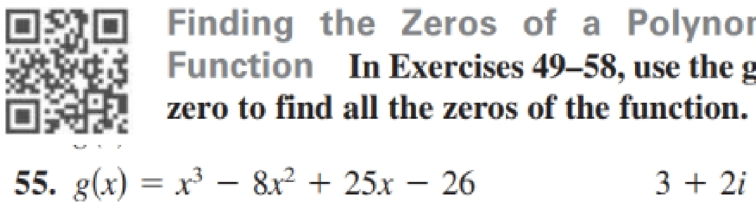


Finding the Zeros of a Polynomial Function In Exercises 49–58, use the given zero to find all the zeros of the function.

55. $g(x) = x^3 - 8x^2 + 25x - 26$ $3 + 2i$ $\Longrightarrow$ $(x-(3+2i))$ is factor.

Divide by $x-(3+2i)$

$$\begin{array}{r|rrrr} 3+2i & 1 & -8 & 25 & -26 \\ & & 3+2i & -19-4i & 26 \\ \hline & 1 & -5+2i & 6-4i & 0 \end{array} \text{ sweet!}$$

Dillon sez

$(3+2i)(-5+2i)$

$= -15+6i-10i+4i^2$

$= -19-4i$ DB

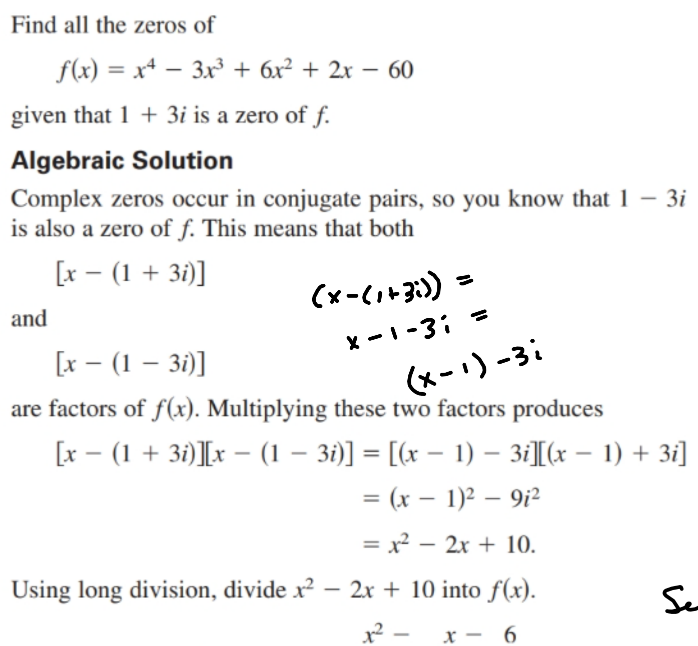
$(3+2i)(6-4i) =$

$= 18-12i+12i-8i^2$

$= 26$

$$\begin{array}{r|rrrr} 3+2i & 1 & -8 & 25 & -26 \\ & & 3+2i & -19-4i & 26 \\ \hline 3-2i & 1 & -5+2i & 6-4i & 0 \text{ sweet!} \\ & & 3-2i & -6+4i & \\ \hline & 1 & -2 & 0 \text{ sweet!} & \\ & x' & c & r & \end{array}$$

This says $f(x) = (x-(3+2i))(x-(3-2i))(x-2)$



Find all the zeros of

$$f(x) = x^4 - 3x^3 + 6x^2 + 2x - 60$$

given that $1 + 3i$ is a zero of f.

