Grade Reports this afternoon.

Probably won't do e-mail settings. To get checked off, reply to one of my e-mails and I'll see if your settings are correct. We'll check that off with the next report.

This week: 2.2 - 2.4

Next week: 2.4 - 2.5

Then Test 2.

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$\cot^2\theta + 1 = \csc^2\theta$$

See a $1 - \sim$?

Think $(1 - \sim)(1 + \sim)$

$= 1 - \sim^2$

CLAIM

$$\frac{\cos\theta\cot\theta}{1-\sin\theta} - 1 = \csc\theta$$

PROOF

$$\frac{\cos\theta\cot\theta}{1-\sin\theta} - \left(\frac{1}{1}\cdot\frac{1-\sin\theta}{1-\sin\theta}\right) = \frac{\cos\theta\cot\theta - 1 + \sin\theta}{1-\sin\theta}$$

$$= \frac{\frac{\cos\theta\cos\theta}{\sin\theta} - 1 + \sin\theta}{1-\sin\theta} = \frac{\frac{\cos^2\theta}{\sin\theta} - \frac{\sin\theta}{\sin\theta} + \frac{\sin^2\theta}{\sin\theta}}{1-\sin\theta}$$

$$= \frac{\frac{\cos^2\theta + \sin^2\theta - \sin\theta}{\sin\theta}}{1-\sin\theta} = \frac{\frac{1-\sin\theta}{\sin\theta}}{\frac{1-\sin\theta}{1}} = \frac{1-\sin\theta}{\sin\theta}\cdot\frac{1}{1-\sin\theta} = \frac{1}{\sin\theta}$$

$$= \csc\theta \quad \blacksquare$$

Claim:

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \frac{1+\sin\theta}{|\cos\theta|}$$

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} =$$

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}\cdot\frac{1+\sin\theta}{1+\sin\theta}} = \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}} = \frac{\sqrt{(1+\sin\theta)^2}}{\sqrt{\cos^2\theta}}$$

$$= \frac{|1+\sin\theta|}{|\cos\theta|} = \boxed{\frac{1+\sin\theta}{|\cos\theta|}}$$ why?

■

$\sqrt{3^2} = 3$

$\sqrt{(-3)^2} = \sqrt{9} = 3$

$\left.\begin{matrix} 3 \mapsto 3 \\ -3 \mapsto 3 \end{matrix}\right\}$ That's what absolute value does.

$\sqrt{x^2} = |x|$

$\sqrt{3^2} = 3$

$\sqrt{(-3)^2} = 3$

Solve $|x| = A \Rightarrow$

$x = \pm A$

Positive -vs- nonnegative

>0 ≥ 0

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Now, $\sin\theta \leq 1 \Rightarrow 0 \leq 1-\sin\theta$

Not what I want.

$-1 \leq \sin\theta \Rightarrow 0 \leq 1+\sin\theta$, so $|1+\sin\theta| = 1+\sin\theta$

$$|1+\sin\theta| = \begin{cases} 1+\sin\theta & \text{if } 1+\sin\theta \geq 0 \\ -(1+\sin\theta) & \text{if } 1+\sin\theta < 0 \end{cases} = 1+\sin\theta\,!$$

Never happen.

Claim:

51 $\tan^5(x) = \tan^3(x)\sec^2(x) - \tan^3(x)$

Proof

$\tan^5(x) = \tan^3(x)\tan^2(x) = \tan^3(x)\left[\sec^2(x) - 1\right]$

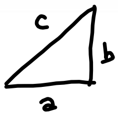
$= \tan^3(x)\sec^2(x) - \tan^3(x)$ ∎

Trigonometric Substitution headed your way in Calc II.

