MAT 122
100 Points Covers Chapter 1

Test 1 – Fall, 2020

Name KEY

NO GRAPHING CALCULATORS!!!

Do all your work and submit answers with your work, on the separate paper provided. Organize your work for efficient grading and feedback. Leave a margin, especially in the top left, where the staple goes!

Leave space between problems. No prizes for saving paper, here. Figure this stuff out, and use your smarts to plant trees! Only use one column of work. Don't start a 2[nd] column to save paper. ALL I WANT ON THIS PAGE IS YOUR NAME.

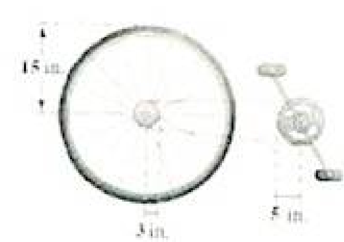
1. (10 pts) Find two angles, between -2π and 2π (i.e., -360^0 and 360^0) that are coterminal with $\frac{155\pi}{24}$. Give exact answers in degrees and radians.

2. Arc Length and Area of Sector. Suppose we have a circle of radius $r = 20$ cm.
   a. (5 pts) Find the arc length on the circle, that is intercepted by an angle of 33754^0. Round to 3 decimal places.
   b. (5 pts) Find the *exact* area of the sector that is intercepted (swept through) by an angle of $\theta = \frac{17\pi}{9}$

3. Basic concept: Draw the doggone pictures!
   a. (5 points) Sketch two triangles that satisfy $\tan(\theta) = \frac{5}{\sqrt{13}}$.
   b. (5 pts) Assume the terminal side of the angle θ lies in the 3[rd] quadrant. Find the other five trigonometric functions of θ.
   c. (5 pts) Again, assuming θ's terminal side lies in Q III, and $0 \le \theta < 2\pi$, find θ, in radians *and* degrees, rounded to 3 decimal places.
   d. (5 pts) Give *all* solutions to the equation $\tan(\theta) = \frac{5}{\sqrt{13}}$, in degrees *and* radians, rounded to three (3) decimal places. (Good one to skip and come back to, if you're moving slowly on this.).

4. (10 pts) Sketch one period of the graphs of $y = \sin(x)$ and $y = \csc(x)$ on the same set of coordinate axes.

5. The radii of the pedal sprocket, the rear wheel sprocket, and the wheel of the bicycle in the figure are 5 inches, 3 inches and 15 inches, respectively. A cyclist is pedaling at a rate of 1.7 revolutions per second.



   a. (5 pts) Find the speed of the bicycle in feet per second. Round final answer to 3 decimal places.
   b. (5 pts) Find the speed in miles per hour. Round final answer to 3 decimal places.

6. (10 pts) Sketch the graph of $f(x) = -25\sin\left(\frac{\pi}{24}x - \frac{13\pi}{12}\right) + 20$.

7. (10 pts) Write the cosine function that achieves its maximum height of $y = 177$ feet at time $t = 3$ seconds and its minimum height of $y = -7$ feet at $t = 27$ seconds.

8. (5 pts) Solve the triangle on the right. That means, find all lengths and angles. Exact answers required.

9. Find the exact value of…

   a. … (5 pts) $\cos\left(\arcsin\left(\frac{-7}{13}\right)\right)$.

   b. … (5 pts) $\arctan\left(\tan\left(\frac{3\pi}{4}\right)\right)$

10. (5 pts) Draw the sketch and use it to find an algebraic expression that is equivalent to $\sin\left(\arctan\left(\frac{7}{\sqrt{4x^2-49}}\right)\right)$. Assume that everything is taking place in the 1st quadrant.

Bonus: Answer *two* of the following 3 problems, for *up to* 10 points:

11. (5 pts) Sketch the triangles corresponding to the following. One or more may have only one triangle. Most will have two possible triangles.

   a. $\cos(x) = 0$

   b. $\sin(x) = 1$

   c. $\sin(x) = \frac{-\sqrt{3}}{2}$

   d. $\tan(x) = \sqrt{3}$

   e. $\csc(x) = 0$

12. (5 pts) Sketch the graph of one period of $y = \cos(x)$ (restricted to make it 1-to-1) *and* $y = \arccos(x)$ (i.e., $y = \cos^{-1}(x)$) on the same set of coordinate axes. I want to see the function and its inverse in the same picture. Label key points as ordered pairs (ALWAYS). State the domain and range of the restricted tangent function and its inverse.

