Section 1.6 and 1.7, today? We'll see how far we get.

$a \sin(b(x-c)) + d$

Wednesday's video:

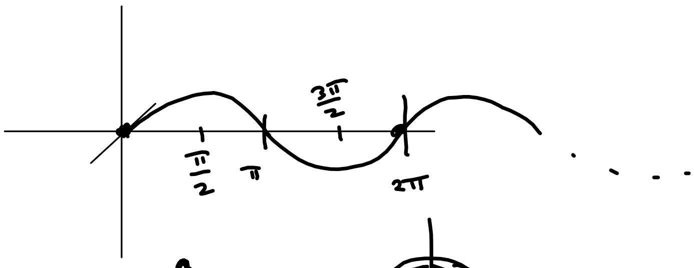
https://harryzaims.com/122/122-fall-22/lectures/220907-1-5.mp4

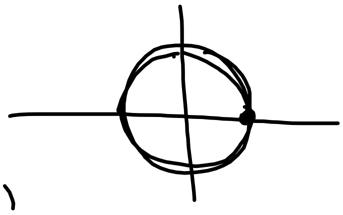
Wednesday's notes:

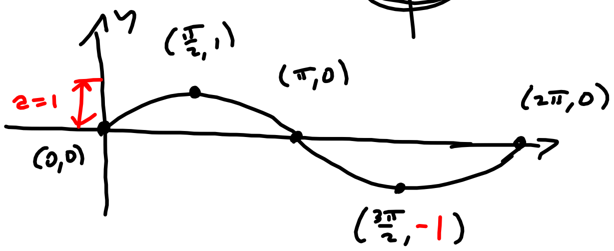
https://harryzaims.com/122/122-fall-22/notes/220907-1-5.pdf

Basic Graph of Sine :

From Last Wednesday







$y = \sin(x)$.
One period.

$y = a\sin(b(x-c)) + d$

Amplitude $\frac{\text{HIGH} - \text{LOW}}{2}$

Period of $\sin(*)$ is $* = 2\pi$

$\boxed{bx = * = 2\pi}$ $\Rightarrow$ I remember this.

$x = \frac{2\pi}{b}$ = period of $\sin(bx)$.

$a\sin(b(x-c)) + d$ → $y = d$ is midline.

starting point.

§1.5 #30

Build Cosine to fit the data.

All we need is high & low

(1, 57.1) Low

(7, 104.1)

PERIOD = ?

Let t = # of the month, starting with January = 1.

PERIOD = 12 months (Xander)

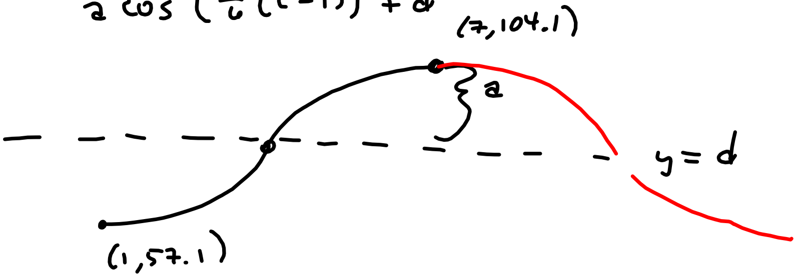
$bx = 2\pi$, when $x = 12$

$12b = 2\pi$, so

$b = \frac{2\pi}{12} = \frac{\pi}{6} \longrightarrow a\cos\left(\frac{\pi}{6}(x-c)\right)+d.$

January = 1 = t = start, so:

$a\cos\left(\frac{\pi}{6}(t-1)\right)+d$



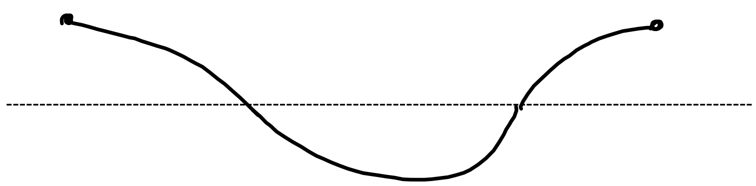
Midline: $\frac{57.1+104.1}{2} = \frac{161.2}{2} = 80.6$

