Test 3 Bonus is up. It's based on 3.3 #94. Similar to Test 3 #4 (?) that I didn't think y'all were ready for.

Test 5 will be available soon. Let me know if there's a test you wish to re-take previous tests!

In-person tests require in-person re-take.

In-person or online testing? Inquiring minds want to know!
Teacher sez: "Let me check. I WANT in-person, but that may have been taken away.
Will check."

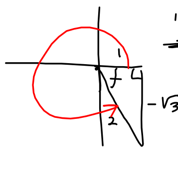
Test 4 questions

$x^5 - (1-\sqrt{3}i) = 0$

$x^5 = 1-\sqrt{3}i$

Principle Root!

$\sqrt[5]{x^5} = x = \sqrt[5]{1-\sqrt{3}\,i}$



$1^2+\sqrt{3}^2 = 1+3=4=c^2$

$\Rightarrow c = \pm\sqrt{4} = 2$

$\theta = -60^\circ = -\frac{\pi}{3}$

OR

$\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$= 300^\circ$

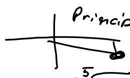
Generally prefer (convention)

$0 \le \theta < 2\pi$

300° OR $\frac{5\pi}{3}$

$r = 2$

$\theta = -\frac{\pi}{3}$ OR $\frac{5\pi}{3}$



$\boxed{\frac{\theta}{5} = -\frac{\pi}{15}}$ 1

inc. $= \frac{2\pi}{5} = \frac{6\pi}{15}$

Increment is $\frac{2\pi}{5} = \frac{6\pi}{15}$

Principle Root:

$\frac{\frac{5\pi}{3}}{5} = \frac{5\pi}{3}\cdot\frac{1}{5}$

$= \frac{\pi}{3}$

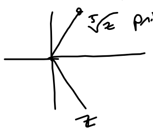
$-\frac{\pi}{15} + \frac{6\pi}{15} = \frac{5\pi}{15} = \boxed{\frac{\pi}{3}}$ 2

$\frac{5\pi}{15} + \frac{6\pi}{15} = \boxed{\frac{11\pi}{15}}$ 3

$\frac{11\pi}{15} + \frac{6\pi}{15} = \boxed{\frac{17\pi}{15}}$ 4

$\frac{17+6}{15}\pi = \boxed{\frac{23\pi}{15}}$ 5

$\frac{23+6}{15}\pi = \boxed{\frac{29\pi}{15}}$ 6 coterminal w/ $-\frac{\pi}{15}$



$\frac{\pi}{3} + \frac{2\pi}{5} = \frac{5\pi+6\pi}{15} = \frac{11\pi}{15}$

$\frac{11+6}{15}\pi = \frac{17\pi}{15}$

$\frac{17+6}{15}\pi = \frac{23\pi}{15}$

$\frac{23+6}{15}\pi = \frac{29\pi}{6}$

$z = 2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$

$\sqrt[5]{z} = \sqrt[5]{2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right),$

Then $\sqrt[5]{2}\left(\cos\frac{11\pi}{15} + i\sin\frac{11\pi}{15}\right)$

etc.

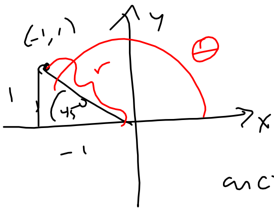
$\sqrt[5]{\sqrt{2}}$

$\sqrt[5]{\sqrt{2}} = \left(2^{\frac{1}{2}}\right)^{\frac{1}{5}}$

$= 2^{\frac{1}{2}\cdot\frac{1}{5}} = 2^{\frac{1}{10}}!$

§6.7 - Polar Coordinates

$(x, y) \longmapsto (r, \theta)$



$x = r\cos\theta$ $\quad x^2 + y^2 = r^2$

$y = r\sin\theta$ $\quad \frac{y}{x} = \tan\theta$

$\arctan\left(\frac{y}{x}\right) = \arctan\left(\frac{1}{-1}\right) = -45^\circ = -\frac{\pi}{4}$

