Last Identity from Section 2.4:

**Double-Angle Formulas** ***(p. 243)***

$$\sin 2u = 2 \sin u \cos u$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

$$\cos 2u = \cos^2 u - \sin^2 u$$
$$= 2\cos^2 u - 1$$
$$= 1 - 2\sin^2 u$$

These are proved by the angle sum formulas, covered previously.

**Sum and Difference Formulas**

$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} \qquad \tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

Here's a Cheat Sheet with all that stuff and more, EXCEPT for the tangent formula, above, because I wanted students to bootstrap from cosine and sine. But you may certainly add that one to your cheat sheet, because it can be faster.....

https://harryzaims.com/122/122-fall-22/cheat-sheet-test-2.pdf

$$\sqrt{\frac{1-\cos(8x)}{2}} = |\sin(4x)| \ ?!$$

$\cos(u+v) = \cos(u)\cos(v) - \sin(u)\sin(v)$

$\cos(2u) = \cos(u+u) = \cos(u)\cos(u) - \sin(u)\sin(u)$

$= \cos^2(u) - \sin^2(u)$

$= 1-\sin^2(u) - \sin^2(u) = \boxed{1-2\sin^2(u)}$

OR

$= \cos^2(u) - (1-\cos^2(u)) = \boxed{2\cos^2(u) - 1}$

$\cos(2u) = 2\cos^2(u) - 1$

$\Rightarrow 2\cos^2(u) - 1 = \cos(2u)$

$2\cos^2(u) = \cos(2u) + 1$

$\cos^2(u) = \frac{\cos(2u)+1}{2}$

$\sqrt{\cos^2(u)} = \sqrt{\frac{\cos(2u)+1}{2}}$

$|\cos(u)| = \sqrt{\frac{1+\cos(2u)}{2}}$

Let $v = 2u$. Then

$|\cos(\frac{v}{2})| = \sqrt{\frac{1+\cos(v)}{2}}$

$\cos(\frac{v}{2}) = \pm\sqrt{\frac{1+\cos(v)}{2}}$

$\cos(2u) = 1-2\sin^2(u)$

$1-2\sin^2(u) = \cos(2u)$

$-2\sin^2(u) = \cos(2u) - 1$

$2\sin^2(u) = 1-\cos(2u)$

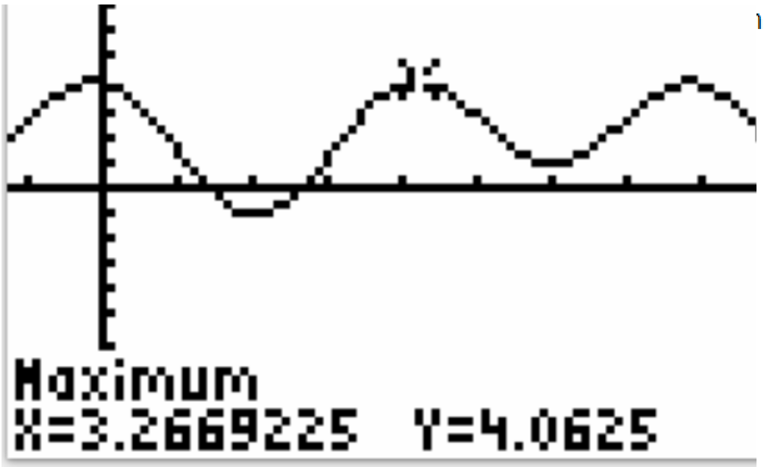
$\sqrt{\sin^2(u)} = \sqrt{\frac{1-\cos(2u)}{2}}$

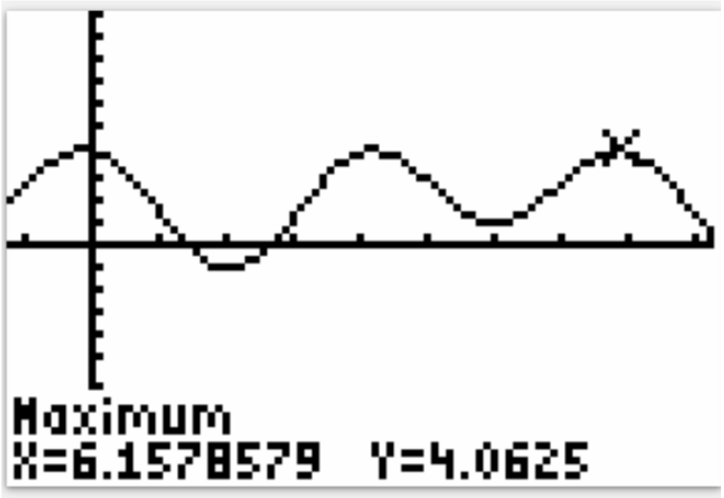
$|\sin(u)| = \sqrt{\frac{1-\cos(2u)}{2}}$

So $\sqrt{\frac{1-\cos(8x)}{2}} = |\sin(4x)|$

Consider the following.

