23. 0/4 points LarTrig10 1.8.052. [3882204]

For the simple harmonic motion described by the trigonometric function, find the maximum displacement, the frequency, the value of d when $t = 7$ and the least positive value of t for which $d = 0$. Use a graphing utility to verify your results.

$d = \frac{1}{4}\cos 20\pi t$ → Amplitude! $\boxed{\frac{1}{4}\ (a)}$

(a) Find the maximum displacement.

(b) Find the frequency. $= \frac{1}{\text{period}}$: (b) $20\pi t = 2\pi \Rightarrow t = \frac{1}{10} = \text{period} \Rightarrow \boxed{\text{frequency} = 10\ \text{(b)}}$

(c) Find the value of d when $t = 7$.

(d) Find the least positive value of t for which $d = 0$.

(c) $\frac{1}{4}\cos\left(20\pi(7)\right) = \boxed{\frac{1}{4}\cos(140\pi) = \frac{1}{4}\ \text{(c)}}$

$\frac{140\pi}{2\pi} = 70$ TIMES AROUND

No Remainder.

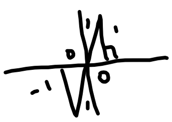
$\cos(140\pi)$

$= \cos(0) = 1$

$\cos(20\pi t) = 0$

Let $u = 20\pi t$

$\cos(u) = 0$



$u = \frac{\pi}{2}, \frac{3\pi}{2}$ if looking for $u \in [0, 2\pi]$

$20\pi t = \frac{\pi}{2} \Rightarrow$

$t = \frac{\pi}{2} \cdot \frac{1}{20\pi} = \boxed{\frac{1}{40} = t}$ (d)

Reciprocal Identities

$\sin u = \dfrac{1}{\csc u}$ $\quad\cos u = \dfrac{1}{\sec u}$ $\quad\tan u = \dfrac{1}{\cot u}$

$\csc u = \dfrac{1}{\sin u}$ $\quad\sec u = \dfrac{1}{\cos u}$ $\quad\cot u = \dfrac{1}{\tan u}$

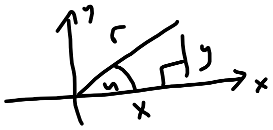
Quotient Identities

$\tan u = \dfrac{\sin u}{\cos u}$ $\quad\cot u = \dfrac{\cos u}{\sin u}$

Pythagorean Identities $\qquad \csc^2(u) - 1 = \cot^2(u)$

$\sin^2 u + \cos^2 u = 1$ $\quad 1 + \tan^2 u = \sec^2 u$ $\quad 1 + \cot^2 u = \csc^2 u$

$\sin(u) = \dfrac{1}{\csc(u)}$? $\qquad \sin(u) = \dfrac{y}{r} = \dfrac{1}{\frac{r}{y}} = \dfrac{1}{\csc(u)}$



$$\tan(u) = \frac{y}{x} = \frac{y}{x}\cdot\frac{\frac{1}{r}}{\frac{1}{r}} = \frac{\frac{y}{r}}{\frac{x}{r}} = \frac{\sin(u)}{\cos(u)}$$

Pythagorus: On the unit circle, $r = 1$, so

$\cos(u) = \frac{x}{r} = x$ $\qquad (x,y)$ on unit circle, so

$\sin(u) = \frac{y}{r} = y$ $\qquad x^2 + y^2 = r^2 = 1$

$\cos^2(u) + \sin^2(u) = 1$ See?

