970-290-0550

No more in-person testing.

Sneaked SOME in, but college is on lock-down as far as we're concerned.

TEST 5 & Comprehensive Final will be available by MONDAY (Sooner for Test 5). WEBASSIGN.

Schedule's messed-up on harryzaims.com.

PLAN: TEST 5 over 6.6-6.9 stuff & Final Test will have Friday, 12/11 as deadline.

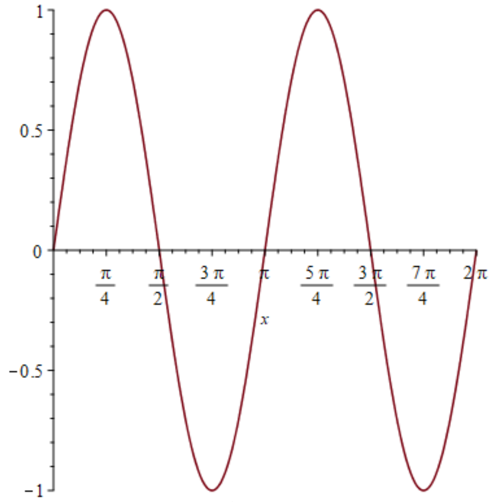
TEST 5 should be up tonight/tomorrow.

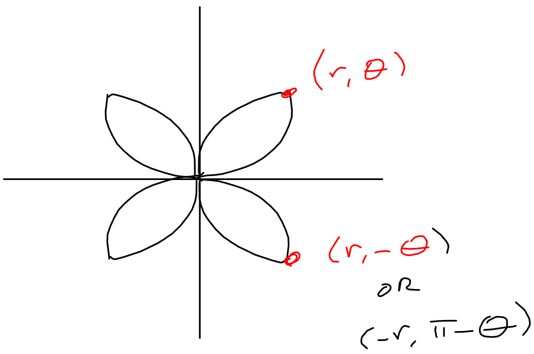
FINAL " " " by Mondy

I want you to have 2 tries, so you can review your 1st attempt.

One more section of new material: 6.9. I'll test on it, lightly. Will lecture over it next week.

More 6.8 graphs:





Symmetry $\theta = \frac{\pi}{2}$ (y-axis)

$(-r, -\theta)$:

$-r = \sin(2(-\theta)) = -\sin(2\theta)$

$r = \sin(2\theta)$ = ORIGINAL

Polar axis: (x-axis)

$(r, -\theta)$ OR $(-r, \pi - \theta)$

give equivalent eq'n.

$-r = \sin(2(\pi - \theta))$

$-r = \sin(2\pi - 2\theta)$

$= \boxed{\sin(2\pi)\cos(-2\theta)} + \sin(-2\theta)\cos(2\pi)$

$-r = -\sin 2\theta$

$r = \sin(2\theta)$ SAME!

$r = \frac{ep}{1 \pm e\cos\theta}$ $r = \frac{ep}{1 \pm e\sin\theta}$

$0 < e < 1 \Rightarrow$ ellipse

$e = 1 \Rightarrow$ parabola

$e > 1 \Rightarrow$ hyperbola

$$\frac{ep}{1 \pm e\cos\theta}$$

$$\frac{2}{2-\cos\theta} = \frac{2}{2(1-\frac{1}{2}\cos\theta)}$$

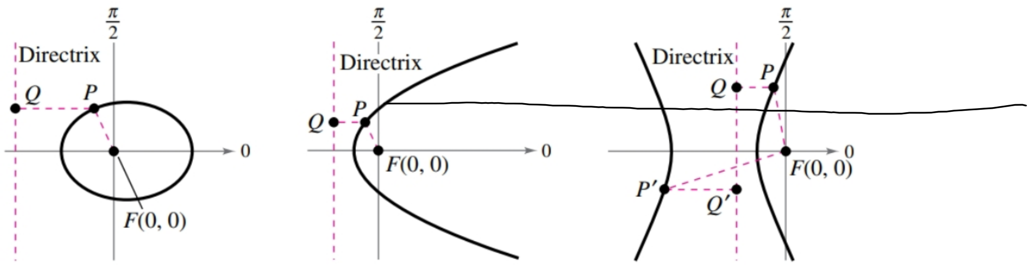
$$= \frac{1}{1-\frac{1}{2}\cos\theta} = \frac{(\frac{1}{2})(2)}{1-\frac{1}{2}\cos\theta} \rightarrow p = 2$$

