Test 4 Open Dates:

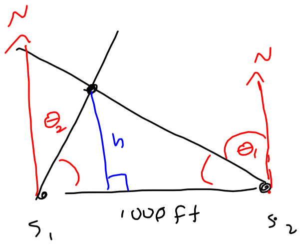
Wednesday-Friday, 11/18-11/20

Use WebAssign.

I think I'll do a separate vector "test" that has something like #4 from Test 3.

Navigation question or the hanging-weight problem.

I also want to put a fire-spotter-triangulation question as bonus on Test 4.



ASA situation

Law of sines slays it.

$h = ?$

4.1

11. 0/1 points

Perform the operation and write the result in standard form.

$$-\left(\frac{3}{2}+\frac{5}{2}i\right)+\left(\frac{5}{3}+\frac{10}{3}i\right)$$

$$=\left(-\frac{3}{2}+\frac{5}{3}\right)+\left(-\frac{5}{2}+\frac{10}{3}\right)i$$

$$=\frac{-9+10}{6}+\left(\frac{-15+20}{6}\right)i$$

$$=\frac{1}{6}+\frac{5}{6}i \quad = \frac{1}{6}+i\cdot\frac{5}{6}$$

$$=\frac{1}{6}+\left(\frac{5}{6}\right)i$$

12. 0/1 points

Perform the operation and write the result in standard form.

$13i(1-9i)$

$$=13i-13\cdot 9\cdot i^2 = 13i+117$$

$$\boxed{=117+13i}$$

14. 0/1 points

Perform the operation and write the result in standard form.

$(6+7i)^2$

$$(a+b)^2 = a^2+2ab+b^2$$

$$(a-b)^2 = a^2-2ab+b^2$$

$$=6^2+2(6)(7i)+(7i)^2$$

$$=36+84i-49$$

$$\boxed{=-13+84i}$$

17. 0/1 points

Write the quotient in standard form.

$$\left(\frac{9}{i}\right)\left(\frac{-i}{-i}\right) = \frac{-9i}{-i^2} = \frac{-9i}{1} = -9i$$

18. 0/1 points

Write the quotient in standard form.

$$\left(\frac{6+i}{6-i}\right)\left(\frac{6+i}{6+i}\right) = \frac{6^2+2\cdot 6i+i^2}{6^2+1^2} = \frac{36+12i-1}{36+1}$$

$6+i = 6+1i$

$$= \boxed{\frac{35}{37} + \frac{12}{37}i}$$

22. 0/1 points

Write the complex number in standard form. (Simplify your answer completely.)

$(2+\sqrt{-7})(8-\sqrt{-14})$ ~~$= 16 - 2i\sqrt{14}$~~

$$16 + 7\sqrt{2} + i\left(8\sqrt{7} - 2\sqrt{14}\right)$$

$\sqrt{7\cdot 7\cdot 2} = 7\sqrt{2}$

$= (2+i\sqrt{7})(8-i\sqrt{14}) = 16 - 2i\sqrt{14} + 8i\sqrt{7} - i^2\sqrt{7}\sqrt{14}$

$\sqrt{7}i$

$\sqrt{7i}$ is BAD

$= 16 + 6i\sqrt{14} + 7\sqrt{2}$ No! $\sqrt{14}$ & $\sqrt{7}$ are NOT like terms.

$= 16 + 7\sqrt{2} +$

Try again:

$= 16 + -2i\sqrt{14} + 8i\sqrt{7} + 7\sqrt{2}$

$= 16 + 7\sqrt{2} + (-2\sqrt{14} + 8\sqrt{7})\,i$

5. 0/1 points

Determine the number of solutions of the equation in the complex number system.

$x^4 + 5x^2 + 14 = 0$

$4 = n =$ degree

By FTA.

It's like pullin' teeth to get students to develop a strong quadradic-formula style. DO THE DISCRIMINANT, FIRST, IF YOU'RE GOING TO USE THE QUADRATIC FORMULA!