57. $\tan(\sin^{-1} x) = \dfrac{x}{\sqrt{1-x^2}}$ **58.** $\cos(\sin^{-1} x) = \sqrt{1-x^2}$

59. $\tan\left(\sin^{-1}\dfrac{x-1}{4}\right) = \dfrac{x-1}{\sqrt{16-(x-1)^2}}$

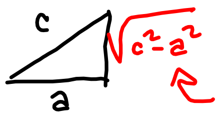
$a^2+b^2=c^2$



$a^2+b^2=c^2 \longrightarrow$

$c^2 = a^2+b^2$

$c = \pm\sqrt{a^2+b^2}$



fill it in

$a^2+b^2=c^2$

$a^2 = c^2-b^2$

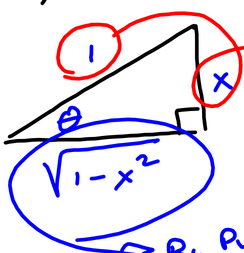
$a = \pm\sqrt{c^2-b^2}$

$\tan(\sin^{-1}(x)) = \tan\theta$

Angle, θ, whose sine is x. Assume QI $\longrightarrow$

$c = \sqrt{a^2+b^2}$ ← Hypotenuse!

$a = \sqrt{c^2-b^2}$ ← One of the legs!

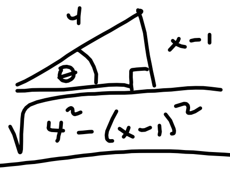


From $\theta = \sin^{-1}(x)$

By Pythagorus.

$\therefore \tan\theta = \dfrac{x}{\sqrt{1-x^2}} = \tan(\sin^{-1}(x))$

$\tan\left(\sin^{-1}\left(\frac{x-1}{4}\right)\right) = \tan\theta$



$\longrightarrow \tan\theta = \dfrac{x-1}{\sqrt{16-(x-1)^2}}$

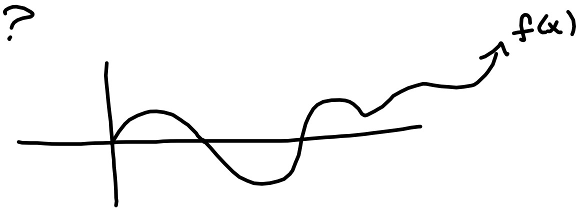
If $\tan^2\theta = \sec^2\theta - 1$, why isn't $\tan\theta = \sqrt{\sec^2\theta - 1}$?

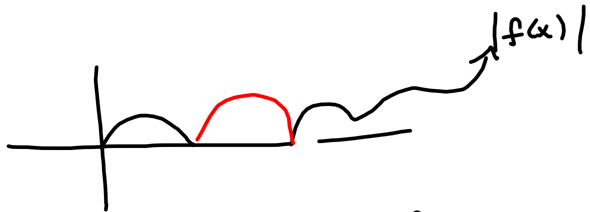
$\sqrt{\tan^2\theta} = \sqrt{\sec^2\theta - 1}$

$|\tan\theta| = \sqrt{\sec^2\theta - 1}$

$\tan\theta = \pm\sqrt{\sec^2\theta - 1}$

Missing!





$|f(x)| = \begin{cases} f(x) & \text{if } f(x) \geq 0 \\ -f(x) & \text{if } f(x) < 0 \end{cases}$

2.3

$2\sin^2 x + 5\cos x = 4$

$2(1-\cos^2 x) + 5\cos x = 2 - 2\cos^2 x + 5\cos x = 4$

$-2\cos^2 x + 5\cos x = 2$

$-2\cos^2 x + 5\cos x - 2 = 0$

$2\cos^2 x - 5\cos x + 2 = 0$

Let $u = \cos x \Longrightarrow$

$2u^2 - 5u + 2 = 0$

$2u^2 - 4u - 1u + 2 = 0$

$2u(u-2) - 1(u-2) = 0$

$(u-2)(2u-1) = 0 \Longrightarrow$

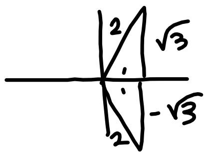
$u = \cos x = 2$ or $2u - 1 = 0 = 2\cos x - 1$

Nevah!

