Sum and Difference Formulas

4 $\sin(u + v) = \sin u \cos v + \cos u \sin v$

3 $\sin(u - v) = \sin u \cos v - \cos u \sin v$

2 $\cos(u + v) = \cos u \cos v - \sin u \sin v$

1 $\boxed{\cos(u - v) = \cos u \cos v + \sin u \sin v}$

5 $\tan(u + v) = \dfrac{\tan u + \tan v}{1 - \tan u \tan v}$ $\qquad$ $\tan(u - v) = \dfrac{\tan u - \tan v}{1 + \tan u \tan v}$

$\sin(u+v) = \sin u \cos v + \sin v \cos u$

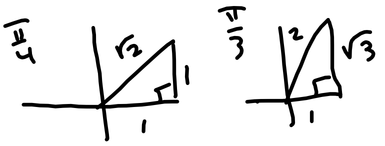
$\frac{\pi}{4} + \frac{\pi}{3} = \frac{3\pi + 4\pi}{12} = \frac{7\pi}{12}$

Use a sum identity to find the exact value of

$\sin\left(\frac{7\pi}{12}\right)$

$7 = 1 + 6$
$= 2 + 5 \qquad \frac{2\pi}{12} + \frac{5\pi}{12}$
$= 3 + 4 \qquad \frac{3\pi}{12} + \frac{4\pi}{12} = \frac{\pi}{4} + \frac{\pi}{3}$!

$\Rightarrow \sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) + \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right)$



$= \frac{1}{\sqrt{2}} \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}$

$= \frac{1+\sqrt{3}}{2\sqrt{2}}$ = (Rationalized)

$= \frac{\sqrt{2}}{\sqrt{2}}\left(\frac{1+\sqrt{3}}{2\sqrt{2}}\right) = \frac{\sqrt{2} + \sqrt{2}\sqrt{3}}{2 \cdot 2} = \frac{\sqrt{2} + \sqrt{2 \cdot 3}}{4}$

$= \boxed{\frac{\sqrt{2} + \sqrt{6}}{4}}$

use $\cos(u-v)$ ident. to prove $\sin(u-v)$

Recall: $\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$ cofunction identity

Then $\sin(u-v) = \cos\left(\frac{\pi}{2}-(u-v)\right)$

$= \cos\frac{\pi}{2}\cos(u-v) + \sin\left(\frac{\pi}{2}\right)\sin(u-v)$

$= 0\cdot\cos(u-v) + \sin(u-v) = \sin(u-v)$. No help. See last page!

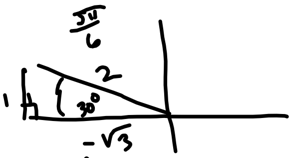
$\sin\left(\frac{13\pi}{12}\right)$ $u=?,\ v=?$

$u=\frac{10\pi}{12}=\frac{5\pi}{6}$,

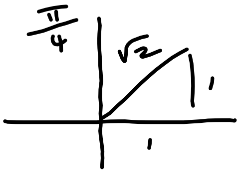
$v=\frac{3\pi}{12}=\frac{\pi}{4}$

$\sin\left(\frac{13\pi}{12}\right)=\sin\left(\frac{5\pi}{6}+\frac{\pi}{4}\right)=\sin\left(\frac{5\pi}{6}\right)\cos\left(\frac{\pi}{4}\right)+\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{5\pi}{6}\right)$

$\frac{5\pi}{6}$



$\frac{\pi}{4}$

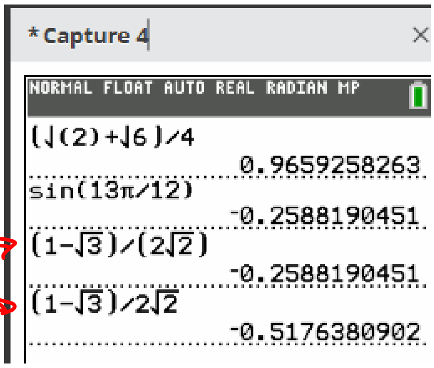


$= \frac{1}{2}\cdot\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\cdot\left(-\frac{\sqrt{3}}{2}\right)$

$= \boxed{\frac{1-\sqrt{3}}{2\sqrt{2}} = \sin\left(\frac{13\pi}{12}\right)}$

$\cos\left(\frac{13\pi}{12}\right)=\cos\left(\frac{5\pi}{6}+\frac{\pi}{4}\right)=\cos\left(\frac{5\pi}{6}\right)\cos\left(\frac{\pi}{4}\right)-\sin\left(\frac{5\pi}{6}\right)\sin\left(\frac{\pi}{4}\right)$

$= -\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}-\frac{1}{2}\cdot\frac{1}{\sqrt{2}} = \boxed{\frac{-\sqrt{3}-1}{2\sqrt{2}} = \cos\left(\frac{17\pi}{12}\right)}$ OR $-\frac{\sqrt{3}+1}{2\sqrt{2}}$ OR $\frac{-(\sqrt{3}+1)}{2\sqrt{2}}$



See?