6. 0/1 points

Use the discriminant to determine the number of real solutions of the quadratic equation.

$4x^2 - x - 4 = 0$

$a = 4,\ b = -1,\ c = -4 \Rightarrow$

$D = b^2 - 4ac = (-1)^2 - 4(4)(-4)$

$= 1 + 64 = 65 > 0 \Rightarrow \boxed{2 \text{ real sol'ns}}$

8. 0/1 points

Solve the quadratic equation. (Enter your answers as a comma-separated list.)

$10x^2 - 1 = 0$

$x =$ ✖ $\frac{1}{\sqrt{10}}, -\frac{1}{\sqrt{10}}$

$10x^2 = 1$

$x^2 = \frac{1}{10}$

$x = \pm \frac{1}{\sqrt{10}}$

Didn't rationalize denominator in their answers.

$10x^2 - 1 = 0$

$10x^2 + 0x - 1 = 0$

$a = 10,\ b = 0,\ c = -1$

$b^2 - 4ac =$

$0^2 - 4(10)(-1)$

$= 40$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{0 \pm \sqrt{40}}{2(10)} = \frac{\pm 2\sqrt{10}}{20} = \boxed{\pm \frac{\sqrt{10}}{10}} = \pm \frac{1}{\sqrt{10}}$

is rationalized solution

$2 \mid 40$
$2 \mid 20$
$2 \mid 10$
5

9. 0/1 points

Solve the quadratic equation. (Enter your answers as a comma-separated list.)

$(x + 9)^2 - 7 = 0$

m1

$(x+9)^2 = 7$

$x+9 = \pm\sqrt{7}$

$\boxed{x = -9 \pm \sqrt{7}}$

Book/webassign wants $-9+\sqrt{7}, -9-\sqrt{7}$

10. 0/1 points

Solve the quadratic equation. (Enter your answers as a comma-separated list.)

$16x^2 + 8x + 1 = 0$

$(4x)^2 + 2(4x)(1) + 1^2 = (4x+1)^2$ SET 0

Perfect-Square Trinomial

$\Rightarrow 4x+1 = 0$

$4x = -1$

$\boxed{x = -\frac{1}{4}}$

$16x^2 + 8x + 1 = 0$

$a = 16,\ b = 8,\ c = 1$

$b^2 - 4ac = 8^2 - 4(16)(1)$

$= 64 - 64 = 0$

$x = \frac{-8 \pm \sqrt{0}}{2(16)} = \frac{-8}{32} \boxed{= -\frac{1}{4} = x}$

QUADRATIC IN FORM

12. 0/1 points

Solve the polynomial equation. (Enter your answers as a comma-separated list.)

$$x^4 - 4x^2 - 5 = 0 \qquad u = x^2$$

$u^2 - 4u - 5 = (u-5)(u+1) = 0 \Rightarrow u \in \{-1, 5\}$

$a = 1, b = -4, c = -5$

$b^2 - 4ac = (-4)^2 - 4(1)(-5)$

$= 16 + 20 = 36$

$u = \frac{4 \pm \sqrt{36}}{2(1)} = \frac{4 \pm 6}{2}$ $\frac{10}{2} = 5 = u$, $\frac{-2}{2} = -1 = u$

$u = x^2 = -1 \Rightarrow$

$x = \pm\sqrt{-1} = \boxed{\pm i = x}$

$u = x^2 = 5 \Rightarrow$

$\boxed{x = \pm\sqrt{5}}$

Completing the Square.

$u^2 - 4u - 5 = 0$

$u^2 - 4u = 5$

$u^2 - 4u + (2)^2 = 5 + 4$

$(u-2)^2 = 9$

$u - 2 = \pm 3$

$u = 2 \pm 3$ → 5, -1

15. 0/2 points

Write the polynomial as the product of linear factors.

$$f(x) = 2x^3 - x^2 + 96x - 48$$

$f(x) =$ $\left(x - 4i\sqrt{3}\right)\left(x + 4i\sqrt{3}\right)(2x - 1)$

List all the zeros of the function. (Enter your answers as a comma-separated list.)