$e = \frac{1}{2}$

$1 = \frac{1}{2} \cdot 2$

Alternative Definition of a Conic

The locus of a point in the plane that moves such that its distance from a fixed point (focus) is in a constant ratio to its distance from a fixed line (directrix) is a **conic.** The constant ratio is the *eccentricity* of the conic and is denoted by e. Moreover, the conic is an **ellipse** when $0 < e < 1$, a **parabola** when $e = 1$, and a **hyperbola** when $e > 1$. (See the figures below.)



Ellipse: $0 < e < 1$ $\quad \frac{PF}{PQ} < 1$

Parabola: $e = 1$ $\quad \frac{PF}{PQ} = 1$

Hyperbola: $e > 1$ $\quad \frac{PF}{PQ} = \frac{P'F}{P'Q'} > 1$

$$r = \frac{ep}{1 \pm e\cos\theta}$$ Vertical directrix

$$r = \frac{ep}{1 \pm e\sin\theta}$$ Horizontal directrix

The locus of a point in the plane that moves such that its distance from a fixed point (focus) is in a constant ratio to its distance from a fixe line (directrix) is a ---Select--- 1a **LarTrig10 6.9.001.** (3884513) 34513)

The constant ratio is the ---Select--- 2a **2. LarTrig10 6.9.002.** ne conic and is denoted by ---Select--- 2b

An equation of the form $r = \frac{ep}{1 - e\cos(\theta)}$ has a ---Select--- 3a vertical directrix ---Select--- 3b to the left of the pole .

Match the conic with its eccentricity.

(a) $0 < e < 1$

(b) $e = 1$

(c) $e > 1$

3. LarTrig10 6.9.003.

Write the polar equation of the conic for each value of e. Identify the type of conic represented by each equation. Verify your answers with a graphing utility.

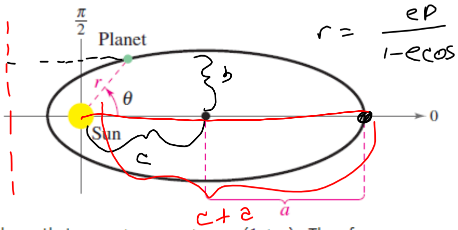
$$r = \frac{2e}{1 - e\cos(\theta)}$$

$e = 1$ $\quad r = \frac{2}{1-\cos\theta}$ parabola

$e = 0.5$ $\quad r = \frac{2e}{1-.5\cos\theta} = \frac{1}{1-.5\cos\theta}$ ellipse

$e = 1.5$ $\quad \frac{2e}{1-1.5\cos\theta} = \frac{3}{1-1.5\cos\theta}$ hyperbola

The planets travel in elliptical orbits with the sun at one focus. Assume that the focus is at the pole, the major axis lies on the polar axis, and the length of the major axis is $2a$ (see figure) Show that the polar equation of the orbit is $r = a(1 - e^2) / (1 - e \cos \theta)$, where e is the eccentricity.



$r = \frac{ep}{1 - e\cos\theta}$

#52 WebAssign

#53 in book

want: $r = \frac{a(1-e^2)}{1 - e\cos\theta}$

When $\theta = 0$, we know that $r = c + a = ea + a = a(1 + e)$. Therefore,

Want: $ep = a(1-e^2) = a - ae^2$

$a^2 - b^2 = c^2 = a^2e^2$

$e = \frac{c}{a}$

$ae = c$

$a^2 - b^2 = c^2$

$r = \frac{ep}{1 - \cos\theta}$

$ep = a(1-e^2)$?

