$$16x^2 + 9y^2 - 160x + 72y = -400$$

$$16x^2 - 160x \quad + 9y^2 + 72y \quad = -400$$

$$16(x^2 - 10x \quad) + 9(y^2 + 8y \quad) = -400$$

$$16(x^2 - 10x + 5^2) + 9(y^2 + 8y + 4^2) = -400 + 16(25) + 9(16)$$

$\frac{10}{2} = 5 \rightsquigarrow 5^2 = 25$ $\qquad$ $\frac{8}{2} = 4 \rightsquigarrow 4^2 = 16$

$$16(x-5)^2 + 9(y+4)^2 = -400 + 400 + 144 = 144$$

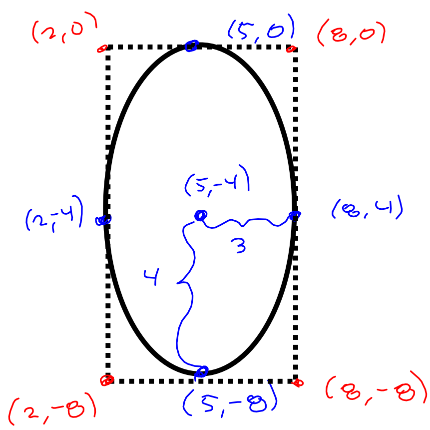
$$16(x-5)^2 + 9(y+4)^2 = 144$$

$$\frac{16(x-5)^2}{144} + \frac{9(y+4)^2}{144} = \frac{144}{144}$$

$$\frac{(x-5)^2}{9} + \frac{(y+4)^2}{16} = 1$$

$a = 3$ $\quad$ $b = 4$

$(h, k) = (5, -4)$



$r = \frac{ep}{1 \pm e\cos\theta}$ $r = \frac{ep}{1 \pm e\sin\theta}$

$0 < e < 1 \Rightarrow$ ellipse

$e = 1 \Rightarrow$ parabola

$e > 1 \Rightarrow$ hyperbola

$$\frac{ep}{1 \pm e\cos\theta}$$

$$\frac{2}{2-\cos\theta} = \frac{2}{2(1-\frac{1}{2}\cos\theta)}$$

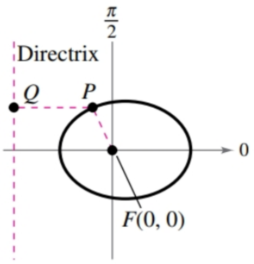
$$= \frac{1}{1-\frac{1}{2}\cos\theta} = \frac{(\frac{1}{2})(2)}{1-\frac{1}{2}\cos\theta} \rightarrow p = 2$$

$e = \frac{1}{2}$

$1 = \frac{1}{2} \cdot 2$

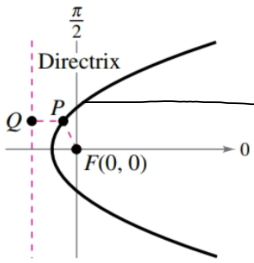
Alternative Definition of a Conic

The locus of a point in the plane that moves such that its distance from a fixed point (focus) is in a constant ratio to its distance from a fixed line (directrix) is a **conic.** The constant ratio is the *eccentricity* of the conic and is denoted by e. Moreover, the conic is an **ellipse** when $0 < e < 1$, a **parabola** when $e = 1$, and a **hyperbola** when $e > 1$. (See the figures below.)



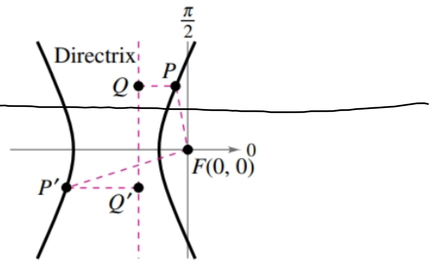
Ellipse: $0 < e < 1$

$\frac{PF}{PQ} < 1$



Parabola: $e = 1$

$\frac{PF}{PQ} = 1$



Hyperbola: $e > 1$

$\frac{PF}{PQ} = \frac{P'F}{P'Q'} > 1$

Polar Equations of Conics

The graph of a polar equation of the form

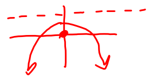
1. $r = \dfrac{ep}{1 \pm e\cos\theta}$ or **2.** $r = \dfrac{ep}{1 \pm e\sin\theta}$

is a conic, where $e > 0$ is the eccentricity and $|p|$ is the distance between the focus (pole) and the directrix.