We're in QII, so

$\arctan\left(\frac{y}{x}\right) + \pi = \frac{3\pi}{4}$

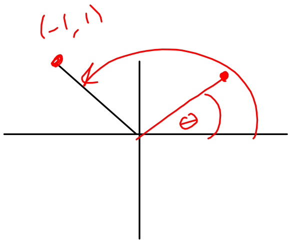
$\arctan\left(\frac{y}{x}\right) + 180^\circ = 135^\circ$

$r^2 = x^2 + y^2 = 1^2 + 1^2 = 2 \implies r = \pm\sqrt{2}$

$(r; \theta) = \left(\sqrt{2}, \frac{3\pi}{4}\right)$ is not unique!

$= \left(-\sqrt{2}, -\frac{\pi}{4}\right)$ Negative length!

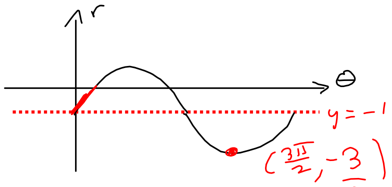
$= \left(\sqrt{2}, \frac{3\pi}{4} + 2\pi n\right) \quad n \in \mathbb{Z}$

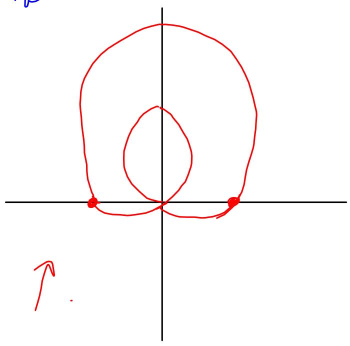


oops! Not quite ready for symmetry!

$r = 2\sin\theta - 1$

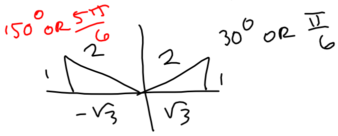
1. Make rectangular graph.
2. " polar graph

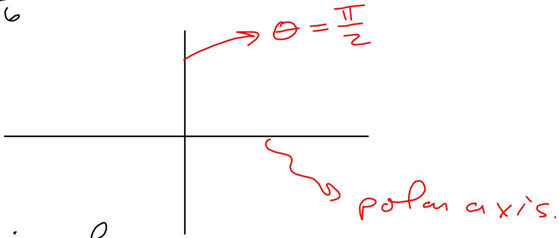




$2\sin\theta = 1$

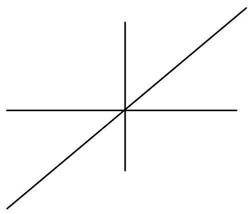
$\sin\theta = \frac{1}{2}$





Line thru the origin in polar coordinates:

θ = constant, e.g., $\theta = \frac{\pi}{4}$



$\tan\frac{\pi}{4} = \frac{y}{x} = 1$

$\boxed{y = x}$ Rectangular.

Circle

$\underbrace{x^2+y^2}_{} = 4$

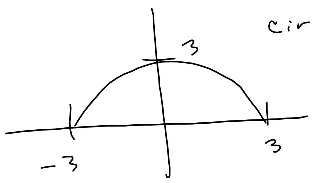
$r^2 = 4$

$r = \pm 2$

Polar coords are GREAT for circular objects.

$r = 2$ captures it all!

circle of radius 3



rectangular

$x^2+y^2=3^2 \Longrightarrow y = \pm\sqrt{9-x^2}$

Top half. $= \sqrt{9-x^2}$ TOP HALF

Find the area.

$r = 3$

Area from $x=-3$ to $x=+3$ of $\sqrt{9-x^2}$ is hard in calculus.

In polars:

$\left.\begin{array}{l} 0 \le r \le 3 \\ 0 \le \theta \le \pi \end{array}\right\}$ Noice!

$\left(3\left(\cos(12^\circ)+i\sin(12^\circ)\right)\right)^4 =$

$3^4\left(\cos(48^\circ)+i\sin(48^\circ)\right)$

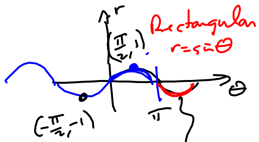
Re-takes for Test 4 coming soon!