$ep = a\left(1 - \frac{c^2}{a^2}\right)$

$a(1-e^2) = a\left(1 - \frac{c^2}{a^2}\right)$

$= a\left(\frac{a^2-c^2}{a^2}\right) = \frac{a^2-c^2}{a} = \frac{b^2}{a} \overset{?}{=} ep$

Hint: $\theta = 0 \Rightarrow$ on major axis & distance from focus to point on ellipse is $r = c + a = ea + a = a(1+e)$

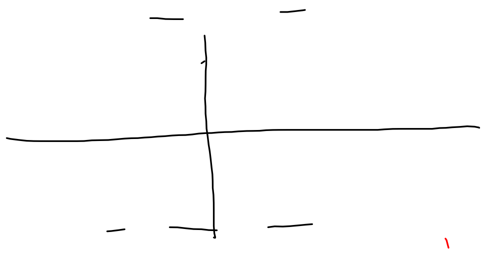
$\therefore \quad a(1+e) = \frac{ep}{1 - e\cos 0} = \frac{ep}{1-e} \Rightarrow$

$a(1+e)(1-e) = ep \Rightarrow$

$a(1-e^2) = ep$

$$r = \frac{2}{2+6\sin\left(\theta+\frac{2\pi}{3}\right)}$$

$\cos\left(\theta+\frac{2\pi}{3}\right)$ what's happening?



$$\frac{2}{2+6\sin\theta} = \frac{2}{2(1+3\sin\theta)}$$

$$= \frac{1}{1+3\sin\theta} = \frac{3\cdot\frac{1}{3}}{1+3\sin\theta}$$

$$\frac{1}{1+3\sin\theta}$$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ gives you the endpoints of the major axis for an ellipse. or vertices for a hyperbola.

$$\frac{1}{1+\frac{1}{2}\sin\theta}$$

$ep = \frac{1}{2}p = 1$

$p = 2$

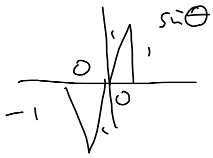
$\theta = \frac{\pi}{2}$: $\frac{1}{1+\frac{1}{2}} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$

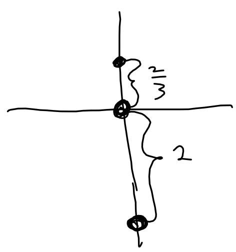
$\theta = \frac{3\pi}{2} = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$

$$\frac{ep}{1 \pm e\cos\theta}$$

$\theta = 0$ & π give you the same as above.

$\sin\theta = \pm 1$





Length of major axis

$= 2 + \frac{2}{3} = \frac{6+2}{3} = \frac{8}{3} = 2a$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$$r = \frac{2}{2+6\sin\left(\theta+\frac{2\pi}{3}\right)}$$

Identify the conic algebraically

$$\boxed{r = \frac{2}{2\left(1+3\sin\left(\theta+\frac{2\pi}{3}\right)\right)} = \frac{1}{1+3\sin\left(\theta+\frac{2\pi}{3}\right)}}$$

$$r = \frac{ep}{1 \pm e\sin\theta}$$

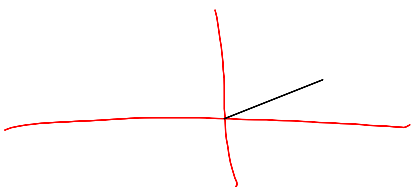
$e = 3 > 1 \Rightarrow$ hyperbola

$$\frac{ep}{1 \pm e\sin(\theta+\phi)}$$

Rotation clockwise by ϕ

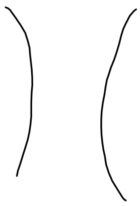
$$\frac{ep}{1 \pm e\sin(\theta-\phi)}$$

Rotate counter-clockwise by ϕ



Rotating in Rectangular coords is TOUGH!

$$\frac{3}{1-1.5\cos\theta}$$



Graphing Calculator

MODE: Polar

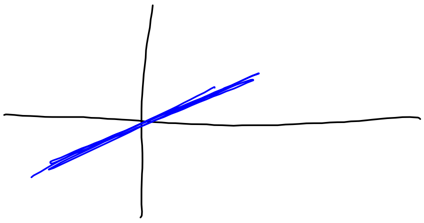
connected -vs- dot Mode

Includes stuff you don't want

Leaves out stuff you want.

$$\frac{ep}{1 \pm \cos\theta} \quad -vs- \quad \frac{ep}{1 \pm \cos(\theta + \frac{\pi}{4})}$$

Rotate clockwise $\frac{\pi}{4}$ radians



$$\frac{ep}{1 \pm \cos(\theta - \frac{\pi}{4})}$$

Counter-clockwise rotation by $\frac{\pi}{4}$