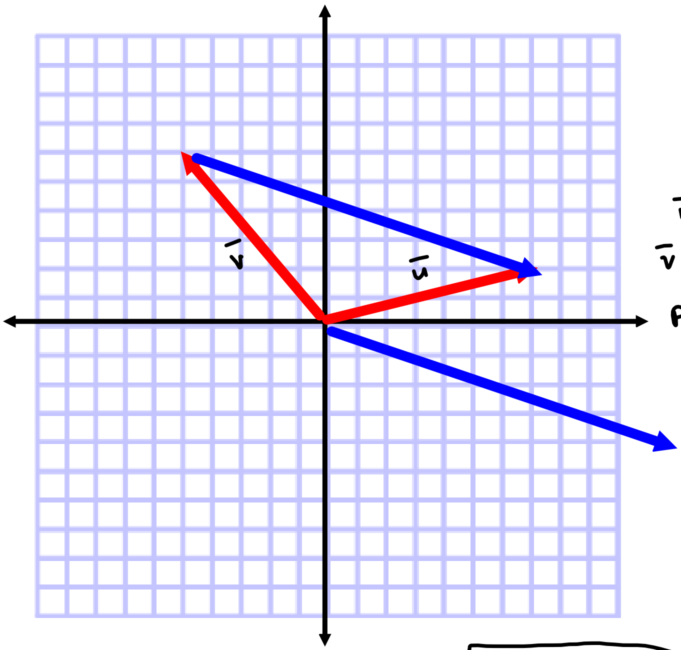



Test 3 postponed 1 week to Monday, 10/31.

Recall:
$\bar{u}-\bar{v}$ is the vector from $\bar{v}$'s terminus to $\bar{u}$'s terminal point (terminus)

$\bar{u} = \langle 7,2\rangle = 7\bar{i} + 2\bar{j}$
$= 7\langle 1,0\rangle + 2\langle 0,1\rangle$

$\bar{v} = \langle -5,6\rangle$

$\bar{u}-\bar{v} = \langle 7,2\rangle - \langle -5,6\rangle$

$= \langle 7+5, 2-6\rangle = \langle 12,-4\rangle$

$= \bar{u} + (-\bar{v})$

Dot Product: $\bar{u} = \langle u_1, u_2\rangle$, $\bar{v} = \langle v_1, v_2\rangle$

$\Rightarrow \bar{u}\bullet\bar{v} = u_1v_1 + u_2v_2$

$= (7)(-5) + (2)(6) = -35 + 12 = -23$

Formally, Dot product is a function from $\underbrace{\mathbb{R}\times\mathbb{R}}_{\mathbb{R}^2}$ into $\mathbb{R}$

$\bullet : \mathbb{R}^2 \longrightarrow \mathbb{R}^1$

Vector space Scalar Field

Distance in $\mathbb{R}^2$:

$D: \mathbb{R}^2 \times \mathbb{R}^2 \longrightarrow \mathbb{R}$

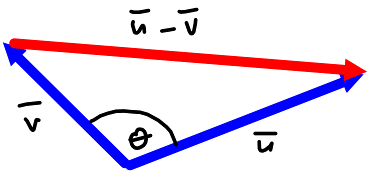
Distance between $P(2,7)$ & $Q(5,6)$ is

$D = \sqrt{(u_1-v_1)^2 + (u_2-v_2)^2}$

$D(P,Q) = \sqrt{(2-5)^2 + (7-5)^2}$

$\mathbb{R}^2$

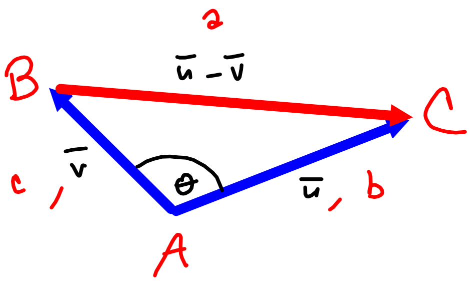
Want to prove $\cos\Theta = \dfrac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\|\bar{v}\|}$.




See Homework Notes for §3.4.