$x =$ $\frac{1}{2}, 4i\sqrt{3}, -4i\sqrt{3}$

Book way is clever, but not as generally useful.

16. 0/1 points LarTrig10 4.2.054. [3883162]

Use the given zero to find all the zeros of the function. (Enter your answers as a comma-separated list. Include the given zero in your answer.)

Function	Zero
$g(x) = 5x^3 + 19x^2 + 21x - 5$	$-2 + i$

$\Rightarrow -2-i$

$$\begin{array}{r|rrrr} -2+i & 5 & 19 & 21 & -5 \\ & & -10+5i & -23-i & 5 \\ \hline -2-i & 5 & 9+5i & -2-i & 0 \\ & & -10-5i & 2+i & \\ \hline & 5 & -1 & 0 & \end{array}$$

$5x - 1 = 0$

$5x = 1$

$x = \frac{1}{5}$

$x = \frac{1}{5}, -2+i, -2-i$

$(9+5i)(-2+i)$
$= -18 + 9i - 10i + 5i^2$
$= -18 - i - 5$
$= -23 - i$

$(-2-i)(-2+i) = 2^2 + 1^2 = 5$

FACTORED FORM $(x-\frac{1}{5})(x-(-2+i))(x-(-2-i))$ is what I like

BOOK WANTS IT EXPANDED.

18. 0/1 points LarTrig9 4.2.080. [2456305]

Find a cubic polynomial function f with real coefficients that has the given complex zeros and x-intercept. (There are many correct answers.)

Complex Zeros	x-Intercept
$x = -3 \pm \sqrt{3}i$	$(-2, 0)$

$x = -2$ is a zero.

$x + 2$ is a factor.

$f(x) =$ ✗ $\boxed{x^3 + 8x^2 + 24x + 24}$

$$f(x) = (x+2)\underbrace{(x-(-3+\sqrt{3}i))(x-(-3-\sqrt{3}i))}_{\text{Book's trick:}}$$

$(x-(-3+\sqrt{3}i))(x-(-3-\sqrt{3}i))$

$= (x+3-\sqrt{3}i)(x+3+\sqrt{3}i)$

$= ((x+3) - \sqrt{3}i)((x+3) + \sqrt{3}i)$

$(a - bi)(a+bi) = a^2 + b^2$

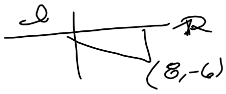
$= (x+3)^2 + (\sqrt{3})^2 = x^2 + 6x + 9 + 3$

$= x^2 + 6x + 12$

$f(x) = (x+2)(x^2+6x+12)$

$$\begin{array}{r} = x^3 + 6x^2 + 12x \\ 2x^2 + 12x + 24 \\ \hline x^3 + 8x^2 + 24x + 24 \end{array} = f(x)$$

4.3



$|8 - 6i| =$ [ ] 10

$|a+bi| = \sqrt{a^2+b^2} = \sqrt{(a+bi)(a-bi)} = \sqrt{8^2+6^2} = \sqrt{64+36}$

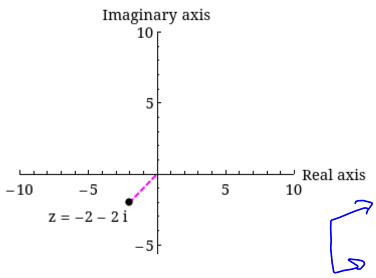
$= \sqrt{100} = \boxed{10 = |8-6i|}$

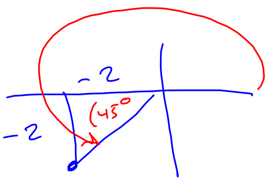
7. 0/1 points

Write the complex number in trigonometric form. (Enter your angle measures in radians.)