$a\cos\left(\frac{\pi}{6}(t-1)\right)+80.6$

Amplitude: $\frac{104.1-57.1}{2} = \frac{47.0}{2} = 23.5$

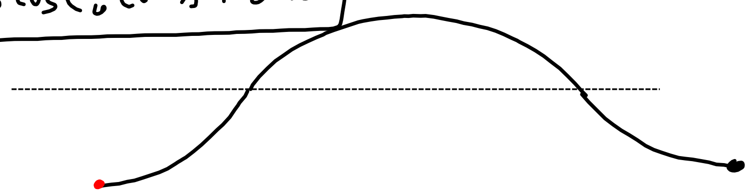
So, our function is $y = 23.5\cos\left(\frac{\pi}{6}(t-1)\right)+80.6$

Upside-Down!



To achieve this: Multiply by −1!

$\boxed{y = -23.5\cos\left(\frac{\pi}{6}(t-1)\right)+80.6}$



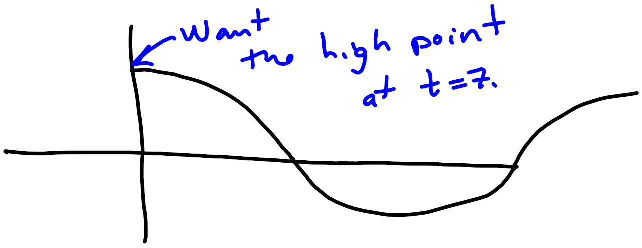
Book Answer:

$$23.5\cos\left(\frac{\pi t}{6} - 3.67\right) + 80.6 \ !?$$

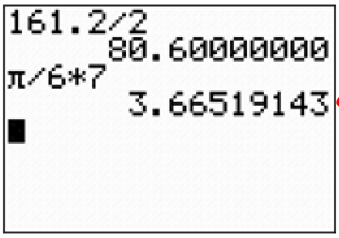
What'd they do?

They took a right-side-up cosine & shifted it!

Want the high point at $t=7$.



$23.5\cos\left(\frac{\pi}{6}(t-7)\right) + 80.6$ will put high point at $t=7$



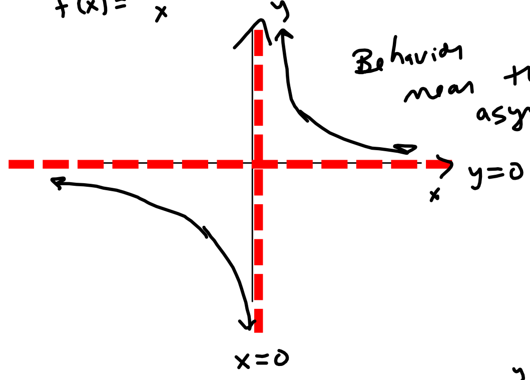
There's the book's 3.67.

§1.5/1.6 Questions?

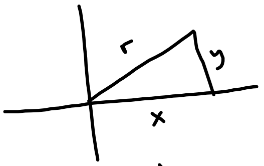
Are you surfing harryzaims.com successfully?

Things not done in lecture:
tangent's basic graph.
Questions on your graphs.

