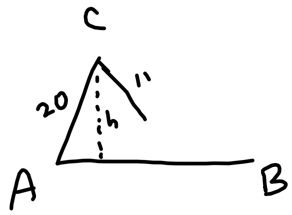
Boyd's Question $A = 76^0$, $a=11$, $b=20$



$\frac{h}{20} = \sin 76^0 \Rightarrow$

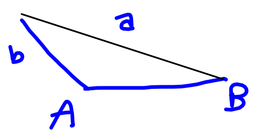
$h = 20 \sin 76^0 \approx 19.4 > 11$, so a is too short to reach.

Last time: ASS

$A = \ldots$, $a = \ldots$, $b = \ldots$

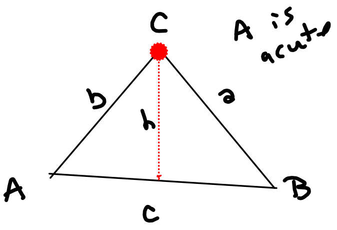
If A is obtuse?

AT MOST ONE SOLUTION, and only if $a > b$

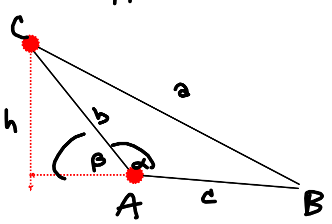


LAW OF SINES

A is acute



A is obtuse



$\boxed{\sin A = \frac{h}{b}}$

$\sin B = \frac{h}{a}$

$\Rightarrow h = b \sin A = a \sin B$

$\Rightarrow \frac{\sin A}{a} = \frac{\sin B}{b}$

obtuse Case

$\alpha = A$

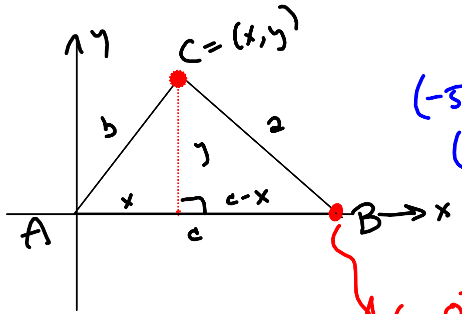
$\alpha + \beta = \pi = 180^0$, so

$\beta = \pi - \alpha = \pi - A$, ∉

$\sin \beta = \sin(\pi - A)$

$= \sin \pi \cos(-A) + \sin(-A) \cos \pi$

$= 0 \quad (-\sin A)(-1) = \sin A$ ∎



LEMMA

$(-5)^2 = ((-1)(5))^2 = (-1)^2 (5^2) = 25$

$(c-x)^2 = (-1(x-c))^2 = \dots = (x-c)^2$

Law of Cosines:

$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = b^2 + c^2$ by Pythagorus,

but wait!