13. (5 pts) Find all solutions θ of the trigonometric polynomial $\tan^2(\theta) - 3 = 0$ in the interval $[0, 2\pi)$

14. (5 pts) Super-duper bonus Find all solutions θ of the trigonometric polynomial $8\cos^3(\theta) - 4\cos^2(\theta) - 6\cos\theta + 3 = 0$ in the interval $[0, 2\pi)$.

1 10 pts $\theta = \frac{155\pi}{4} \cdot \frac{180^\circ}{\pi} = 1162.5^\circ$ Degrees version.

$\frac{1162.5^\circ}{360^\circ} = 3.22916\overline{6}$ revs

$(.22916\overline{6} \text{ revs})\left(\frac{360^\circ}{1 \text{ rev}}\right) = 82.5^\circ$

$82.5^\circ - 360^\circ = -277.5^\circ$

$\boxed{82.5^\circ, -277.5^\circ}$

$(82.5^\circ)\left(\frac{\pi}{180^\circ}\right) = \frac{11\pi}{24}$

$\frac{11\pi}{24} - 2\pi = \frac{11\pi - 48\pi}{24}$

$= -\frac{37\pi}{24}$

$\frac{11\pi}{24}, -\frac{37\pi}{24}$ SEE LAST PAGE FOR SOMETHING MORE RATIONAL

Scratch:

$\frac{82.5^\circ}{180^\circ} = .458\overline{3} = x$

$4.58\overline{3} = 10x$

$4.125 = 9x$

$9000x = 4125$

$x = \frac{4125}{9000} = \frac{11}{24}$, so

$\frac{11}{24}\pi$ & $\frac{11}{24}\pi - 2\pi = -\frac{37\pi}{24}$

2 $r = 20$ cm

a 5 pts $\theta = 33754^\circ \Rightarrow s = r\theta = (20)(33754^\circ)$

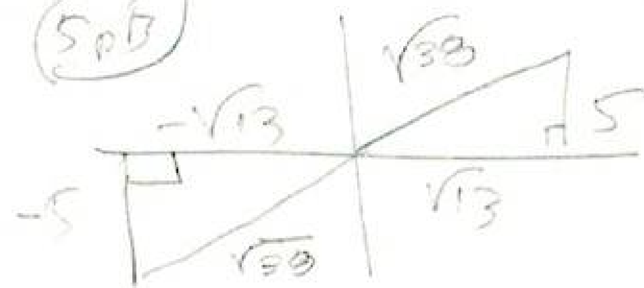
TIMES $\frac{\pi}{180^\circ} \approx 11{,}782.36097$ cm

$\approx \boxed{11{,}782.361 \text{ cm}}$

b 5 pts $\theta = \frac{17\pi}{9} \Rightarrow A = \frac{1}{2}r^2\theta = \frac{1}{2}(20)^2\left(\frac{17\pi}{9}\right)$

$= \frac{1}{2}(400)\left(\frac{17\pi}{9}\right) = (200)\left(\frac{17\pi}{9}\right) = \boxed{\frac{3400\pi}{9} \text{ cm}^2}$

(3) (a) $\tan\theta = \frac{5}{\sqrt{13}}$ (5 pts)

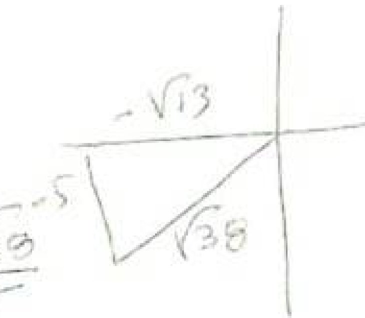


$\cancel{22}\ 5^2 + \sqrt{13}^2$

$= 25 + 13 = 38$

$\longrightarrow \sqrt{38}$

(b) (5 pts) Assume QIII, then



$\sin\theta = \frac{-5}{\sqrt{38}}$ $\qquad$ $\csc\theta = -\frac{\sqrt{38}}{5}$

$\cos\theta = \frac{-\sqrt{13}}{\sqrt{38}}$ $\qquad$ $\sec\theta = \frac{-\sqrt{38}}{\sqrt{13}}$