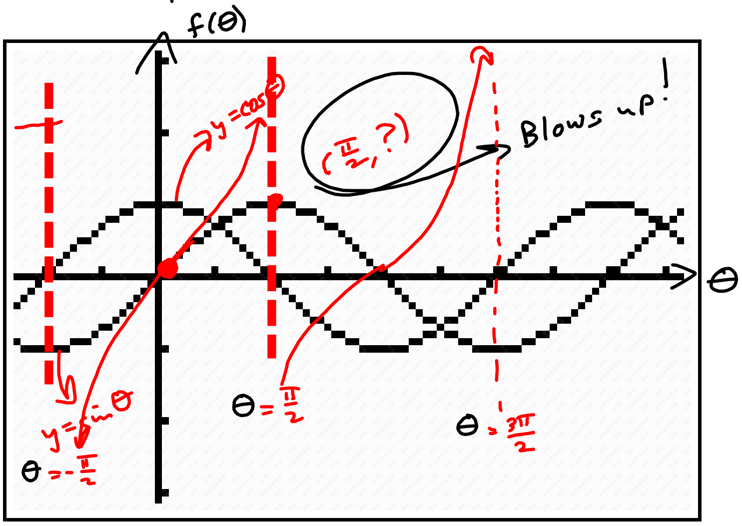
$f(x) = \frac{1}{x}$

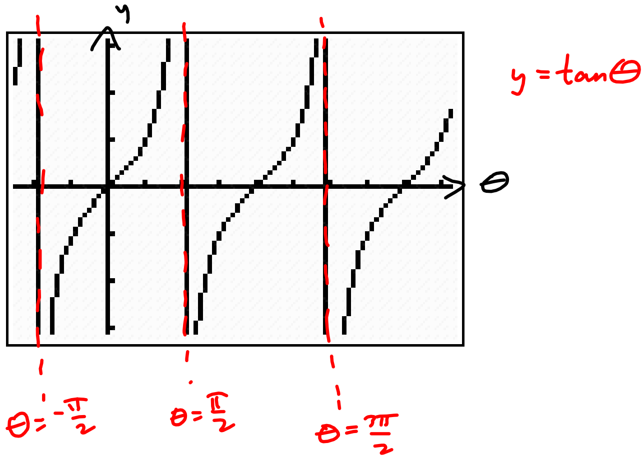


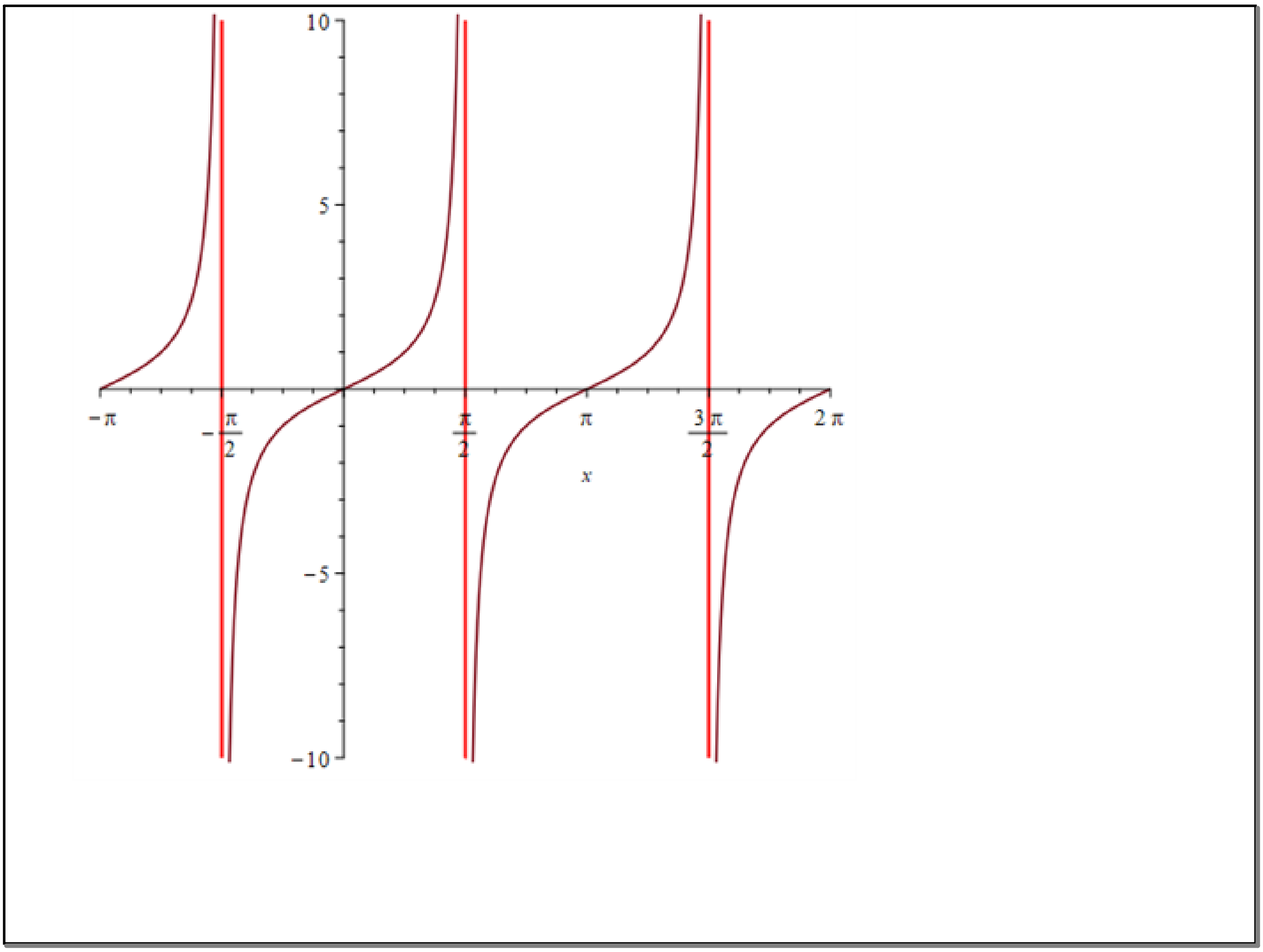
Behavior near the asymptote $x=0$ (vertical)