$z =$ [ ] $2\sqrt{2}\left(\cos\left(\frac{5\pi}{4}\right) + i\sin\left(\frac{5\pi}{4}\right)\right)$

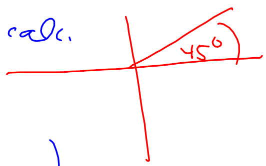
$180°+45° = 225° = \frac{5\pi}{4}$





$\arctan\left(\frac{-2}{-2}\right) = 45°$

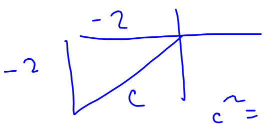
calc.



$(-2,-2)$

$-2-2i$

$(-2)^2+(-2)^2$

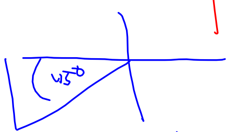


$c^2 = 2^2 + 2^2$

$= 8$

$c = \sqrt{8} = \sqrt{2 \cdot 2 \cdot 2}$

$\boxed{2\sqrt{2} = r}$

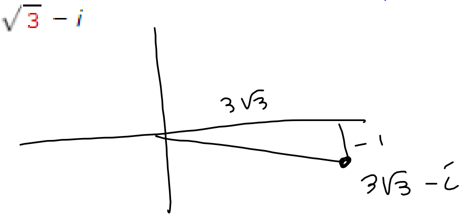


$z = 2\sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$

9. 0/2 points

Plot the complex number.

$3\sqrt{3} - i$



14. 0/1 points

Find the product. Leave the result in trigonometric form. (Let $0 \le \theta < 2\pi$.)

$$\left[2\left(\cos\left(\frac{\pi}{8}\right) + i\sin\left(\frac{\pi}{8}\right)\right)\right]\left[6\left(\cos\left(\frac{\pi}{24}\right) + i\sin\left(\frac{\pi}{24}\right)\right)\right]$$

$$12\left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right)$$

$$\frac{\pi}{8}\cdot\frac{3}{3} + \frac{\pi}{24} = \frac{4\pi}{24} = \frac{\pi}{6}$$

$$= 12\left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right)$$

16. 0/1 points

Find the quotient. Leave the result in trigonometric form. (Let $0 \le \theta < 2\pi$.)

$$\frac{8(\cos(40°) + i\sin(40°))}{16(\cos(10°) + i\sin(10°))}$$

$$\frac{1}{2}(\cos(30°) + i\sin(30°))$$

$$\frac{8}{16} = \frac{1}{2}$$

$$\begin{array}{r} 40° \\ -\ 10° \\ \hline 30° \end{array}$$

$$\frac{1}{2}\left(\cos(30°) + i\sin(30°)\right)$$

19. 0/4 points LarTrig10 4.4.057. [3883325]

Consider the following.

$$\frac{8 + 15i}{1 - \sqrt{3}i}$$

(a) Write the trigonometric forms of the complex numbers. (Let $0 \le \theta < 2\pi$. Round your angles to three decimal places.) ✓

$8 + 15i$ = ✗ $17(\cos(1.081) + i\sin(1.081))$

$1 - \sqrt{3}i$ = ✗ $2\left(\cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{5\pi}{3}\right)\right)$

$\sqrt{8^2+15^2} = \sqrt{64+225} = \sqrt{289} = 17$

$\arctan\left(\frac{15}{8}\right)$ = exact value.

(b) Perform the indicated operation using the trigonometric forms. (Let $0 \le \theta < 2\pi$. Round your angles to three decimal places.)

✗ $\frac{17}{2}(\cos(2.128) + i\sin(2.128))$

```
tan⁻¹(15/8)
     1.080839001
```

$\to \theta \approx 1.081$

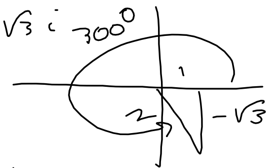
(c) Perform the indicated operation using the standard forms, and check [Round all numerical values to three decimal places.)