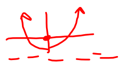
$$r = \frac{ep}{1 \pm e\cos\theta}$$ Vertical directrix

$$r = \frac{ep}{1 \pm e\sin\theta}$$ Horizontal directrix

1. Horizontal directrix above the pole: $r = \dfrac{ep}{1 + e\sin\theta}$



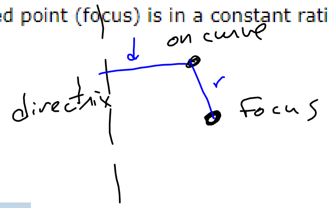
2. Horizontal directrix below the pole: $r = \dfrac{ep}{1 - e\sin\theta}$



3. Vertical directrix to the right of the pole: $r = \dfrac{ep}{1 + e\cos\theta}$
4. Vertical directrix to the left of the pole: $r = \dfrac{ep}{1 - e\cos\theta}$

The locus of a point in the plane that moves such that its distance from a fixed point (focus) is in a constant ratio to its distance from a fixe line (directrix) is a ---Select--- 1a **LarTrig10 6.9.001.** (3884513) 3884513)

$0<e<1 \Rightarrow$ ellipse
$e=1 \Rightarrow$ parabola
$e>1 \Rightarrow$ hyperbola



$\frac{r}{d}=e$

The constant ratio is the ---Select--- 2a **2. LarTrig10 6.9.002.** conic and is denoted by ---Select--- 2b

e = eccentricity
(Not Euler's 'e'.)

An equation of the form $r = \dfrac{ep}{1 - e\cos(\theta)}$ has a ---Select--- 3a vertical directrix

---Select--- 3b to the left of the pole .

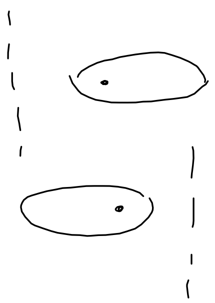
$\dfrac{ep}{1 \pm e\sin\theta}$ Horizontal directrix

Match the conic with its eccentricity.

(a) $0 < e < 1$ ellipse

(b) $e = 1$ parabola

(c) $e > 1$ Hyperbola



3. LarTrig10 6.9.003.

Write the polar equation of the conic for each value of e. Identify the type of conic represented by each equation. Verify your answers with a graphing utility.

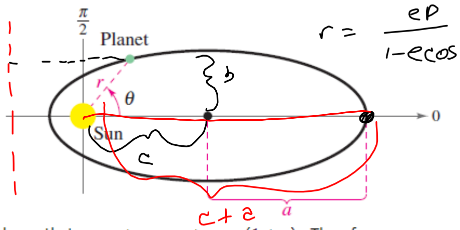
$$r = \frac{2e}{1 - e\cos(\theta)}$$

$e = 1$ $\quad r = \frac{2}{1-\cos\theta}$ parabola

$e = 0.5$ $\quad r = \frac{2e}{1-.5\cos\theta} = \frac{1}{1-.5\cos\theta}$ ellipse

$e = 1.5$ $\quad \frac{2e}{1-1.5\cos\theta} = \frac{3}{1-1.5\cos\theta}$ hyperbola

The planets travel in elliptical orbits with the sun at one focus. Assume that the focus is at the pole, the major axis lies on the polar axis, and the length of the major axis is $2a$ (see figure) Show that the polar equation of the orbit is $r = a(1 - e^2) / (1 - e \cos \theta)$, where e is the eccentricity.