$2\cos x = 1$

$\cos x = \frac{1}{2} \Longrightarrow x = 60^\circ, 300^\circ$

$= \boxed{\frac{\pi}{3}, \frac{5\pi}{3}} = x$



Solution-set presentation:

$x \in \left\{ \frac{\pi}{3}, \frac{5\pi}{3} \right\}$

We found all sol'ns in $[0; 2\pi)$, but not all sol'ns!

$\mathbb{Z}$ = set of integers $= \{ \dots, -3, -2, -1, 0, 1, 2, 3, \dots \}$

$x = \frac{\pi}{3} + 2n\pi$, $n \in \mathbb{Z}$

or $x = \frac{5\pi}{3} + 2n\pi$, $n \in \mathbb{Z}$.

WebAssign's looking for this.

$x \in \left\{ \theta + 2n\pi \;\middle|\; n \in \mathbb{Z} \;\&\; \theta = \frac{\pi}{3} \text{ or } \frac{5\pi}{3} \right\}$

$= \left\{ \theta + 2n\pi \;\middle|\; n \in \mathbb{Z}, \theta \in \left\{ \frac{\pi}{3}, \frac{5\pi}{3} \right\} \right\}$

27. 0/1 points

Solve the multiple-angle equation. (Enter your answers as a comma-separated list. Use n as an inte
response in radians.)

$$2\sin 2x + \sqrt{3} = 0$$

$x =$ ✗ $\pi n + \frac{2\pi}{3}, \pi n + \frac{5\pi}{6}$

$2\sin(2x) + \sqrt{3} = 0$

Double Angle $\rightarrow$ FIND ALL $x \in [0, 2\pi)$

To find ALL solutions, 1st find all solutions in $[0, 2\pi)$

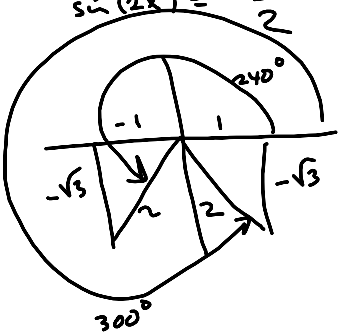
$0 \le x < 2\pi \Rightarrow$

$0 \le 2x < 4\pi$

Find all $2x$'s in $[0, 4\pi)$

$2\sin(2x) = -\sqrt{3} \Rightarrow$

$\sin(2x) = -\frac{\sqrt{3}}{2}$



$2x = \frac{4\pi}{3} \Rightarrow x = \frac{4\pi}{6} = \frac{2\pi}{3}$

$2x = \frac{5\pi}{3} \Rightarrow x = \frac{5\pi}{6}$

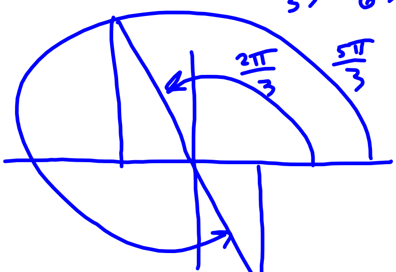
Need to run the 2x's out to 4π!

$2x = \frac{4\pi}{3}, 2\pi + \frac{4\pi}{3} = \frac{4\pi}{3}, \frac{10\pi}{3}$

$2x = \frac{5\pi}{3}, 2\pi + \frac{5\pi}{3} = \frac{5\pi}{3}, \frac{11\pi}{3}$

$\Longrightarrow 2x = \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3}$

$x = \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6}$



I'm happy with

$x = \boxed{\frac{2\pi}{3} + 2n\pi}, \frac{5\pi}{6} + 2n\pi, \boxed{\frac{5\pi}{3} + 2n\pi}, \frac{11\pi}{6} + 2n\pi$

$\frac{2\pi}{3} + n\pi$

$\boxed{\frac{5\pi}{6} + n\pi}$ ⟹ WebAssign wants