✗ $-4.495 + 7.214i$

$z_1 \approx 17(\cos(1.081) + i\sin(1.081))$

$\frac{z_1}{z_2} = \frac{17}{2}\left(\cos\left(1.08... - \frac{5\pi}{3}\right) + i\sin\left(1.08... - \frac{5\pi}{3}\right)\right)$

$1 - \sqrt{3}i$ 300°



$(300^\circ)\left(\frac{\pi}{180^\circ}\right) = \frac{5\pi}{3}$

$z_2 = 2\left(\cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{5\pi}{3}\right)\right)$

```
tan⁻¹(15/8)
     1.080839001
17/2*(cos(Ans-5π
/3)+isin(Ans-5π/
3))
-4.495190528+7.…
```

```
tan⁻¹(15/8)
     1.080839001
17/2*(cos(Ans-5π
/3)+isin(Ans-5π/
3))
…28+7.214101615i
```

```
/3)+isin(Ans-5π/
3))
…28+7.214101615i
tan⁻¹(15/8)
     1.080839001
Ans-5π/3
    -4.155148755
```

$\to \theta \approx -4.155 \notin [0, 2\pi)$

$2\pi - 4.155...$

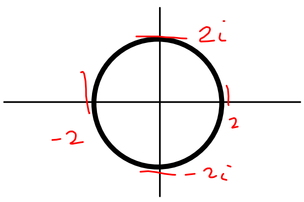
≈ 2.128036552

```
…28+7.214101615i
tan⁻¹(15/8)
     1.080839001
Ans-5π/3
    -4.155148755
Ans+2π
     2.128036552
```

20. 0/1 points

Sketch the graph of all complex numbers z satisfying the given condition.

$|z| = 2$



4.4

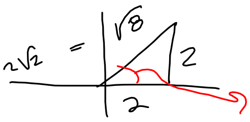
5. 0/1 points LarTrig10 4.5.0

Use DeMoivre's Theorem to find the indicated power of the complex number. Write the result in standard form.

$(2 + 2i)^6$

✗ $-512i$

$2\sqrt{2} = \sqrt{8}$



$\arctan\left(\frac{2}{2}\right) = 45^\circ$ OR $\frac{\pi}{4}$

$$\left(2\sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)\right)^6$$

$$= 2^6\left(2^{\frac{1}{2}}\right)^6\left(\cos\left(\frac{3\pi}{2}\right) + i\sin\left(\frac{3\pi}{2}\right)\right) = 64 \cdot 2^3 \cdot (0 - i) = \boxed{= -512i}$$

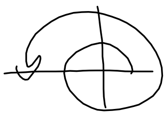
$6\left(\frac{\pi}{4}\right) = \frac{3\pi}{2}$

8. 0/1 points LarTrig10 4.5.

Use DeMoivre's Theorem to find the indicated power of the complex number. Write the result in standard form.

$$\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^6 = \cos(3\pi) + i\sin(3\pi) = \boxed{-1}$$

$6\left(\frac{\pi}{2}\right) = 3\pi$



11. 0/7 points LarTrig10 4.5.041. [3883400]

Consider the following.

Cube roots of $27\left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}\right)$

3rd root: $\frac{\theta}{3}$, $\frac{2\pi}{3}$

(a) Use the formula $z_k = \sqrt[n]{r}\left(\cos \frac{\theta + 2\pi k}{n} + i \sin \frac{\theta + 2\pi k}{n}\right)$ to find the indicated roots of the complex number.

(Enter your answers in trigonometric form. Let $0 \le \theta < 2\pi$.)