Algebraic Solution

Complex zeros occur in conjugate pairs, so you know that $1 - 3i$ is also a zero of f. This means that both

$$[x - (1 + 3i)]$$

and

$$[x - (1 - 3i)]$$

are factors of $f(x)$. Multiplying these two factors produces

$$\begin{aligned}[x - (1 + 3i)][x - (1 - 3i)] &= [(x - 1) - 3i][(x - 1) + 3i] \\ &= (x - 1)^2 - 9i^2 \\ &= x^2 - 2x + 10.\end{aligned}$$

Using long division, divide $x^2 - 2x + 10$ into $f(x)$.

$$\begin{array}{r} x^2 - x - 6 \\ x^2 - 2x + 10 \overline{) x^4 - 3x^3 + 6x^2 + 2x - 60} \\ x^4 - 2x^3 + 10x^2 \end{array}$$

$(x-(1+3i)) =$

$x-1-3i =$

$(x-1)-3i$

$(a-b)^2 = a^2-2ab+b^2$

$(a+b)^2 = a^2+2ab+b^2$

$(a-b)(a+b) = a^2-b^2$

$(a-bi)(a+bi) = a^2-(bi)^2 = a^2-b^2i^2$

$= a^2+b^2$

$(x-(1+3i))(x-(1-3i))$

$= ((x-1)-3i)((x-1)+3i)$

$= (x-1)^2-(3i)^2$

$= x^2-2x+1-9i^2$

$= x^2-2x+1+9$

$= x^2-2x+10$

See Example for more details

$(x^2-2x+10)(x^2-x-6)$

$= (x^2-2x+10)(x-3)(x+2)$

$(x-(1+3i))(x-(1-3i))(x-3)(x+2)$

Split into linear factors.

Caity's question:

Split $x^2-2x+10$ into linear factors:

SLEDGEHAMMER

$a=1,\ b=-2,\ c=10$

$b^2-4ac = (-2)^2-4(1)(10) = 4-40 = -36$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{2 \pm \sqrt{36}}{2} = \frac{2 \pm 6}{2} = \frac{(1 \pm 3i)(2)}{2} = 1 \pm 3i \longrightarrow$$

$x^2-2x+10 = (x-(1+3i))(x-(1-3i))$ is factored

Find all the zeros of

$$f(x) = x^4 - 3x^3 + 6x^2 + 2x - 60$$

given that $1 + 3i$ is a zero of f.

My Method for this example

$$\begin{array}{r|rrrrr} 1+3i & 1 & -3 & 6 & 2 & -60 \\ & & 1+3i & 5+45i & -124+78i & \\ \hline & 1 & 14+3i & 11+45i & -122+78i & \end{array}$$

?! Ernando

$(1+3i)(14+3i)$

$= 14 + 3i + 42i + 9i^2$

$= 5 + 45i$

$(11+45i)(1+3i)$

$= 11 + 33i + 45i + 135i^2$

$= -124 + 78i$

$$\begin{array}{r|rrrrr} 1+3i & 1 & -3 & 6 & 2 & -60 \\ & & 1+3i & -11-3i & 4-18i & 60 \\ \hline 1-3i & 1 & -2+3i & -5-3i & 6-18i & 0 \text{ sweet!} \\ & & 1-3i & -1+3i & -6+18i & \\ \hline & 1 & -1 & -6 & 0 \text{ sweet!} & \\ & x^2 & x^1 & c & r & \end{array}$$

$(1+3i)(-2+3i)$

$= -2 + 3i - 6i + 9i^2$

$= -11 - 3i$

$(-5-3i)(1+3i)$

$= -5 - 15i - 3i - 9i^2$

$= 4 - 18i$

$6(1-3i)$

$= 6 - 18i$

$6(1-3i)(1+3i) = 6(1^2+3^2) = 60$

This says $f(x) = (x-(1+3i))(x-(1-3i))(x^2-x-6)$

$=$

$x^2 - x - 6 = 0$

$(x-3)(x+2)$

$a=1, b=-1, c=-6$

$b^2 - 4ac = 1 - 4(1)(-6) = 25$

$x = \dfrac{1 \pm \sqrt{25}}{2} = \dfrac{1 \pm 5}{2}$ $\to \frac{6}{2} = 3 = x$, $\frac{-4}{2} = -2 = x$

$\Rightarrow x^2 - x - 6 = (x-3)(x+2)$

Complete the Square Method.