Graphing in Polar Coordinates.

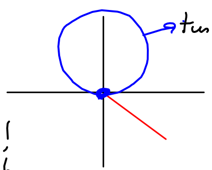
KEY: Graphing in Rectangular coordinates & seeing the "loops!"

$r=\sin\theta$ in polar coords.

First graph in rectangular coords



r is negative! Flip through the pole! (origin)



turns out it's a circle!

See Symmetry, S6.8 pg 2 (Pg 478 in 10^{th} Ed.)

Symmetry about $\theta=\frac{\pi}{2}$ (Symmetry about y-axis)

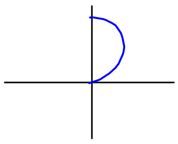
$r=\sin\theta \qquad (r,\theta)\mapsto(-r,-\theta)$

$-r=\sin(-\theta)=-\sin\theta$

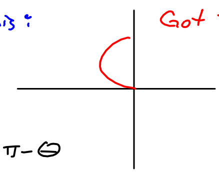
$\Rightarrow r=\sin\theta=$ Same!

(Equivalent Equation!)

That means all I needed to graph was from 0 to $\frac{\pi}{2}$ for this one.



gives us this:

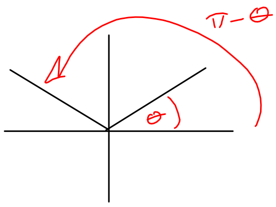


Got this for free!

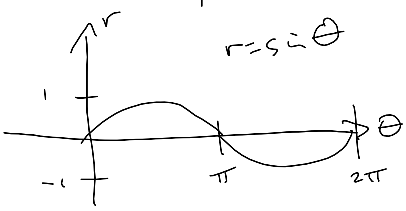
Alternate: Replace θ by $\pi-\theta$

$r=\sin\theta \mapsto r=\sin(\pi-\theta)=\sin\pi\cos(-\theta)+\sin(-\theta)\cos\pi$

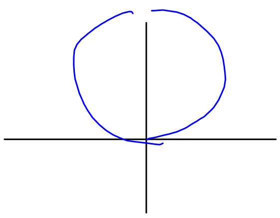
$=0\cdot\overset{\text{EVEN}}{\cos\theta}-(\sin\theta)(-1)=\sin\theta$! Same!



Mirrors across $\theta=\frac{\pi}{2}$ axis (which used to be y-axis)



$r=\sin\theta$

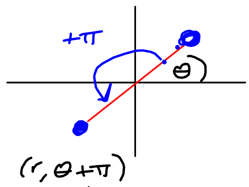


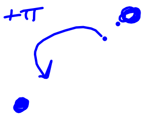
Symmetry

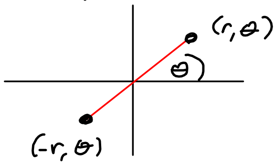
Symmetry through the pole:

$(r,\theta) \longmapsto (r, \theta+\pi)$

$(r,\theta) \longmapsto (-r, \theta)$







You see this kind of symmetry in something like

$r^2 = \sin\theta$

$(-r)^2 = \sin\theta$

$(-1)^2 r^2 = r^2 = \sin\theta$ Same!

Symmetry:

Pole: $r^2 = \sin\theta$

$r^2 = \sin(\theta+\pi)$

$= \sin\theta\cos\pi + \cancel{\sin\pi\cos\theta}$

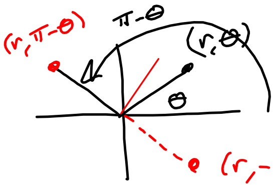
$r^2 = -\sin\theta$ NO

$(-r)^2 = \sin\theta$

$r^2 = \sin\theta$ = Same as original! So, symmetric thru origin.