$z_0 =$ $3\left(\cos \frac{5\pi}{18} + i \sin \frac{5\pi}{18}\right)$

$z_1 =$ $3\left(\cos \frac{17\pi}{18} + i \sin \frac{17\pi}{18}\right)$

$z_2 =$ $3\left(\cos \frac{29\pi}{18} + i \sin \frac{29\pi}{18}\right)$

$\sqrt[n]{r}\left(\cos\left(\frac{\theta+2k\pi}{n}\right) + i\sin\left(\frac{\theta+2k\pi}{n}\right)\right)$

$k = 0, 1, \ldots, n-1$

Increment: $\frac{2\pi}{3}$. $\theta = \frac{5\pi}{6}$

z_0: $\sqrt[3]{27}\left(\cos\frac{5\pi}{18} + i\sin\frac{5\pi}{18}\right) = z_0$

$\frac{5\pi}{6} \div 3 = \frac{5\pi}{18}$

(b) Write each of the roots in standard form. (Round all numerical values to four decimal places.)

BORING!

$z_0 =$ $1.9284 + 2.2981i$

$z_1 =$ $-2.9544 + 0.5209i$

$z_2 =$ $1.0261 - 2.8191i$

$\frac{2\pi}{3} \cdot \frac{6}{6} = \frac{12\pi}{18}$

$\frac{5\pi}{18} + \frac{12\pi}{18} = \frac{17\pi}{18}$

$\frac{17\pi}{18} + \frac{12\pi}{18} = \frac{29\pi}{18}$

12. 0/11 points LarTrig10 4.5.044. [388318

Consider the following.

Fifth roots of $243\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$

$r = 243 = 3^5$

$$\begin{array}{r|l} 3 & 243 \\ 3 & 81 \\ 3 & 27 \\ 3 & 9 \\ & 3 \end{array}$$

(a) Use the formula $z_k = \sqrt[n]{r}\left(\cos \frac{\theta + 2\pi k}{n} + i \sin \frac{\theta + 2\pi k}{n}\right)$ to find the indicated roots of the complex number.

(Enter your answers in trigonometric form. Let $0 \le \theta < 2\pi$.)

$z_0 =$ $3\left(\cos \frac{3\pi}{20} + i \sin \frac{3\pi}{20}\right)$

$z_1 =$ $3\left(\cos \frac{11\pi}{20} + i \sin \frac{11\pi}{20}\right)$

$z_2 =$ $3\left(\cos \frac{19\pi}{20} + i \sin \frac{19\pi}{20}\right)$

$z_3 =$ $3\left(\cos \frac{27\pi}{20} + i \sin \frac{27\pi}{20}\right)$

$z_4 =$ $3\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right)$

$\sqrt[5]{r} = 3$

$\frac{2\pi}{5}$ is increment

$\frac{3\pi}{4} \div 5 = \frac{3\pi}{20}$: z_0

z_1: $\frac{3\pi}{20} + \frac{2\pi}{5}\cdot\frac{4}{4} = \frac{(3+8)\pi}{20} = \frac{11\pi}{20}$

z_2: $\frac{11+8}{20} = \frac{19}{20}$

(b) Write each of the roots in standard form. (Round all numerical values to four decimal places.)

z_3: $19+8 = 27$

z_4: $27+8 = 35$

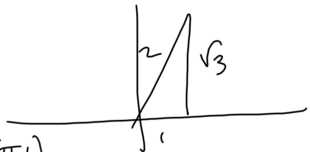
$\frac{35\pi}{20} = \frac{7\pi}{4}$

10. 0/1 points

Find the square roots of the complex number. (Enter your answers as a comma-separated list.)

$1 + \sqrt{3}i$

$-\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}i, \frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}i$



$60°$ or $\frac{\pi}{3}$

$r = 2$

So $\sqrt{2}\left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right)$

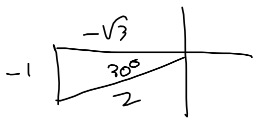
$\left(\frac{\pi}{3} \div 2 = \frac{\pi}{6}\right)$

$= \sqrt{2}\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = \frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}i$

$\frac{\pi}{6} + \frac{2\pi}{2} = \frac{\pi + 6\pi}{6} = \frac{7\pi}{6}$

Increment

$\sqrt{2}\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$



$= \sqrt{2}\left(-\frac{\sqrt{3}}{2} + -\frac{1}{2}i\right)$

$= -\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}i$