$r = \frac{ep}{1-e\cos\theta}$

#52 WebAssign

#53 in book

want: $r = \frac{a(1-e^2)}{1-e\cos\theta}$

When $\theta = 0$, we know that $r = c + a = ea + a = a(1 + e)$. Therefore,

Want:

$ep = a(1-e^2) = a - ae^2$

$a^2 - b^2 = c^2 = a^2e^2$

$e = \frac{c}{a}$

$ae = c$

$a^2 - b^2 = c^2$

$r = \frac{ep}{1-\cos\theta}$

$ep = a(1-e^2)$?

$ep = a\left(1 - \frac{c^2}{a^2}\right)$

$a(1-e^2) = a\left(1 - \frac{c^2}{a^2}\right)$

$= a\left(\frac{a^2-c^2}{a^2}\right) = \frac{a^2-c^2}{a} = \frac{b^2}{a} \overset{?}{=} ep$

Hint: $\theta = 0 \Rightarrow$ on major axis & distance from focus to point on ellipse is $r = c + a = ea + a = a(1+e)$

$\therefore \quad a(1+e) = \frac{e0}{1-e\cos 0} = \frac{ep}{1-e} \Rightarrow$

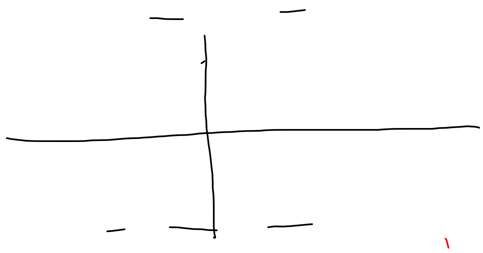
$a(1+e)(1-e) = ep \Rightarrow$

$a(1-e^2) = ep$

$r = \dfrac{2}{2+6\sin\left(\theta+\frac{2\pi}{3}\right)}$ $\cos\left(\theta+\frac{2\pi}{3}\right)$ what's happening?

$$\frac{2}{2+6\sin\theta} = \frac{2}{2(1+3\sin\theta)}$$

$$= \frac{1}{1+3\sin\theta} = \frac{3\cdot\frac{1}{3}}{1+3\sin\theta}$$



$\dfrac{1}{1+3\sin\theta}$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ gives you the endpoints of the major axis for an ellipse. or vertices for a hyperbola.

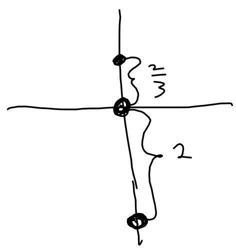
$\dfrac{ep}{1\pm e\cos\theta}$ $\theta = 0$ & π give you the same as above.

$\dfrac{1}{1+\frac{1}{2}\sin\theta}$ $ep = \frac{1}{2}p = 1$

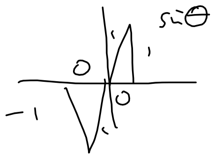
$p = 2$

$\theta = \frac{\pi}{2}$: $\frac{1}{1+\frac{1}{2}} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$

$\theta = \frac{3\pi}{2} = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$



$\sin\theta = \pm 1$



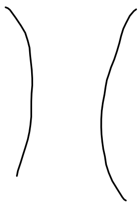
Length of major axis

$= 2 + \frac{2}{3} = \frac{6+2}{3} = \frac{8}{3} = 2a$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$$\frac{3}{1-1.5\cos\theta}$$



Graphing Calculator

MODE: Polar

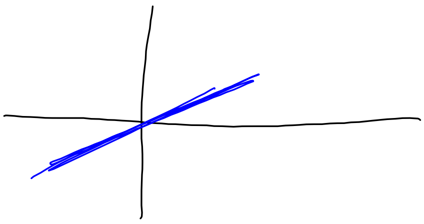
connected-vs-dot Mode

Includes stuff you don't want

Leaves out stuff you want.

$$\frac{ep}{1 \pm \cos\theta} \quad -\text{vs}- \quad \frac{ep}{1 \pm \cos(\theta + \frac{\pi}{4})}$$

Rotate clockwise $\frac{\pi}{4}$ radians



$$\frac{ep}{1 \pm \cos(\theta - \frac{\pi}{4})}$$

Counter-clockwise rotation by $\frac{\pi}{4}$