Magnitude of $\bar{u}$ is $\|\bar{u}\|$ = distance from the tip of $\bar{u}$ to $\langle 0,0\rangle$.

$$D = \sqrt{(u_1-0)^2+(u_2-0)^2}$$
$$= \sqrt{u_1^2+u_2^2}$$
$$= \sqrt{u_1 u_1 + u_2 u_2}$$
$$= \sqrt{\bar{u}\cdot\bar{u}}$$
, i.e.,
$$\|\bar{u}\|^2 = \bar{u}\cdot\bar{u}$$




Law of Cosines Says: $a^2 = b^2 + c^2 - 2bc\cos\theta$

$$\|\bar{u}-\bar{v}\|^2 = \|\bar{u}\|^2 + \|\bar{v}\|^2 - 2\|\bar{u}\|\|\bar{v}\|\cos\theta$$

$$2\|\bar{u}\|\|\bar{v}\|\cos\theta = \|\bar{u}\|^2 + \|\bar{v}\|^2 - \|\bar{u}-\bar{v}\|^2$$

$$= \bar{u}\cdot\bar{u} + \bar{v}\cdot\bar{v} - \left((\bar{u}-\bar{v})\cdot(\bar{u}-\bar{v})\right)$$

$$= \bar{u}\cdot\bar{u} + \bar{v}\cdot\bar{v} - \left[(u_1-v_1)(u_1-v_1) + (u_2-v_2)(u_2-v_2)\right]$$

$$= \bar{u}\cdot\bar{u} + \bar{v}\cdot\bar{v} - \left[u_1^2 - 2u_1v_1 + \underline{v_1^2} + u_2^2 - 2u_2v_2 + \underline{v_2^2}\right]$$

$$= \bar{u}\cdot\bar{u} + \bar{v}\cdot\bar{v} - \left[\bar{u}\cdot\bar{u} + \bar{v}\cdot\bar{v} - 2u_1v_1 - 2u_2v_2\right]$$

$$= \bar{u}\cdot\bar{u} + \underline{\bar{v}\cdot\bar{v}} - \bar{u}\cdot\bar{u} - \underline{\bar{v}\cdot\bar{v}} + 2\bar{u}\cdot\bar{v}$$

$$-2u_1v_1 - 2u_2v_2 = -2\left[u_1v_1 + u_2v_2\right]$$
$$= -2\,\bar{u}\cdot\bar{v}$$

$$\Rightarrow 2\|\bar{u}\|\|\bar{v}\|\cos\theta = 2\bar{u}\cdot\bar{v}$$

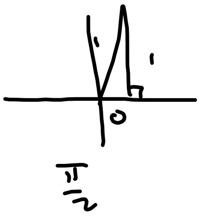
$$\Rightarrow \cos\theta = \frac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\|\bar{v}\|}$$ Woo Hoo!

Find angle in radians between

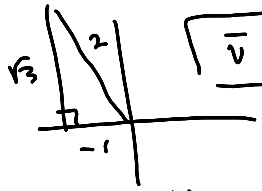
$\bar{u} = \cos\left(\frac{\pi}{2}\right)\bar{i} + \sin\left(\frac{\pi}{2}\right)\bar{j} = \cos\left(\frac{\pi}{2}\right)\langle 1,0\rangle + \sin\left(\frac{\pi}{2}\right)\langle 0,1\rangle$

$= \langle \cos\frac{\pi}{2}, \sin\frac{\pi}{2}\rangle$ Note $\|\bar{u}\| = 1$, because $\sin^2\theta + \cos^2\theta = 1$

& $\cos\left(\frac{2\pi}{3}\right)\bar{i} + \sin\left(\frac{2\pi}{3}\right)\bar{j} = \bar{v} = \langle \cos\left(\frac{2\pi}{3}\right), \sin\left(\frac{2\pi}{3}\right)\rangle$




$\boxed{\bar{u} = \langle 0, 1\rangle}$




$\boxed{\bar{v} = \langle -\frac{1}{2}, \frac{\sqrt{3}}{2}\rangle}$

$$\cos\theta = \frac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\|\bar{v}\|} = \frac{(0)(-\frac{1}{2}) + (1)(\frac{\sqrt{3}}{2})}{\sqrt{0^2+1^2}\sqrt{(-\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}}$$

$$= \frac{\frac{\sqrt{3}}{2}}{(1)(\sqrt{\frac{1}{4}+\frac{3}{4}})} = \frac{\frac{\sqrt{3}}{2}}{\sqrt{1}} = \frac{\sqrt{3}}{2}$$