$\tan\theta = \frac{5}{\sqrt{13}}$ $\qquad$ $\cot\theta = \frac{\sqrt{13}}{5}$

(c) (5 pts) $\arctan\left(\frac{5}{\sqrt{13}}\right) \approx 54.20424009^\circ \longrightarrow$

We want $180^\circ + 54..^\circ = 234.20424009^\circ$

$\boxed{234.204^\circ}$ OR $\boxed{4.088 \text{ Radians}}$

(d) (5 pts) $\{54.204^\circ + 180^\circ n \mid n \in \mathbb{Z}\}$

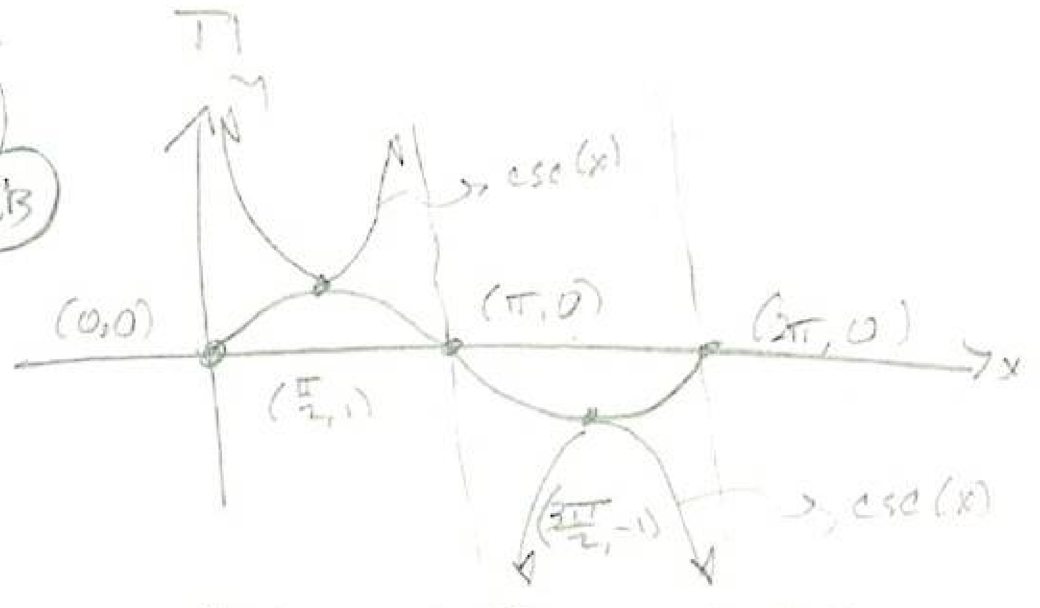
$\{4.088 + \pi n \mid n \in \mathbb{Z}\}$

ALT: $\{x + 360^\circ n \mid x = 54.204^\circ, 234.204^\circ, n \in \mathbb{Z}\}$

$\{x + 2\pi n \mid x = 4.088, .946, n \in \mathbb{Z}\}$

4

10 pts



5 $\left(\frac{1.7 \text{ revs front}}{1 \text{ sec}}\right)\left(\frac{5 \text{ revs rear}}{3 \text{ rev front}}\right)\left(\frac{2\pi \text{ radians}}{1 \text{ rev rear}}\right)\left(\frac{15 \text{ in}}{\text{rear}}\right)\cdot$

$\left(\frac{1 \text{ ft}}{12 \text{ in}}\right) = \frac{(1.7)(5)(2\pi)(15)}{(3)(12)} \approx 22.25294796 \frac{\text{ft}}{\text{sec}}$

$\approx \boxed{22.253 \text{ ft/sec}}$

a 5 pts

b $\left(22.252\ldots \text{ ft/sec}\right)\left(\frac{60 \frac{\text{mi}}{\text{hr}}}{88 \frac{\text{ft}}{\text{sec}}}\right) \approx \boxed{15.172 \frac{\text{mi}}{\text{hr}}}$