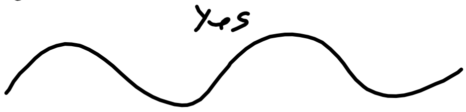
$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{y}{r}}{\frac{x}{r}} = \frac{y}{r}\cdot\frac{r}{x} = \frac{y}{x} = \tan\theta$$

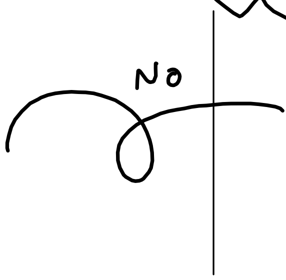





10
5
−5
−10
−π
−π/2
π/2
π
3π/2
2π
x

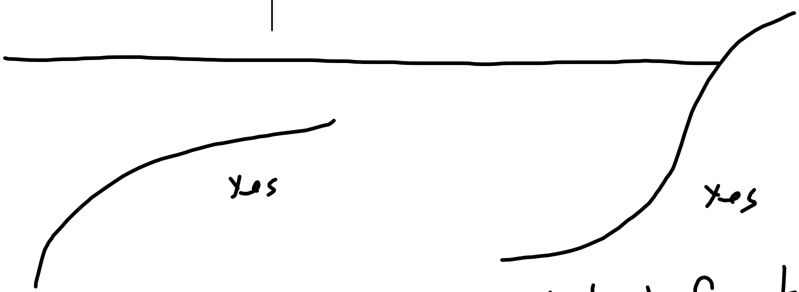
Functions?

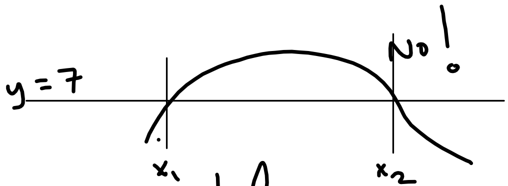




function: $f(x)$ is well-defined if for each $x \in \mathcal{D}$, there's one (and only one) $y \in \mathcal{R}$

$\mathcal{D}$ = Domain, $\mathcal{R}$ = Range.





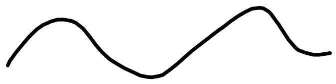
Horizontal Line Test.

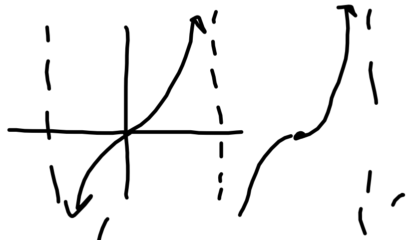
1-to-1 function is a function (one x? one y.) that for each $y \in \mathcal{R}$, there's exactly one $x \in \mathcal{D}$

$$f(x_1) = f(x_2)$$
$$\Longrightarrow x_1 = x_2$$

FOR THE INVERSE RELATION TO BE A FUNCTION, THE ORIGINAL MUST BE 1-to-1.

The problem with inverse trig functions is trig functions are not 1-to-1.





Solution: Restrict the Domain!