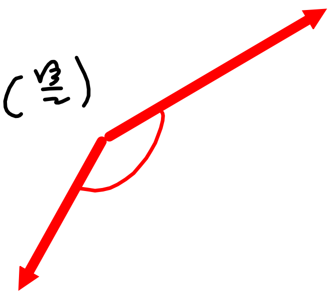
So $\cos\theta = \frac{\sqrt{3}}{2} \Rightarrow$

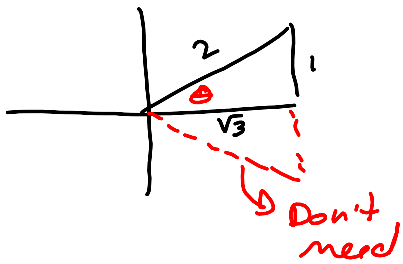
$0 \le \theta \le 180^0 \Rightarrow$ arccosine gives $\theta$ w/o having to decide quadrants.

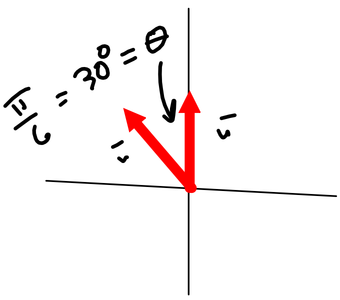
So $\arccos(\cos\theta) = \arccos\left(\frac{\sqrt{3}}{2}\right)$

$\theta = \arccos\left(\frac{\sqrt{3}}{2}\right)$

$= 30^0 = \frac{\pi}{6}$

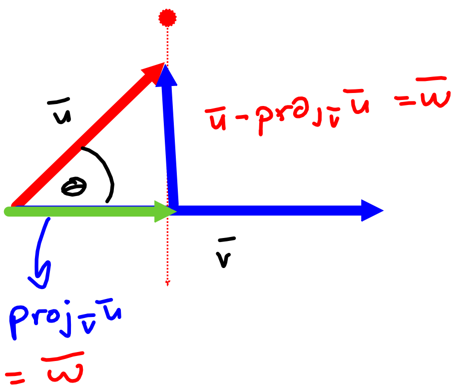


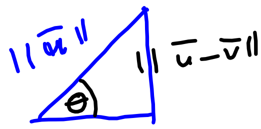


## Projections

$\text{proj}_{\bar v}\bar u$ = Projection of $\bar u$ onto $\bar v$.




$$\|\text{proj}_{\bar v}\bar u\| = |\cos\theta|\,\|\bar u\|$$

$$= \frac{|\bar u \cdot \bar v|}{\|\bar u\|\|\bar v\|}\|\bar u\| = \text{Length of projection.}$$

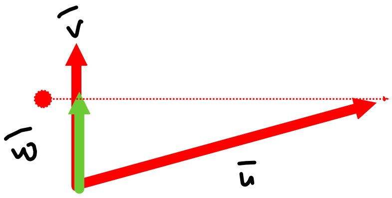
Now, to find the vector we multiply this scalar times a UNIT VECTOR in the direction of $\bar v$

$$\text{proj}_{\bar v}\bar u = \left(\frac{|\bar u \cdot \bar v|\,\|\bar u\|}{\|\bar u\|\,\|\bar v\|}\right)\frac{1}{\|\bar v\|}\bar v$$

In general,

$$\text{proj}_{\bar v}\bar u = \left(\frac{\bar u \cdot \bar v}{\|\bar v\|^2}\right)\bar v$$

$\bar{u} = \langle 7, 2 \rangle$ , $\bar{v} = \langle 0, 3 \rangle$




$$\bar{w} = \text{proj}_{\bar{v}} \bar{u}$$

$$= \frac{\bar{u} \cdot \bar{v}}{\|\bar{v}\|^2} \bar{v}$$

$$= \frac{0 + 6}{3^2} \langle 0, 3 \rangle$$

$$= \frac{6}{9} \langle 0, 3 \rangle = \langle 0, 2 \rangle$$

$$= \text{proj}_{\bar{v}} \bar{u}$$

**26.** 0/1 points

Find $\mathbf{u} \cdot \mathbf{v}$, where $\theta$ is the angle between $\mathbf{u}$ and $\mathbf{v}$.