5 pts

15.17246452

122 T1

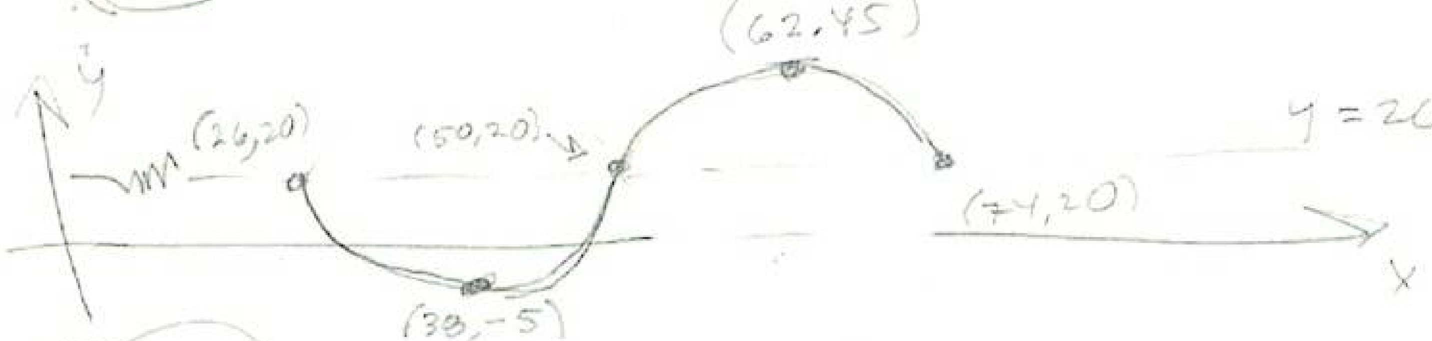
③

⑥ $f(x) = -25 \sin\left(\frac{\pi}{24}(x-26)\right) + 20$

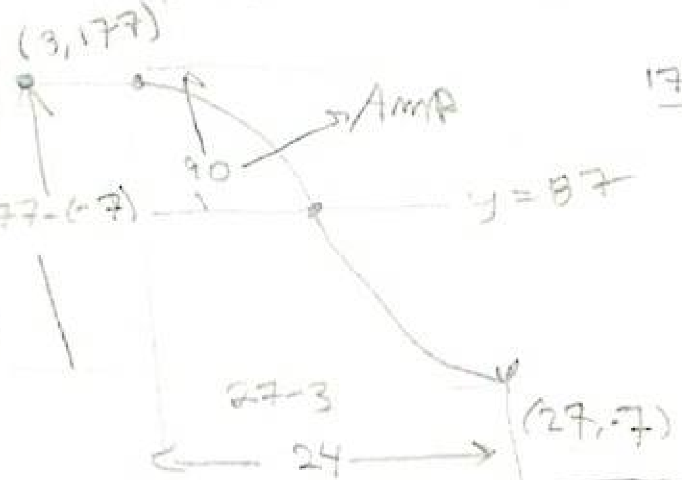
Flip, AMP, START

$\frac{\pi}{24}x - \frac{13\pi}{12} = \frac{\pi}{24}(x-26)$ $T = 48$ from $x\frac{\pi}{24} = 2\pi \rightarrow x = 48$

10pts



⑦ 10pts



$184 = 177 - (-7)$

$\frac{184}{2} = 92 = a$

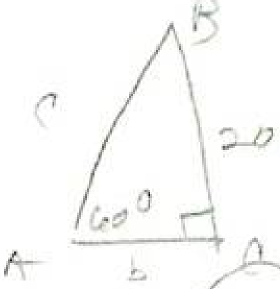
$\frac{177 + (-7)}{2} = \frac{170}{2} =$

Mid

$$\boxed{f(x) = 92\cos\left(\frac{\pi}{24}(x-3)\right) + 85}$$

122 T1 (3)

(8)



5 pts

$$\frac{20}{c} = \sin 60° = \frac{\sqrt{3}}{2}$$

$$20 = \frac{\sqrt{3}}{2} c$$

$$c = 20\left(\frac{2}{\sqrt{3}}\right) = \frac{40}{\sqrt{3}} \text{ OR } \boxed{\frac{40\sqrt{3}}{3} = c}$$

11.5470053$\dot{8}$

$$\frac{20}{b} = \tan 60° \longrightarrow b = \frac{20}{\tan 60°}$$

$$B = 90° - 60° \boxed{= 30° = B} = \frac{20}{(\sqrt{3})} \text{ OR } \boxed{\frac{20\sqrt{3}}{3} = b}$$