$\tan^2(u) + 1$

$$= \frac{\sin^2(u)}{\cos^2(u)} + \frac{1}{1}\cdot\frac{\cos^2(u)}{\cos^2(u)} = \frac{\sin^2(u) + \cos^2(u)}{\cos^2(u)} = \frac{1}{\cos^2(u)}$$

$$= \frac{1}{\left(\frac{1}{\sec^2(u)}\right)} = 1\cdot\frac{\sec^2(u)}{1} = \sec^2(u)$$

$\tan^2(u) + 1 = \sec^2(u)$

Cofunction Identities

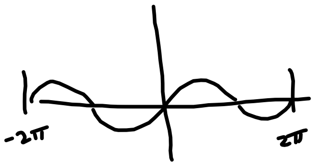
$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \qquad \cos\left(\frac{\pi}{2} - u\right) = \sin u$$

$$\tan\left(\frac{\pi}{2} - u\right) = \cot u \qquad \cot\left(\frac{\pi}{2} - u\right) = \tan u$$

$$\sec\left(\frac{\pi}{2} - u\right) = \csc u \qquad \csc\left(\frac{\pi}{2} - u\right) = \sec u$$

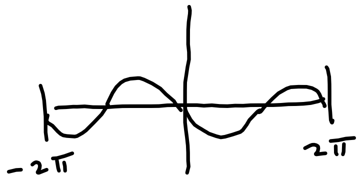
$\sin\left(\frac{\pi}{2} - u\right) = \cos(u)$?

$\sin(u)$:

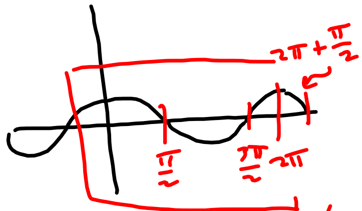


$\frac{\pi}{2} - u = -\left(u - \frac{\pi}{2}\right)$

$\sin(-u)$ $\quad x \to -x$



$\sin\left(-\left(u - \frac{\pi}{2}\right)\right)$



Looks like cosine!

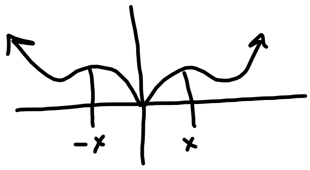
It is!

Even/Odd Identities

$\sin(-u) = -\sin u \qquad \cos(-u) = \cos u \qquad \tan(-u) = -\tan u$

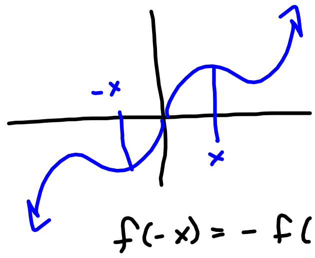
$\csc(-u) = -\csc u \qquad \sec(-u) = \sec u \qquad \cot(-u) = -\cot u$

Even



$f(-x) = f(x)$

ODD



$f(-x) = -f(x)$

Analyzing messes:

x^{2n} Even : $+$

x^{2n+1} Odd : $-$

ODD + ODD = ODD

EVEN + EVEN = EVEN

EVEN + ODD = Neither!

$$\frac{\sin(x)\cos(x) + x^3}{7x^5} = \frac{(-)(+)+(-)}{-} = \frac{-((+)+(+))}{-}$$

$= + \Rightarrow$ Even!

$$\frac{\sin(x)\cos(x) + x^2}{7x^5}$$ Neither $$\frac{(-)+(+)}{(-)} = \text{Neither}$$

$$\frac{\sin(x)\cos(x) + x^3}{7x^4} = \frac{-}{+} = \text{ODD}$$

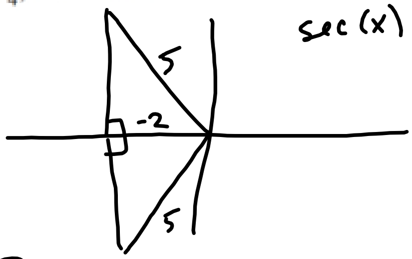
Using Identities to Evaluate a Function

In Exercises 7–12, use the given conditions to find the values of all six trigonometric functions.

