Section 4.2 Stuff. Book has some tricks for some of the messier ones.

17. 0/1 points LarTrig10 4.2.061. [3883013]

Find a polynomial function with real coefficients that has the given zeros. (There are many correct answers.)

3, 3, 1 + i

$f(x) =$ ✖ $x^4 - 8x^3 + 23x^2 - 30x + 18$

Conjugate Pairs says $1-i$ is Also a root.

If I asked this, I'd just want the factored form

$(x-3)^2(x-(1+i))(x-(1-i))$ DONE.

Book wants it expanded. But at least it shares a good technique for the expansion.

I do distributive law:

$(x-(1+i))(x-(1-i))$

$= (x-1-i)(x-1+i) = x^2 - x + ix - x + 1 - i - ix + i - i^2 = x^2 - 2x + 2$

$-i^2 \to -(-1) = +1$

Book exploits:

$(a+bi)(a-bi) = a^2 - (bi)^2 = a^2 + b^2$

$(x-(1+i))(x-(1-i))$

$= (x-1-i)(x-1+i) = ((x-1)-i)((x-1)+i) = (x-1)^2 + 1^2$

$(a-bi)(a+bi) = a^2+b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$(a+b)^2 = a^2 + 2ab + b^2$

$= x^2 - 2x + 1 + 1 = x^2 - 2x + 2$

$\Rightarrow f(x) = (x-3)^2(x^2-2x+2) = (x-3)(x-3)(x^2-2x+2)$

$= (x-3)[x^3 - 2x^2 + 2x - 3x^2 + 6x - 6]$

$= (x-3)(x^3 - 5x^2 + 8x - 6)$

$= x^4 - 5x^3 + 8x^2 - 6x$
$\quad - 3x^3 + 15x^2 - 24x + 18$

$x^4 - 8x^3 + 23x^2 - 30x + 18 = f(x)$

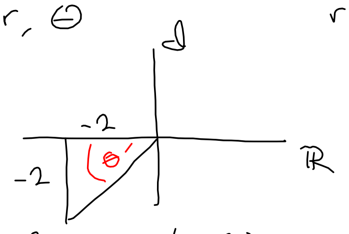
Section 4.3

write $z = -2-2i$ in trig. form:

Need: r, θ

$r = \sqrt{2^2+2^2} = \sqrt{8}$
$= 2\sqrt{2}$

$2 \lfloor 8$
$2 \lfloor 4$
2



$\arctan\left(\frac{-2}{-2}\right) = \arctan(1) = 45^\circ = \theta'$ = Ref. angle.

$\Rightarrow \theta = 180^\circ + 45^\circ = 225^\circ =$
$\pi + \frac{\pi}{4} = \frac{5\pi}{4}$ Prefer (insist) to see these in radians (π- radians), which are easy to get if you're in degrees & convert at the end.

$(225^\circ)\left(\frac{\pi}{180^\circ}\right)$

$\left(\frac{225}{180}\right)\pi = \frac{5\pi}{4}$

$3 \lfloor 225$
$3 \lfloor 75$
$5 \lfloor 25$
5

$2 \lfloor 180$
$2 \lfloor 90$
$3 \lfloor 45$
$3 \lfloor 15$
5

$\frac{\cancel{3^2} \cdot 5}{2^2 \cdot \cancel{3^2} \cdot \cancel{5}} = \frac{5}{4}$

$z = 2\sqrt{2}\left(\cos\left(\frac{5\pi}{4}\right) + i\sin\left(\frac{5\pi}{4}\right)\right)$

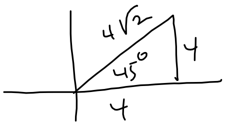
other terms $\begin{cases} \text{argument is } \frac{5\pi}{4} \\ \text{modulus is } 2\sqrt{2} \end{cases}$

DeMoivre: $\left(r_1(\cos\theta_1 + i\sin\theta)\right)\left(r_2(\cos\theta_2 + i\sin\theta_2)\right)$

$= r_1 r_2\left(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\right)$

$\frac{\pi}{6} + \frac{\pi}{12} = \frac{2\pi+\pi}{12} = \frac{3\pi}{12} = \frac{\pi}{4}$

$(4+4i)^6$ using De Moivre



$\frac{\pi}{4}$

$\sqrt{32} = \sqrt{2\cdot2\cdot2\cdot2\cdot2}$

$= 2\cdot2\sqrt{2}$

$z = 4+4i = 4\sqrt{2}\left(\cos 45^0 + i\sin 45^0\right)$

$= 4\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$\sin\frac{3\pi}{2} = -1, \cos\frac{3\pi}{2} = 0$

$z^6 = (4\sqrt{2})^6(\cos \sim\sim\sim = 32768\left(\cos\left(\frac{3\pi}{2}\right) + i\sin\left(\frac{3\pi}{2}\right)\right)$

$(4\sqrt{2})^6 = (4\cdot 2^{\frac{1}{2}})^6 = 4^6(2^{\frac{1}{2}})^6 = 4^6\cdot 2^3 = \quad = -32768i$?!