Recall, if $f(x) = x^2 = y \Rightarrow$

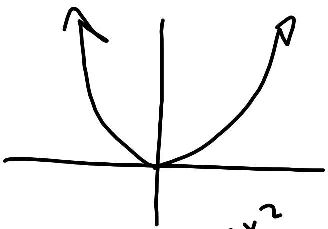
$\boxed{|x| = \sqrt{x^2}} = \sqrt{y} \Rightarrow$

$x = \pm\sqrt{y}$

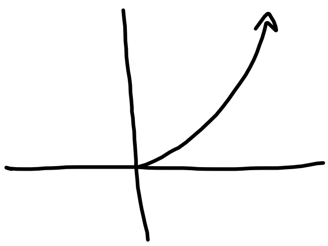
$f^{-1}(x) = \sqrt{y}$?

$f^{-1}(x) = -\sqrt{y}$?

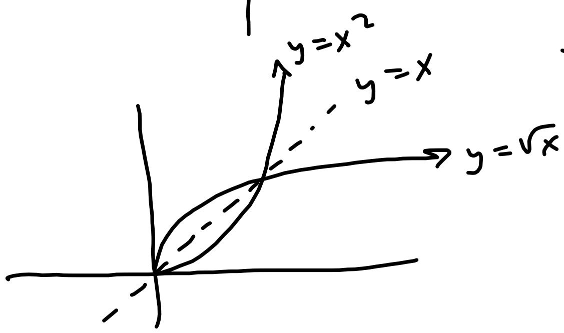
Depends what piece of x^2 you take!



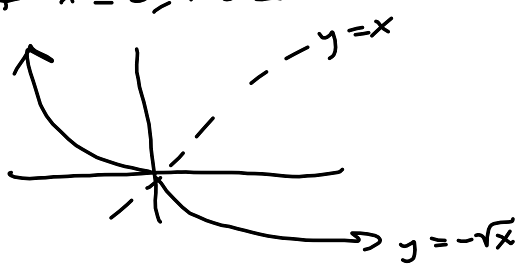
Restrict Domain to $\{x \mid x \geq 0\}$:



Then $x = \pm\sqrt{y}$ becomes $x = \sqrt{y}$



(If $x \leq 0$, then



Conventions:

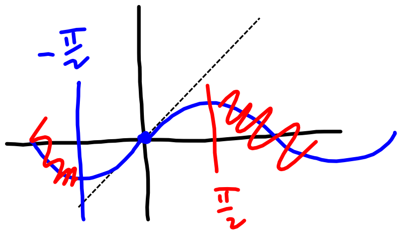
$$\sin^{-2}(x) = \frac{1}{\sin^2(x)}$$

$$\sin^2(x) = (\sin(x))^2$$

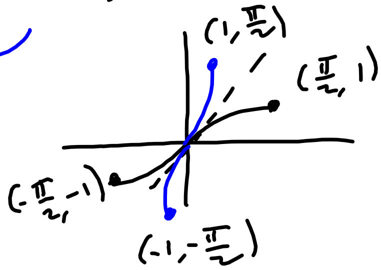
$$\cos^3(x) = (\cos(x))^3$$

But $\sin^{-1}(x)$ means inverse sine, NOT $\frac{1}{\sin(x)} = (\sin(x))^{-1}$, which sucks.

So, I use arcsin(x) to mean inverse sine, etc.



Restricted domain to keep sine 1-to-1 is $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$



sine

arcsin

Your calculator sees angles between -Pi/2 and Pi/2 when it's calculating inverse sine.

Find all solutions $x \in [0°, 360°]$ that solve

$\sin(x) = \frac{2}{3}$, to 3 decimal places

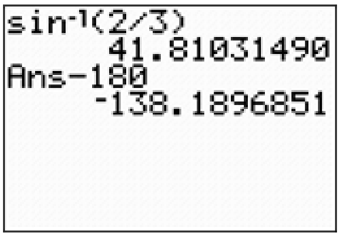
USE SIN^{-1} key

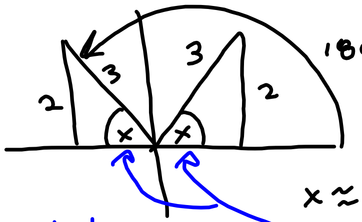
(virtue key!

$\sin(x) = \frac{2}{3} \implies$

$x = \sin^{-1}\left(\frac{2}{3}\right)$

is NOT Quite COMPLETE!