23.09401077

$$\boxed{C = 90°}$$

(9) (a) 5 pts $\cos\left(\arcsin\left(-\frac{7}{13}\right)\right) = \cos\Theta$

$\arcsin\left(-\frac{7}{13}\right)$ $13^2 - 7^2 = 169 - 49 = 120$

$\sqrt{120} = 2\sqrt{30}$

$$\begin{array}{r|l} 2 & 120 \\ 2 & 60 \\ 2 & 30 \\ & 15 \end{array} \longrightarrow \cos\Theta = \boxed{\frac{2\sqrt{30}}{13}} \approx .8426500885$$

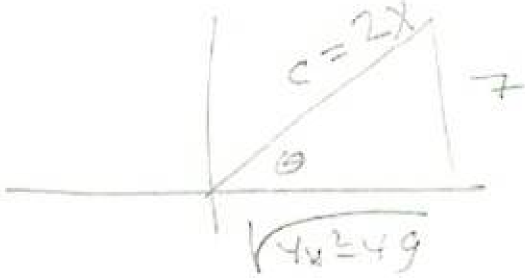
(b) 5 pts $\arctan\left(\tan\left(\frac{3\pi}{4}\right)\right) = \arctan(-1) = \boxed{-\frac{\pi}{4}}$

OR $-45°$

122 Σ1

4

10 5pts $\sin\left(\arctan\left(\frac{7}{\sqrt{4x^2-49}}\right)\right) = \sin\theta$



