + i\sin\left(\frac{17\pi}{30}\right)\right)$

$3\left(\cos\left(\frac{29\pi}{30}\right) + i\sin\left(\frac{29\pi}{30}\right)\right)$

$3\left(\cos\left(\frac{41\pi}{30}\right) + i\sin\left(\frac{41\pi}{30}\right)\right)$

$3\left(\cos\left(\frac{53\pi}{30}\right) + i\sin\left(\frac{53\pi}{30}\right)\right)$