$$\|\mathbf{u}\| = 4,\ \|\mathbf{v}\| = 9,\ \theta = \frac{2\pi}{3}$$

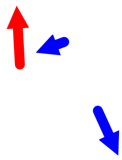
$$\cos\left(\frac{2\pi}{3}\right) = \frac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\|\bar{v}\|} \Rightarrow$$

$$\frac{1}{2} = \frac{\bar{u}\cdot\bar{v}}{(4)(9)} \Rightarrow \frac{36}{2} = \boxed{18 = \bar{u}\cdot\bar{v}}$$

FACT $\bar{u} \perp \bar{v}$ means $\bar{u}$ is orthogonal to $\bar{v}$

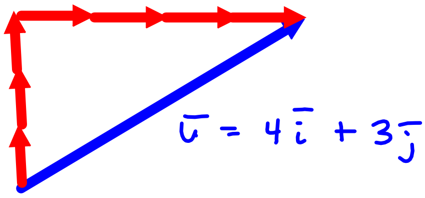
if and only if $\bar{u}\cdot\bar{v} = 0$ (b/c $\cos 90^\circ = 0$)

$$0 = \cos\theta = \frac{\bar{u}\cdot\bar{v}}{\sim\!\sim} \Rightarrow \qquad \frac{A}{B} = 0 \Rightarrow A = 0$$



$$0 = \bar{u}\cdot\bar{v}$$

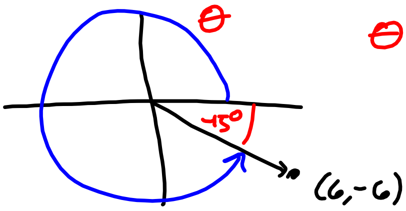
$\bar{i}$ & $\bar{j}$ span the plane.




**34.** 0/2 points

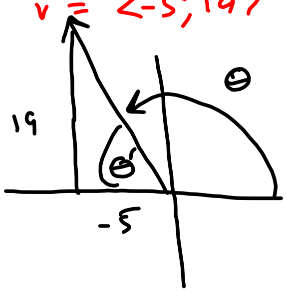
Find the magnitude and direction angle of the vector **v**.

$\mathbf{v} = 6\mathbf{i} - 6\mathbf{j} = \langle 6, -6 \rangle$




$\theta = 315^\circ = \frac{7\pi}{4}$

$\vec{v} = \langle -5, 19 \rangle$




$\tan\theta = \frac{19}{-5} \Longrightarrow$

$\arctan\left(-\frac{19}{5}\right) \approx -1.31 \approx -75.26^\circ$


So $\theta = 180^\circ - 75...^\circ$

$\approx 1.83$ (radians)

$\pi + \arctan\left(-\frac{19}{5}\right)$

$= \pi - \arctan\left(\frac{19}{5}\right)$

$\theta' = 75.26^\circ$

measured from negative $x$-axis = reference angle.

25. 0/3 points

Use vectors to find the interior angles of the triangle with the given vertices. (F

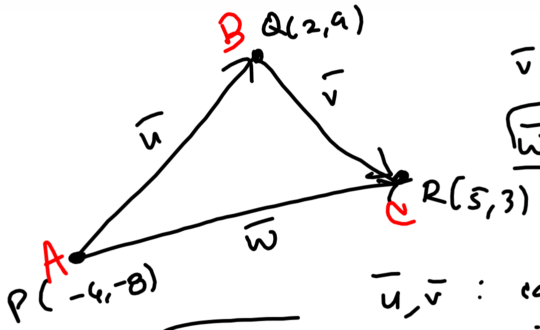
(−6, −8), (2, 9), (5, 3)

| | | | |
|---|---|---|---|
| | 19.80 | ° | (smallest value) |
| | 51.77 | ° | |
| | 108.43 | ° | (largest value) |

$\bar{u} = \langle 2-(-6), 9-(-8) \rangle = \boxed{\langle 8, 17 \rangle = \bar{u}}$