| **Function** | **Trigonometric Equation** |
|---|---|
| $f(x) = 4\cos^2(x) - \sin(x)$ | $-8\sin(x)\cos(x) - \cos(x) = 0$ |




$-8\sin(x)\cos(x) - \cos(x) = 0$

$-\cos(x)\,(8\sin(x) + 1) = 0 \rightarrow$

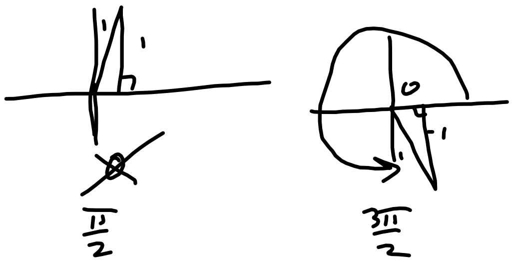
$-\cos(x) = 0$ or $8\sin(x) + 1 = 0$

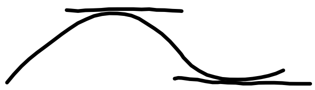
$\cos(x) = 0$

$f(x) = 4\cos^2(x) - \sin(x)$

$\Longrightarrow f'(x) = 8\cos(x)(-\sin(x)) - \cos(x)$

= slope function in calculus. we find where the slope is zero.



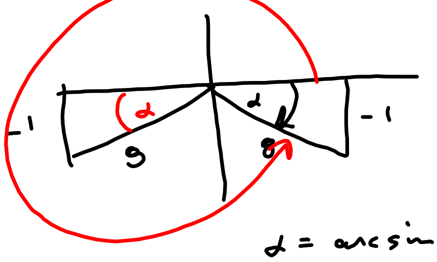



$8\sin(x) + 1 = 0$

$8\sin(x) = -1$

$\sin(x) = -\frac{1}{8}$

$2\pi + \arcsin(-\frac{1}{8})$

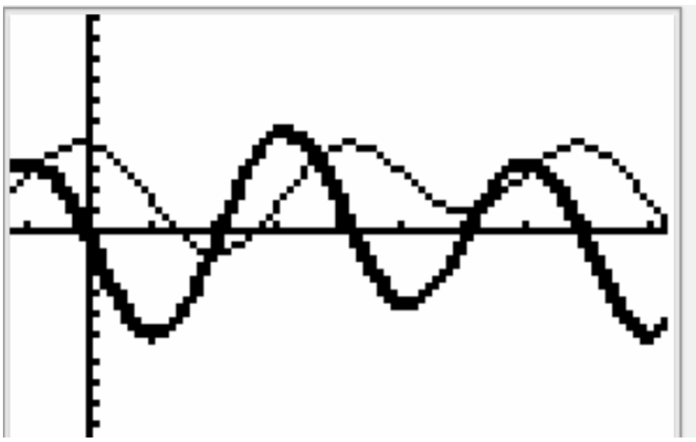



$\sin^{-1}(-\frac{1}{8})$ is a negative angle, due to the restriction on sine that we make to keep sine 1-to-1 & hence make inverse sine a function.

$\alpha = \arcsin(-\frac{1}{8})$ & it's negative.

we want a coterminal angle that's positive & between $0$ & $2\pi$

$\pi - \arcsin(-\frac{1}{8})$ is the one in $Q\overline{IV}$



$x = \pi\left(n - \frac{1}{2}\right)$ and $n \in \mathbb{Z}$

$x = 2\pi n - \sin^{-1}\left(\frac{1}{8}\right)$ and $n \in \mathbb{Z}$

$x = 2\pi n + \pi + \sin^{-1}\left(\frac{1}{8}\right)$ and $n \in \mathbb{Z}$

$\pi(n - \frac{1}{2})$

$n = 1, 2$

$\frac{\pi}{2}, \frac{3\pi}{2}$

$2\pi n - \sin^{-1}(\frac{1}{8})$

$n = 1$

$2\pi - \arcsin(\frac{1}{8})$ (or $2\pi + \arcsin(-\frac{1}{8})$) My soln

$2\pi(2) - \arcsin(\frac{1}{8})$

$4\pi - \arcsin(\frac{1}{8})$ Meh

$2\pi n + \pi + \arcsin(\frac{1}{8})$

$n=0$: $\pi + \arcsin(\frac{1}{8}) = \underline{\pi - \arcsin(-\frac{1}{8})}$ My soln

**Product-to-Sum Formulas**

$$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$$

$$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$

**Sum-to-Product Formulas**

$$\sin u + \sin v = 2 \sin\left(\frac{u + v}{2}\right) \cos\left(\frac{u - v}{2}\right)$$

$$\cos u + \cos v = 2 \cos\left(\frac{u + v}{2}\right) \cos\left(\frac{u - v}{2}\right)$$

$$\cos u - \cos v = -2 \sin\left(\frac{u + v}{2}\right) \sin\left(\frac{u - v}{2}\right)$$

Big for Calculus II

$$\int \boxed{\sin(5x)\cos(2x)}\,dx \quad \text{is HARD}$$

Trig Drill.