Parens make a difference!

Not a big fan of the tangent formula

One method:

use $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\Rightarrow \tan\left(\frac{13\pi}{12}\right)=\frac{\sin\left(\frac{13\pi}{12}\right)}{\cos\left(\frac{13\pi}{12}\right)}$

$$= \frac{\frac{1-\sqrt{3}}{2\sqrt{2}}}{-\left(\frac{1+\sqrt{3}}{2\sqrt{2}}\right)} = -\frac{1-\sqrt{3}}{2\sqrt{2}} \cdot \frac{2\sqrt{2}}{1+\sqrt{3}} = \frac{1-\sqrt{3}}{1+\sqrt{3}} = -\left(\frac{1-\sqrt{3}}{1+\sqrt{3}}\right)\left(\frac{1-\sqrt{3}}{1-\sqrt{3}}\right)$$

$$= -\frac{(1-\sqrt{3})^2}{1^2-\sqrt{3}^2} = -\left(\frac{1-2\sqrt{3}+3}{-2}\right) = \frac{4-2\sqrt{3}}{2} = \frac{2-\sqrt{3}}{1} = \boxed{2-\sqrt{3}}$$

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(1-√3)/(2√2)
                -0.2588190451
(1-√3)/2√2
                -0.5176380902
tan(13π/12)
                 0.2679491924
2-√3
                 0.2679491924
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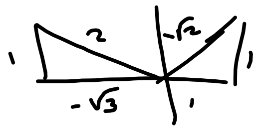
$$\tan\left(\frac{13\pi}{12}\right) = \tan\left(\frac{5\pi}{6} + \frac{\pi}{4}\right)$$

$$= \frac{\tan\left(\frac{5\pi}{6}\right) + \tan\left(\frac{\pi}{4}\right)}{1 - \tan\left(\frac{5\pi}{6}\right)\tan\left(\frac{\pi}{4}\right)}$$

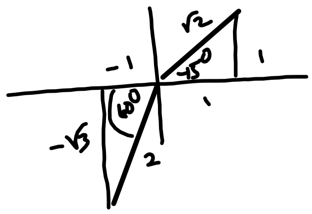


$$= \frac{-\frac{1}{\sqrt{3}} + 1}{1 - \left(\frac{1}{-\sqrt{3}}\right)(1)} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}} = \left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right) \cdot \left(\frac{\sqrt{3}-1}{\sqrt{3}-1}\right)$$

$$= \frac{4-2\sqrt{3}}{2} = \frac{2-\sqrt{3}}{1} = \boxed{2-\sqrt{3}}$$

What about 285° ?

$285 = 240 + 45$



etc.

$$(285)\left(\frac{\pi}{180}\right) = \frac{19\pi}{12}$$

$$= \frac{(16+3)\pi}{12} = \frac{16\pi}{12} + \frac{3\pi}{12} = \frac{4\pi}{3} + \frac{\pi}{4}$$

37. 0/1 points LarTrig10 2.3.071. [3882776

Use the Quadratic Formula to find all solutions of the equation in the interval $[0, 2\pi)$. (Enter your answers as a comma-separated list. Round each answer to four decimal places.)

$$15\sin^2(x) - 17\sin(x) + 4 = 0$$

$x =$ ✗ 0.3398, 0.9273, 2.2143, 2.8018

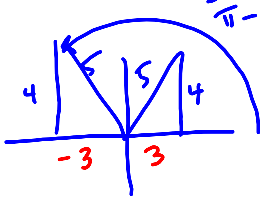
LET $u = \sin(x)$

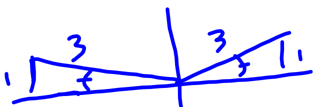
$\Rightarrow 15u^2 - 17u + 4 = 0$

$b^2 - 4ac = (17)^2 - 4(15)(4) = 289 - 240 = 49$

$$\Rightarrow u = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{17 \pm \sqrt{49}}{2(15)} = \begin{cases} \frac{24}{30} = \frac{4}{5} = \sin(x) \\ \frac{10}{30} = \frac{1}{3} = \sin(x) \end{cases}$$