$* \quad \frac{x}{b} = \cos A \Rightarrow x = b\cos A$

$\frac{y}{b} = \sin A \Rightarrow y = b\sin A$

$(u-v)^2 = u^2 - 2uv + v^2$

$a^2 = (c-x)^2 + y^2$

$= (x-c)^2 + y^2 \quad *$

$= x^2 - 2cx + c^2 + y^2$

$= (b\cos A)^2 - 2c\, b\cos A + c^2 + (b\sin A)^2$

$= b^2\cos^2 A - 2cb\cos A + c^2 + b^2\sin^2 A$

$= a^2 = b^2\cos^2 A + b^2\sin^2 A - 2cb\cos A + c^2 + \cancel{b^2}$

$= b^2(\cos^2 A + \sin^2 A) - 2bc\cos A + c^2 + \cancel{b^2}$

$= b^2 - 2bc\cos A + c^2$

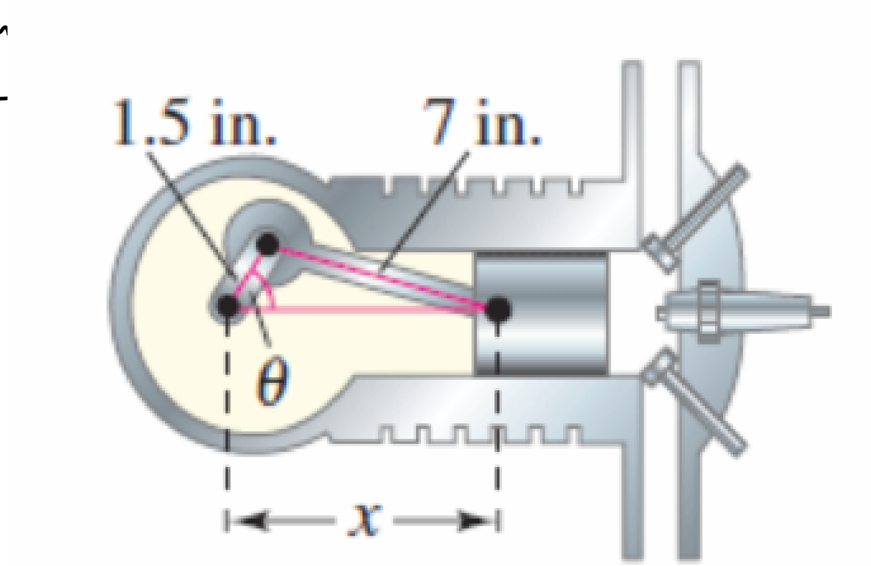
i.e., $a^2 = b^2 + c^2 - 2bc\cos A$

Alternate Form: Solve for $\cos A$

$a^2 - b^2 - c^2 = -2bc\cos A$

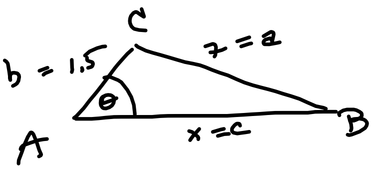
$\frac{a^2 - b^2 - c^2}{-2bc} = \boxed{\frac{b^2 + c^2 - a^2}{2bc} = \cos A}$

3.2 Piston



(a) Use the Law of Cosines to write an equation giving the relationship between x and θ.

✗ $x^2 - 3x\cos(\theta) - 46.75 = 0$



$x^2 = 1.5^2 + 7^2 - 2(1.5)(7)\cos\theta$ No.

$a^2 = b^2 + c^2 - 2bc\cos A$

$7^2 = 1.5^2 + x^2 - 2(1.5)x\cos\theta$

$49 = 2.25 + x^2 - 3x\cos\theta$

a $\boxed{x^2 - (3\cos\theta)x - 46.75 = 0}$

(b) Write x as a function of θ. (Select the sign that yields positive values of x.)

$ax^2+bx+c=0 \Rightarrow x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$a=1$

$b=-3\cos\theta$

$c=-46.75$

$\Rightarrow b^2-4ac = 9\cos^2\theta - 4(1)(-46.75) = 9\cos^2+187$

cosine w/o an argument is short for "costume."

$= 9\cos^2\theta + 187$

$$\Rightarrow x = \frac{3\cos\theta \pm\sqrt{9\cos^2\theta+187}}{2}$$

We want to keep it positive

$$x=\frac{3\cos\theta+\sqrt{9\cos^2\theta+187}}{2}$$

is biggest when $\cos\theta$ is biggest. That happens @ $0 \,\&\, 2\pi$, so @ $\theta=0$

$$\frac{3+\sqrt{9+187}}{2}=\frac{3+\sqrt{196}}{2}=\frac{3+14}{2}=\frac{17}{2}=8.5$$

Book says 4.25? why'm I off by a factor of 2?

No it doesn't!

Online Grapher

https://www.geogebra.org/graphing?lang=en

More precise than Desmos

MISC:

TRIANGLE AREA

$\boxed{\text{Area} = \frac{1}{2} bc \sin A}$ Add to Cheat Sheet.

HERON'S FORMULA

Can be handy. Don't memorize. Cheat sheet is fine.

Heron's Area Formula ***(p. 274)***

Given any triangle with sides of lengths a, b, and c, the area of the triangle is

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } s = \frac{a+b+c}{2}.$$

For next time, make sure you take a look at the 3.3.