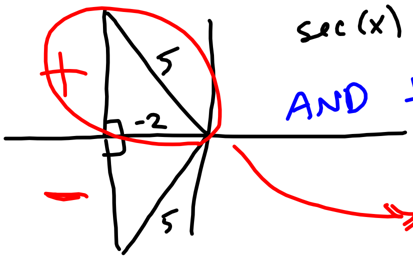
7. $\sec x = -\frac{5}{2}$, $\tan x < 0$
8. $\csc x = -\frac{7}{6}$, $\tan x > 0$
9. $\sin \theta = -\frac{3}{4}$, $\cos \theta > 0$
10. $\cos \theta = \frac{2}{3}$, $\sin \theta < 0$
11. $\tan x = \frac{2}{3}$, $\cos x > 0$
12. $\cot x = \frac{7}{4}$, $\sin x < 0$

#7 $\sec(x) = -\frac{5}{2}$

$\Longrightarrow$ $\cos(x) = -\frac{2}{5}$



$\sec(x) = -\frac{5}{2}$

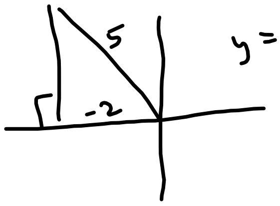


$\sec(x) = -\frac{5}{2}$

AND $\tan(x) < 0$

$\tan(x) = \frac{+}{-2} = -$, i.e., < 0!

So

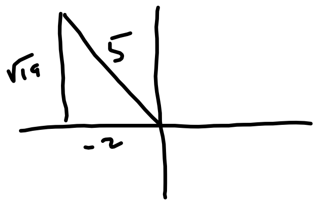


$y = +\sqrt{5^2 - 2^2} = \sqrt{25-4} = \sqrt{19}$

DoH!

"$25-4=21$, idiot!"

Love,
Fernando.



$\sin(x) = \frac{\sqrt{21}}{5}$ $\qquad$ $\csc(x) = \frac{5}{\sqrt{21}} = \frac{5\sqrt{21}}{21}$

$\cos(x) = -\frac{2}{5}$ $\qquad$ $\sec(x) = -\frac{5}{2}$

$\tan(x) = -\frac{\sqrt{21}}{2}$ $\qquad$ $\cot(x) = -\frac{2}{\sqrt{21}} = \frac{-2\sqrt{21}}{21}$

$\sin = \frac{\sqrt{21}}{5}$ is a sin

sine w/o its argument is just "SINFUL."

Simplifying a Trigonometric Expression
In Exercises 19–22, use the fundamental identities to simplify the expression. (There is more than one correct form of each answer).

19. $\dfrac{\tan\theta\cot\theta}{\sec\theta}$ **20.** $\cos\left(\dfrac{\pi}{2} - x\right)\sec x$

21. $\tan^2 x - \tan^2 x\sin^2 x$ **22.** $\sin^2 x\sec^2 x - \sin^2 x$

(19) $$\frac{\frac{\sin\theta}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta}}{\frac{1}{\cos\theta}} = \frac{1}{\left(\frac{1}{\cos\theta}\right)} = \cos\theta$$

WRITE MUCH THINK LITTLE

$$\frac{\tan\theta\cot\theta}{\sec\theta} = \frac{1}{\sec\theta} = \cos\theta$$

(22) $\sin^2(x)\sec^2(x) - \sin^2(x)$

$$= \sin^2(x)\left(\sec^2(x) - 1\right) = \sin^2(x)\left(\tan^2(x)\right)$$

$$= \frac{\sin^2(x)}{1}\cdot\frac{\sin^2(x)}{\cos^2(x)} = \frac{\sin^4(x)}{\cos^2(x)}$$

OR ... $$\frac{\sin^2(x)}{1}\cdot\frac{(1-\cos^2(x))}{\cos^2(x)} = \frac{\sin^2(x)}{\cos^2(x)} - \left(\frac{\sin^2(x)\cos^2(x)}{\cos^2(x)}\right)$$

$$= \tan^2(x) - \sin^2(x)$$

"Results may vary."
Depends on what you want to use it for or "do to it" later.

Factoring a Trigonometric Expression

In Exercises 23–32, factor the expression. Use the fundamental identities to simplify, if necessary. (There is more than one correct form of each answer.)