$\bar{v} = \langle 5-2, 3-9 \rangle = \boxed{\langle 3, -6 \rangle = \bar{v}}$

$\boxed{\bar{w} = \langle 11, 11 \rangle}$



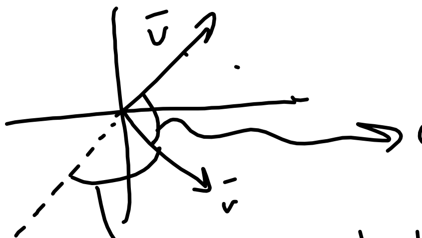

$\bar{u}, \bar{v}$ : $\cos A = \dfrac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\,\|\bar{v}\|} = \dfrac{(8)(3)+(17)(-6)}{\sqrt{353}\sqrt{47}} = \dfrac{24-102}{\sqrt{353}\sqrt{47}} = \dfrac{-78}{\sqrt{353}\sqrt{47}} \Rightarrow A \approx 19.80°$

$\|\bar{u}\| = \sqrt{8^2+17^2} = \sqrt{64+289}$

$= \sqrt{353} = \|\bar{u}\|$

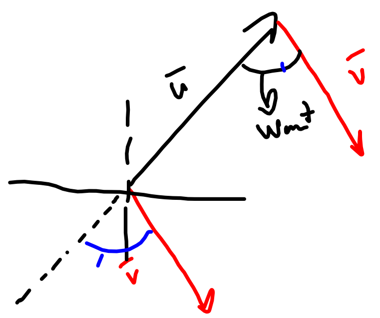
$\|\bar{v}\| = \sqrt{9+36} = \sqrt{47} = \|\bar{v}\|$ 45, idiot

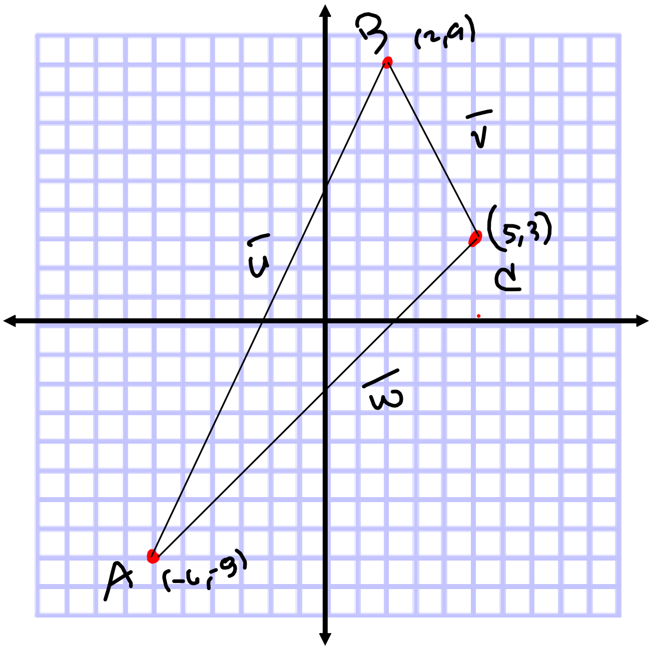
$\|\bar{w}\| = \sqrt{11^2+11^2} = \sqrt{2(11)^2} = 11\sqrt{2} = \|\bar{w}\|$




$\cos\theta = \dfrac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\,\|\bar{v}\|}$

want this ∢ it's the supplement of the angle between $\bar{u}$ & $\bar{v}$.






$\bar{u} = \langle 8, 17 \rangle$

$\|\bar{u}\| = \sqrt{353}$

$\bar{v} = \langle 3, -6 \rangle$

$\|\bar{v}\| = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$

$\begin{array}{r} 170 \\ 17 \\ \hline 187 \end{array}$

$\bar{w} = \langle 11, 11 \rangle$

$\|\bar{w}\| = 11\sqrt{2}$

$$\cos A = \frac{\bar{u} \cdot \bar{w}}{\|\bar{u}\|\|\bar{w}\|} = \frac{88 + 187}{\sqrt{353}\sqrt{242}}$$

$$= \frac{275}{\sqrt{353}\sqrt{242}} =$$