$x^2 - x - 6 = x^2 - x + (\frac{1}{2})^2 \underline{- (\frac{1}{2})^2 - 6} = (x - \frac{1}{2})^2 - \frac{25}{4} = 0$

$\Rightarrow (x - \frac{1}{2})^2 = \frac{25}{4}$

$\Rightarrow x - \frac{1}{2} = \pm\sqrt{\frac{25}{4}} = \pm\frac{\sqrt{25}}{\sqrt{4}} = \pm\frac{5}{2}$

$\Rightarrow x = \frac{1}{2} \pm \frac{5}{2}$ $\to \frac{6}{2} = 3$, $\frac{-4}{2} = -2$, etc.

§4.2 #14

$$x^2 - x + 42 = x^2 - x + \left(\frac{1}{2}\right)^2 - \frac{1}{4} + \overset{16}{\frac{42}{1}} \cdot \overset{8}{\frac{4}{4}} = \left(x - \frac{1}{2}\right)^2 + \frac{167}{4} = 0$$

$$\Rightarrow \left(x - \frac{1}{2}\right)^2 = -\frac{167}{4}$$

$$x - \frac{1}{2} = \pm i \frac{\sqrt{167}}{2} \longrightarrow x = \frac{1 \pm i\sqrt{167}}{2} \longrightarrow$$

$$f(x) = \left(x - \left(\frac{1 + i\sqrt{167}}{2}\right)\right)\left(x - \left(\frac{1 - i\sqrt{167}}{2}\right)\right)$$

$$x^2 - x + 42 = 0$$

$$x^2 - x \qquad = -42$$

$$x^2 - x + \left(\frac{1}{2}\right)^2 = -42 + \frac{1}{4} = \frac{-168}{4} + \frac{1}{4} = -\frac{167}{4}$$

$$\uparrow$$

$$\frac{1}{2} \leadsto \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

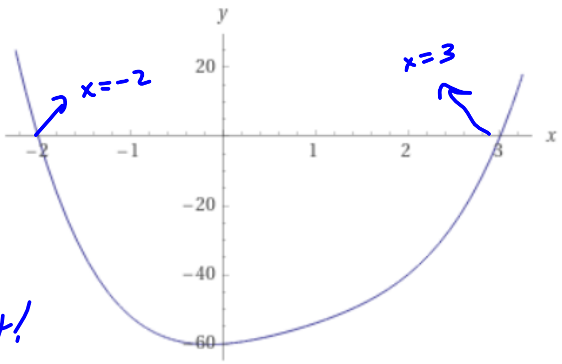
$$\left(x - \frac{1}{2}\right)^2 = -\frac{167}{4}$$, etc.

Find all the zeros of

$$f(x) = x^4 - 3x^3 + 6x^2 + 2x - 60$$

given that 1 + 3i is a zero of f.

Graphical Assist



Rational zeros appear to be $x=-2,3$

-2	1	-3	6	2	-60	
		-2	10	-32	60	
3	1	-5	16	-30	0	sweet!
		3	-6	30		
	1	-2	10	0	sweet!	