23. $\dfrac{\sec^2 x - 1}{\sec x - 1}$ **24.** $\dfrac{\cos x - 2}{\cos^2 x - 4}$

25. $1 - 2\cos^2 x + \cos^4 x$ **26.** $\sec^4 x - \tan^4 x$

27. $\cot^3 x + \cot^2 x + \cot x + 1$

28. $\sec^3 x - \sec^2 x - \sec x + 1$

FACTOR BY GROUPING.

29. $3\sin^2 x - 5\sin x - 2$ **30.** $6\cos^2 x + 5\cos x - 6$

31. $\cot^2 x + \csc x - 1$ **32.** $\sin^2 x + 3\cos x + 3$

23

$\dfrac{\sec^2(x) - 1}{\sec(x) - 1}$

$a^2 - b^2 = (a-b)(a+b)$

$= \dfrac{(\sec(x) - 1)(\sec(x) + 1)}{\sec(x) - 1}$

$\boxed{= \sec(x) - 1}$

26

$\sec^4(x) - \tan^4(x)$

$= (\sec^2(x))^2 - \ldots$

$= (\tan^2(x) + 1)^2 - \tan^4(x)$

$= \boxed{\tan^4(x)} + 2\tan^2(x) + 1 \boxed{- \tan^4(x)}$

$= 2\tan^2(x) + 1$ Legit

$= 2\sec^2(x) - 2 + 1$

$= 2\sec^2(x) - 1$ Legit

$(\sec^2(x) - \tan^2(x))(\sec^2(x) + \tan^2(x))$

$= (\tan^2(x) + 1 - \tan^2(x))$ ——

$= 1\,(\sec^2(x) + \tan^2(x))$

$= \sec^2(x) + \tan^2(x)$

$= 2\tan^2(x) + 1$

$3\sin^2(x) - 5\sin(x) - 2$

$P(u) = 3u^2 - 5u - 2 \quad = \quad 3u^2 - 3u - 2u - 2$

$ac = -6$

$-3 - 2 = -5$

But $(-3)(-2) = +6$!

$= 3u(u-1) - 2(u+1)$?!

Nope!

Try $-6 + 1$

$3u^2 - 6u + 1u - 2$

$= 3u(u-2) + 1(u-2)$

$= (u-2)(3u+1)$

$= (\sin(x) - 2)(3\sin(x) + 1)$

SLEDGE HAMMER

$3u^2-5u-2$

$a=3,\ b=-5,\ c=-2$

$b^2-4ac = (-5)^2-4(3)(-2)$

$= 25+24 = 49$

$= \dfrac{-b\pm\sqrt{b^2-4ac}}{2a} = \dfrac{5\pm\sqrt{49}}{2(3)} = \dfrac{5\pm 7}{6}$ → $\dfrac{12}{6}=2$, $\dfrac{-2}{6}=-\dfrac{1}{2}$

connor says: "Your arithmetic sucks, Steve." Love, Connor

MAT 121 FACTOR THEOREM

That says $=-\frac{1}{2}$, 2 are roots, so

$-\frac{1}{3}$, dummy

$(u-(-\frac{1}{3}))$ & $(u-2)$ are factors:

So, $(u+\frac{1}{3})(u-2)$ is ALMOST $P(u)$!

Just need the 3 out front!

$3(u+\frac{1}{3})(u-2) = (3u+1)(u-2)$

$\Rightarrow P(\sin(x)) = (3\sin(x)+1)(\sin(x)-2)$

Simplifying a Trigonometric Expression

In Exercises 33–40, use the fundamental identities to simplify the expression. (There is more than one correct form of each answer.)

33. $\tan\theta \csc\theta$

34. $\tan(-x)\cos x$

35. $\sin\phi(\csc\phi - \sin\phi)$

36. $\cos x(\sec x - \cos x)$

37. $\sin\beta\tan\beta + \cos\beta$

38. $\cot u \sin u + \tan u \cos u$

39. $\dfrac{1 - \sin^2 x}{\csc^2 x - 1}$

40. $\dfrac{\cos^2 y}{1 - \sin y}$

40

$$\left(\frac{\cos^2(y)}{1-\sin(y)}\right)\left(\frac{1+\sin(y)}{1+\sin(y)}\right) = \frac{(\cos^2(y))(1+\sin(y))}{(1-\sin^2(y))}$$

$$= \frac{\cancel{\cos^2(y)}(\sin(y)+1)}{\cancel{\cos^2(y)}} = \sin(y)+1$$

Multiplying Trigonometric Expressions In Exercises 41 and 42, perform the multiplication and use the fundamental identities to simplify. (There is more than one correct form of each answer.)