But $u = 5x$, $v = 2x$, then we have

$$\int \frac{1}{2}\left[\sin(5x+2x) + \sin(5x-2x)\right]dx$$

$$= \frac{1}{2}\left[\int \boxed{\left(\sin(7x) + \sin(3x)\right)}\,dx\right]$$

$$= \frac{1}{2}\left[\int \sin(7x)\,dx + \int \sin(3x)\,dx\right] \quad \text{is } \underline{\text{EASY}}.$$

§2.5 questions

$\cancel{8}\sin(2x)\sin(x) = \cancel{8}\cos(x)$

$2\sin(x)\cos(x)\sin(x) - \cos(x) = 0$

$\cos(x)\left[2\sin^2(x) - 1\right] = 0$

$\cos(x) = 0$

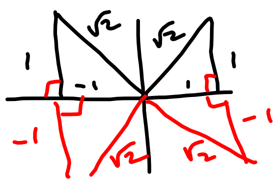
$\frac{\pi}{2}, \frac{3\pi}{2}$

$2\sin^2(x) - 1 = 0$

$2\sin^2(x) = 1$

$\sin^2(x) = \frac{1}{2}$

$\sin(x) = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}}$




$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

0/1 points LarTrig10 2.5.008. [3882736]

Solve the equation. (Find all solutions of the equation in the interval $[0, 2\pi)$. Enter your answers as a comma-separated list.)

$8\sin(2x)\sin(x) = 8\cos(x)$

$x =$ $\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$

**8.** 0/3 points LarTrig10 2.5.024. [38

Use the given conditions to find the exact values of $\sin(2u)$, $\cos(2u)$, and $\tan(2u)$ using the double-angle formulas.

$\sec(u) = -2$, $\pi/2 < u < \pi$

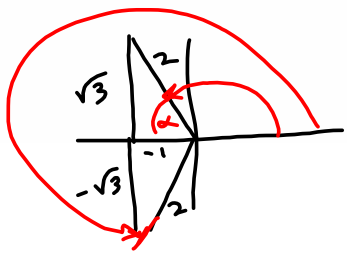
$\sin(2u) =$ ✗ $-\frac{\sqrt{3}}{2}$

$\cos(2u) =$ ✗ $-\frac{1}{2}$

$\tan(2u) =$ ✗ $\sqrt{3}$

$\sec(u) = -2$

$\cos(u) = -\frac{1}{2}$

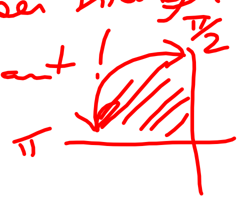


$\alpha = 60^\circ = \frac{\pi}{3}$

So $u = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

OR $u = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

Says the upper triangle is the ~~α~~ u we want!



So $u = \frac{3\pi}{4}$

$\sin(u) = \frac{\sqrt{3}}{2}$

$\cos(u) = -\frac{1}{2}$

$\tan(u) = -\sqrt{3}$

$\sin(2u) = 2\sin(u)\cos(u) = 2\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right) = \boxed{-\frac{\sqrt{3}}{2} = \sin(2u)}$

$\cos(2u) = \cos^2(u) - \sin^2(u) = \left(-\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} - \frac{3}{4} = -\frac{2}{4} = \boxed{-\frac{1}{2} = \cos(2u)}$

$\tan(2u) = \frac{\sin(2u)}{\cos(2u)} = -\frac{\sqrt{3}}{2} \cdot \left(-\frac{2}{1}\right) = \boxed{\sqrt{3} = \tan(2u)}$

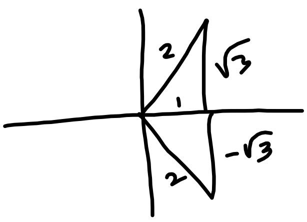
HALF-ANGLES ARE TRICKY. You need to know what quadrant u is in, to know whether it's "+" or "−."

$\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos(u)}{2}}$

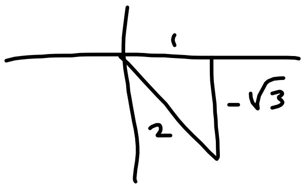
which one depends on the quadrant u is in.

Given $\cos(u) = \frac{1}{2}$ and $\sin(u) < 0$, what's $\sin(2u)$

$\cos(u) = \frac{1}{2}$




we want the one where $\sin(u) < 0$. That's THIS one:




$\frac{3\pi}{2} < u < 2\pi \longrightarrow$

$\frac{3\pi}{4} < \frac{u}{2} < \pi$, so

$\frac{u}{2} \in Q\,II$

cosine $< 0$

sine $> 0$

i.e., $\sin\left(\frac{u}{2}\right) = +\sqrt{\frac{1-\cos(u)}{2}}$

$\cos\left(\frac{u}{2}\right) = -\sqrt{\frac{1+\cos(u)}{2}}$