$\Rightarrow f(x) = (x+2)(x-3)(x^2-2x+10)$

$x^2-2x+10 = 0$

$x^2-\underset{b}{2x}+1^2 = -10+1$

$\frac{2}{2} = 1 \rightsquigarrow 1^2$

$(x-1)^2 = -9$

$x-1 = \pm\sqrt{-9} = \pm i \cdot 3 = \pm 3i$

$x = 1 \pm 3i \Rightarrow$

$f(x) = (x+2)(x-3)(x-(1+3i))(x-(1-3i))$

$= (x+2)(x-3)(x-1-3i)(x-1+3i)$

$\sqrt{-28}$

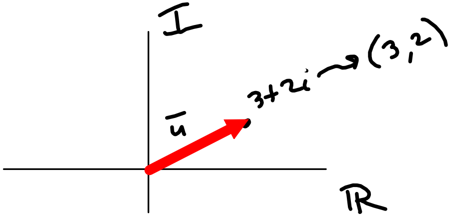
$= 2i\sqrt{7}$

$2 \,\underline{|\, 28}$
$2 \,\underline{|\, 14}$
$\quad 7$

Primes

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37

Magnitude/Modulus of a complex # a+bi



$|3+2i|$ is like magnitude of a vector $\bar{u} = \langle 3,2 \rangle$

$\|\bar{u}\| = \sqrt{\bar{u} \cdot \bar{u}} = \sqrt{3^2+2^2} = \sqrt{13}$

$|3+2i| = \sqrt{3^2+2^2} = \sqrt{13}$

$z = 3+2i \Longrightarrow \bar{z} = 3-2i$ = complex conjugate of z.

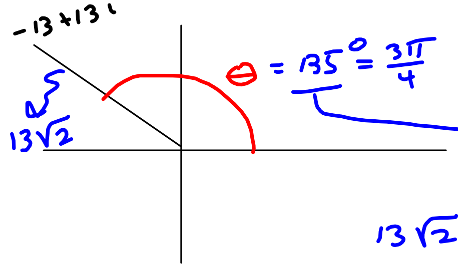
Note $z\bar{z} = (3+2i)(3-2i) = 9+4 = 13$

$= |z|^2$, i.e.,

$|z| = (z\bar{z})^{1/2} = \sqrt{z\bar{z}}$

Trigonemetric Form of $a+bi = z$

$(-13,13)$ $\langle -13, 13 \rangle$



$\theta = 135^\circ = \frac{3\pi}{4}$

$\sqrt{13^2+13^2} = \sqrt{2 \cdot 13^2} = 13\sqrt{2}$

$13\sqrt{2}\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right)$

See connection to chapter 3?

$13\sqrt{2}\left\langle \cos\left(\frac{3\pi}{4}\right), \sin\left(\frac{3\pi}{4}\right)\right\rangle$

ADVANTAGES:

Multiplication is a ROTATION!

$z = 2\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right) = 2(0+i) = 2i$

$w = 3\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right) = 3\left(-\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right) = 3\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)$

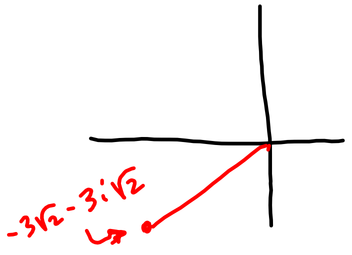
$= \frac{-3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i$

$zw = (2i)\left(-\frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i\right) = -3i\sqrt{2} + 3\sqrt{2}i^2$

$= -3\sqrt{2} - 3\sqrt{2}i = zw$

$|zw|^2 = (3\sqrt{2})^2 + (3\sqrt{2})^2 = 18+18 = 36$

$|zw| = \sqrt{36} = 6$



What's the angle? $\frac{5\pi}{4}$

Check this out!

$$\left(2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)\right)\left(3\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)\right)$$

$$= 6\left(\cos\left(\frac{\pi}{2}+\frac{3\pi}{4}\right) + i\sin\left(\frac{\pi}{2}+\frac{3\pi}{4}\right)\right)$$

$$= 6\left(\cos\left(\frac{5\pi}{4}\right) + i\sin\left(\frac{5\pi}{4}\right)\right)$$

Rotate $7\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$ by $\frac{\pi}{3}$ radians:

$\frac{7\pi}{6}+\frac{2\pi}{6}=\frac{9\pi}{6}=\frac{3\pi}{2}$ $\quad 7\left(\cos\left(\frac{7\pi}{6}\right)+i\sin\left(\frac{7\pi}{6}\right)\right)\left(\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)\right)$

$$= 7\left(\cos\frac{3\pi}{2} + i\sin\frac{7\pi}{2}\right)$$

Next Time Roots & DeMoivre !