3rd Kind of Symmetry:

Thru the polar axis (x-axis)

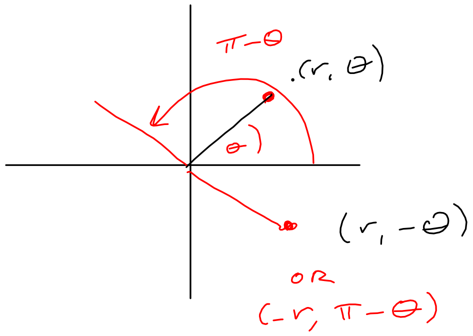


Bottom half of picture from top half.

$(r,\theta) \longmapsto (r, -\theta)$

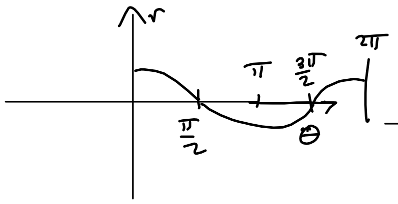
OR

$(r,\theta) \longmapsto (-r, \pi-\theta)$

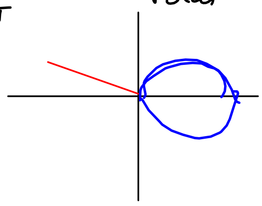


If one holds up, that's good enough.

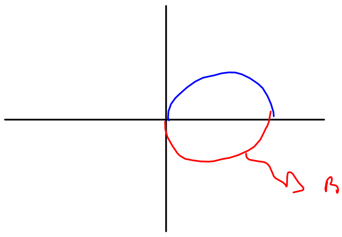
$r = \cos\theta \leadsto r = \cos(-\theta) = \cos\theta$!



Polar



Circle!



$r = \cos\theta$

$r = \cos(-\theta) = \cos\theta$!

$r = \cos\theta$!

It's enough for ONE Test to "work"

$(-r, \pi-\theta)$:

$(-r) = \cos(\pi-\theta)$

$-r = \cos\pi\cos(-\theta) - \cancel{\sin\pi\sin(-\theta)}$

$-r = -1\cos\theta$

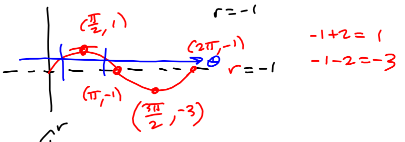
$r = \cos\theta$! Cool!

Amp = 2

$r = 2\sin\theta - 1$

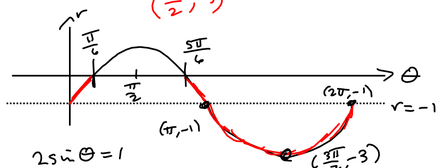
$\to y = -1$ midline

$r = -1$



$-1 + 2 = 1$

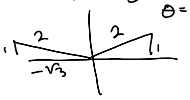
$-1 - 2 = -3$



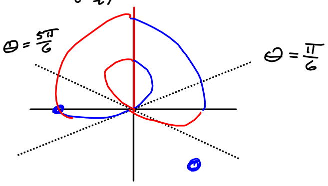
$2\sin\theta = 1$

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$



$\theta = \frac{5\pi}{6}$



$r = 2\sin\theta - 1$

$r = 2\sin(\pi - \theta) - 1$

$= 2(\sin\pi\cos(-\theta) + \sin(-\theta)\cos\pi) - 1$

$= 2\sin\theta - 1$! woo-hoo!

Symmetry thru $\theta = \frac{\pi}{2}$

$r = 2\sin(-\theta) - 1$ Polar Axis?

$= -2\sin\theta - 1$ not the same

$-r = 2\sin(\pi - \theta) - 1$

$-r = 2\sin\theta - 1$ not same.

Thru pole?

$r = 2\sin(\pi + \theta) - 1$

$= 2(\sin\pi\cos\theta + \sin\theta\underbrace{\cos\pi}_{-1}) - 1$

$= 2(-\sin\theta) - 1$ No

Try $(-r, \theta)$

$-r = 2\sin\theta - 1$ not same

Section 6.9 is the last bit.