$7^2 + \left(\sqrt{4x^2-49}\right)^2$

$= 49 + 4x^2 - 49 = 4x^2$

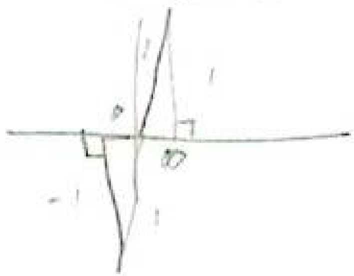
$\Longrightarrow c = 2x$

$$\Longrightarrow \boxed{\sin\theta = \frac{7}{2x}}$$

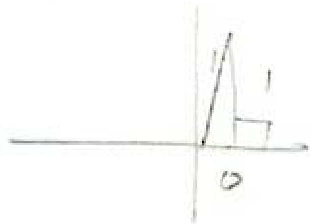
122 T1 Bonus Pages

11 Sols

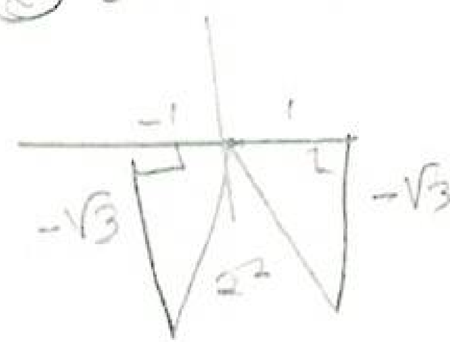
a $\cos(x) = 0$



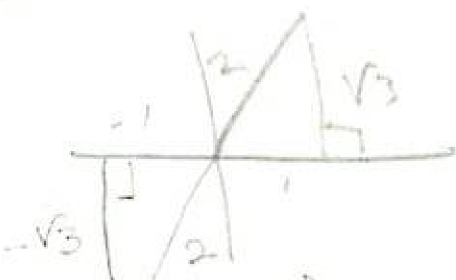
b $\sin(x) = 1$



c $\sin(x) = -\frac{\sqrt{3}}{2}$



d $\tan x = \sqrt{3}$

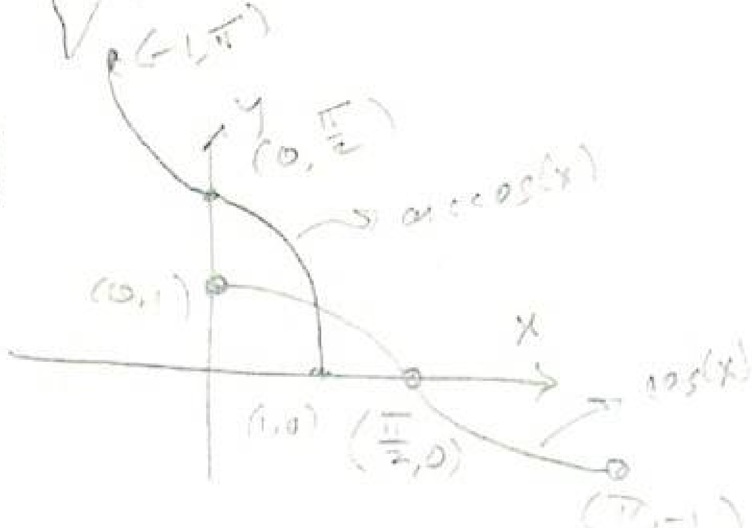


e $\csc(x) = 0$

Impossible

$R(\csc(x)) = (-\infty, -1] \cup [1, \infty)$

12

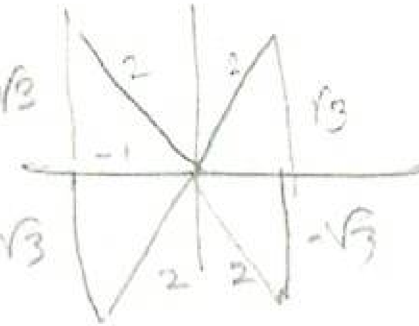


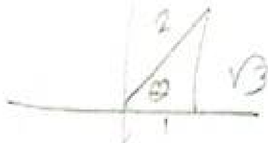
122 TEST 1 Bonus

13 5pts $\tan^2\theta - 3 = 0$

$\tan^2\theta = 3$

$\tan\theta = \pm\sqrt{3}$





$\theta = \frac{\pi}{3}$ OR 60°

So, $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} = \theta$ OR

$\theta = 60^\circ, 120^\circ, 240^\circ, 300^\circ$

14 5pts $8u^3 - 4u^2 - 4u + 3 = 0$, where $u = \cos\theta$

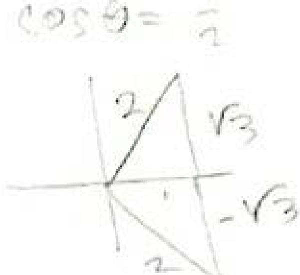
$4u^2(2u-1) - 3(2u-1) = 0$

$(2u-1)(4u^2 - 3) = 0$

$2u - 1 = 0$ $\quad$ $4u^2 - 3 = 0$

$u = \frac{1}{2}$ $\quad$ $4u^2 = 3$

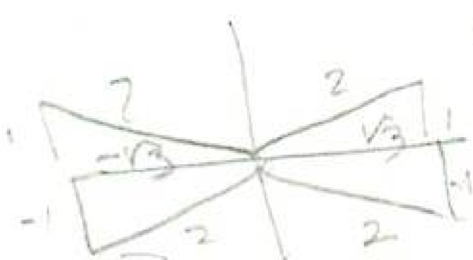
$\cos\theta = \frac{1}{2}$ $\quad$ $u^2 = \frac{3}{4}$



$u = \pm\frac{\sqrt{3}}{2} = \cos\theta$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$



$\theta \in \left\{\frac{\pi}{6}, \frac{\pi}{3}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6}\right\}$

122 Alternate radians approach

$$\frac{155\pi}{24} \Big/ (2\pi) = 3.22916\overline{6} \text{ revs.}$$

$$(3)(2\pi) = 6\pi$$

$$\frac{155\pi}{24} - 6\pi = \frac{155\pi - 144\pi}{24} = \frac{11\pi}{24}$$

$$\frac{11\pi}{24} - 2\pi = \frac{11\pi}{24} - \frac{48\pi}{24} = -\frac{37\pi}{4}$$

Then convert to degrees via the $\frac{180^\circ}{\pi}$ factor.

Alternate degrees approach:

$$\left(\frac{155\pi}{24}\right)\left(\frac{180^\circ}{\pi}\right) = 1162.5^\circ$$

$$\frac{1162.5^\circ}{360^\circ} = 3.22916\overline{6} \text{ revs}$$

$$(3)(360) = 1080$$

$$1162.5^\circ - 1080^\circ = 82.5^\circ$$

$$82.5^\circ - 360^\circ = -277.5^\circ$$