$\sin(x) = \frac{4}{5}$ $\quad \pi - \arcsin\left(\frac{4}{5}\right)$ $\quad \sin(x) = \frac{1}{3}$





$\sin^{-1}\left(\frac{1}{3}\right), \ \pi - \sin^{-1}\left(\frac{1}{3}\right)$

$\sin^{-1}\left(\frac{4}{5}\right), \ \pi - \sin^{-1}\left(\frac{4}{5}\right)$

$\sin(u-v) = \cos\left(\frac{\pi}{2}-(u-v)\right)$

$= \cos\left(\frac{\pi}{2}-u+v\right) = \cos\left(\left(\frac{\pi}{2}-u\right)+v\right)$

$= \cos\left(\frac{\pi}{2}-u\right)\cos(v) - \sin\left(\left(\frac{\pi}{2}-u\right)\right)\sin(v)$

$= \sin(u)\cos(v) - \cos(u)\sin(v)$ ▨

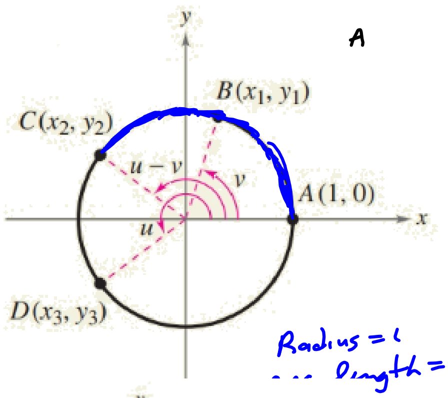
To prove $\cos(u+v) = \cos(u)\cos(v) - \sin(u)\sin(v)$, use this $\cos(u+v) = \cos(u-(-v))$ & Apply $\cos(u-v) = \cos(u)\cos(v)+\sin(u)\sin(v)$

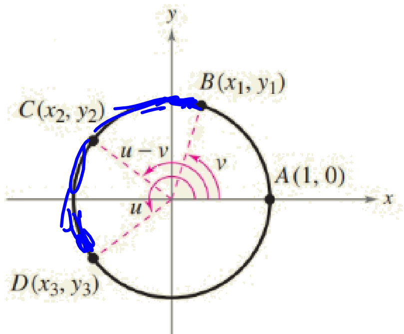
So $\cos(u-(-v)) = \cos(u)\cos(-v)+\sin(u)\sin(-v)$

$= \cos(u)\cos(v) - \sin(u)\sin(v)$, b/c cosine's even & sine's odd.

Proof: $\cos(u-v) = \cos(u)\cos(v)+\sin(u)\sin(v)$

A

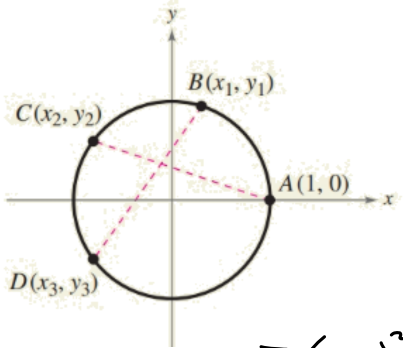




Radius = 1

... length = angle in radians,

Arc AC = Arc BD = $u-v$



$\therefore |\overline{AC}| = |\overline{BD}|$

= length of line segment

$\Rightarrow \sqrt{(x_2-1)^2+(y_2-0)^2} = \sqrt{(x_3-x_1)^2+(y_3-y_1)^2}$

$\Rightarrow (x_2-1)^2+y_2^2 = (x_3-x_1)^2+(y_3-y_1)^2$

$\Rightarrow x_2^2-2x_2+1+y_2^2 = x_3^2-2x_1x_3+x_1^2+y_3^2-2y_1y_3+y_1^2$

$\underbrace{x_2^2+y_2^2}_{1}-2x_2+1 = \underbrace{x_3^2+y_3^2}_{1}+\underbrace{x_1^2+y_1^2}_{1}-2x_1x_3-2y_1y_3$

$-2x_2+1 = 1 + 1 - 2x_3x_1-2y_1y_3$

$-2x_2 = -2x_1x_3-2y_1y_3$

$x_2 = x_1x_3+y_1y_3$

$\cos(u-v) = \cos(u)\cos(v)+\sin(u)\sin(v)$

$x=\cos\theta$
$y=\sin\theta$

The θ's we have are u, v, & $u-v$