41. $(\sin x + \cos x)^2$

42. $(2 \csc x + 2)(2 \csc x - 2)$

Adding or Subtracting Trigonometric Expressions In Exercises 43–48, perform the addition or subtraction and use the fundamental identities to simplify. (There is more than one correct form of each answer.)

43. $\dfrac{1}{1+\cos x}+\dfrac{1}{1-\cos x}$

44. $\dfrac{1}{\sec x+1}-\dfrac{1}{\sec x-1}$

45. $\dfrac{\cos x}{1+\sin x}-\dfrac{\cos x}{1-\sin x}$

46. $\dfrac{\sin x}{1+\cos x}+\dfrac{\sin x}{1-\cos x}$

47. $\tan x-\dfrac{\sec^2 x}{\tan x}$

48. $\dfrac{\cos x}{1+\sin x}+\dfrac{1+\sin x}{\cos x}$

Rewriting a Trigonometric Expression In Exercises 49 and 50, rewrite the expression so that it is *not* in fractional form. (There is more than one correct form of each answer.)

49. $\dfrac{\sin^2 y}{1 - \cos y}$

50. $\dfrac{5}{\tan x + \sec x}$

Trigonometric Functions and Expressions **In Exercises 51–54, use a graphing utility to determine which of the six trigonometric functions is equal to the expression. Verify your answer algebraically.**

51. $\frac{1}{2}(\sin x \cot x + \cos x)$

52. $\sec x \csc x - \tan x$

53. $\dfrac{\tan x + 1}{\sec x + \csc x}$

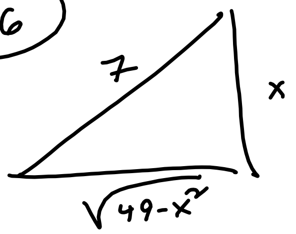
54. $\dfrac{1}{\sin x}\left(\dfrac{1}{\cos x} - \cos x\right)$

Trigonometric Substitution In Exercises 55–58, use the trigonometric substitution to write the algebraic expression as a trigonometric function of θ, where $0 < \theta < \pi/2$.

55. $\sqrt{9 - x^2}$, $x = 3\cos\theta$
56. $\sqrt{49 - x^2}$, $x = 7\sin\theta$
57. $\sqrt{x^2 - 4}$, $x = 2\sec\theta$
58. $\sqrt{9x^2 + 25}$, $3x = 5\tan\theta$

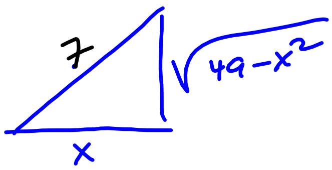
Important

56

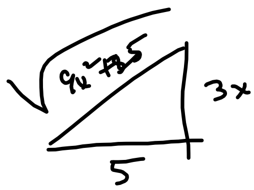


$\frac{x}{7} = \sin\theta \Rightarrow$
$x = 7\sin\theta$

Alternate :



$\frac{x}{7} = \cos\theta \Rightarrow$
$x = 7\cos\theta$



$\frac{3x}{5} = \tan\theta$

$x = \frac{5}{3}\tan\theta$

Solving a Trigonometric Equation **In Exercises 63–66, use a graphing utility to solve the equation for θ, where $0 \le \theta < 2\pi$.**

63. $\sin\theta = \sqrt{1 - \cos^2\theta}$

64. $\cos\theta = -\sqrt{1 - \sin^2\theta}$

65. $\sec\theta = \sqrt{1 + \tan^2\theta}$

66. $\csc\theta = \sqrt{1 + \cot^2\theta}$