$180° - 41.\ldots° = 138.\ldots°$

$x \approx 41.810°, 138.190°$

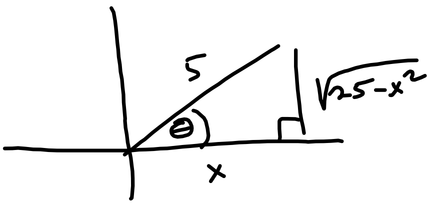
Both solutions have that reference angle of $41.81\ldots°$

33. 0/6 points

Use a graphing utility to graph f and g in the same viewing window to verify that the two functio

$$f(x) = \tan\left(\arccos\left(\frac{x}{5}\right)\right), \quad g(x) = \frac{\sqrt{25 - x^2}}{x}$$

$= \tan\theta$



$\sqrt{5^2 - x^2} = \sqrt{25 - x^2}$

Now, what's $\tan(\theta)$?

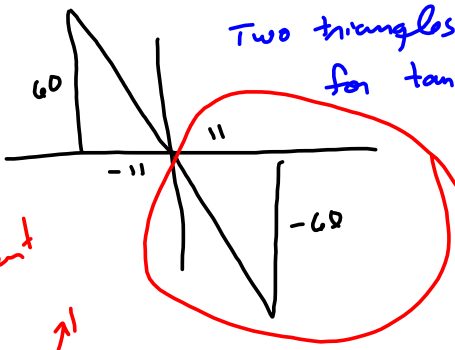
$$\tan\left(\arccos\left(\frac{x}{5}\right)\right) = \frac{\sqrt{25-x^2}}{x} = g(x)!$$

27. 0/1 points

Find the exact value of the expression, if possible. (If not possible, enter IMPOSSIBLE.)

$$\csc\left[\arctan\left(-\frac{60}{11}\right)\right]$$

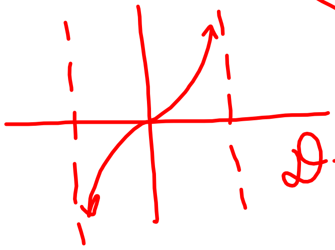
$$\csc\left(\arctan\left(-\frac{60}{11}\right)\right) = \csc(\theta)$$



Two triangles for tangent $= -\frac{60}{11}$

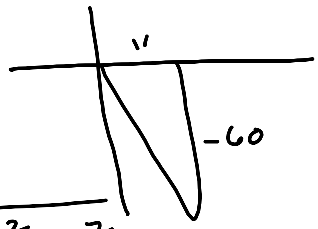
But arctangent only sees this one.

Restricted tangent



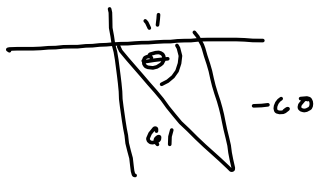
$\theta = (-\frac{\pi}{2}, \frac{\pi}{2})$

So:



$$\sqrt{60^2 + 11^2}$$

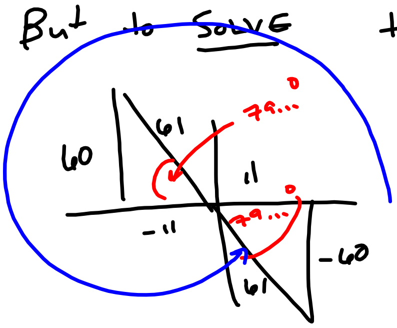
$$= \sqrt{3600 + 121} = \sqrt{3721} = 61$$



Now, $\csc\theta = \dfrac{1}{\sin\theta}$

$$= \frac{1}{-\frac{60}{61}} = \boxed{-\frac{61}{60} = \csc\theta}$$

But to SOLVE $\tan\theta = -\frac{60}{11}$, you'll get 2 sol'ns:




```
tan⁻¹(-60/11)
      -79.61114218
Ans+180
       100.3888578
```

```
      -79.61114218
Ans+180
       100.3888578
tan⁻¹(-60/11)
      -79.61114218
Ans+360
       280.3888578
```

$TAN^{-1}\left(-\frac{60}{11}\right) =$

$\theta \approx -79.611, \; 100.389.$

If angles must be be between $0°$ & $360°$, then

$100.389, \; 360° - 79.611...° \approx 280.389$