$= 4^3\cdot4^3\cdot 8$

$= 64\cdot64\cdot8 = 32768$

$(6)\left(\frac{\pi}{4}\right) = \frac{3\pi}{2}$

$$\begin{array}{r} 64 \\ 64 \\ \hline 256 \\ 3840 \\ \hline 4096 \\ 8 \\ \hline 32768 \end{array}$$

5^{th} roots of z:

There are 5 of them, $\frac{2\pi}{5}$ apart. → The increment

$z = 32(\cos(\pi) + i\sin(\pi))$ $(= -32)$

Principle 5^{th} root: $\sqrt[5]{z} = \sqrt[5]{32}\left(\cos\left(\frac{\pi}{5}\right) + i\sin\left(\frac{\pi}{5}\right)\right)$

Increment $= \frac{2\pi}{5}$

$\frac{\pi}{5} + \frac{2\pi}{5} = \frac{3\pi}{5}$

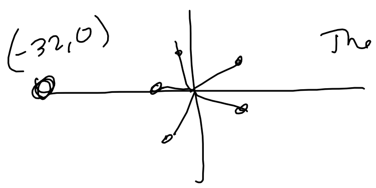
$\frac{3\pi}{5} + \frac{2\pi}{5} = \pi$

$\frac{5\pi}{5} + \frac{2\pi}{5} = \frac{7\pi}{5}$

$\frac{7\pi}{5} + \frac{2\pi}{5} = \frac{9\pi}{5}$

$$\boxed{\begin{aligned} &= 2\left(\cos\tfrac{\pi}{5} + i\sin\tfrac{\pi}{5}\right), \\ &2\left(\cos\tfrac{3\pi}{5} + i\sin\tfrac{3\pi}{5}\right) \\ &2(\cos\pi + i\sin\pi) \\ &2\left(\cos\tfrac{7\pi}{5} + i\sin\tfrac{7\pi}{5}\right) \\ &2\left(\cos\tfrac{9\pi}{5} + i\sin\tfrac{9\pi}{5}\right) \end{aligned}}$$

$\frac{9\pi}{5} + \frac{2\pi}{5} = \frac{11\pi}{5} = \frac{10\pi + \pi}{5} = 2\pi + \frac{\pi}{5}$ coterminal w/ $\frac{\pi}{5}$



The 5 5^{th} roots of -32!

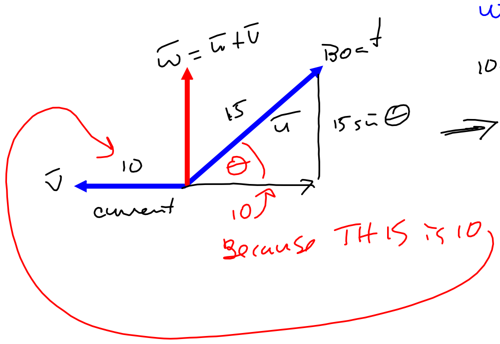
$\sqrt[n]{z}$ reserved for the 1st one, where $k=0$

$$z^{\frac{1}{n}} = r^{\frac{1}{n}}\left(\cos\left(\frac{\theta}{n}+\frac{2\pi}{n}k\right)+i\sin\left(\frac{\theta}{n}+\frac{2\pi}{n}k\right)\right)$$

$k=0,1,\ldots,n-1$

$k=0$ was $\frac{\pi}{5}$ in last example.

$\frac{\theta+2k}{n}$, $k=0,1,\ldots,n-1$



$\vec{w} = \langle \boxed{0}, 15\sin\theta \rangle = \langle -10, 0 \rangle + \langle 15\cos\theta, 15\sin\theta \rangle$

$= \langle \boxed{-10 + 15\cos\theta}, 15\sin\theta \rangle$

$10 + 15\cos\theta = 0$

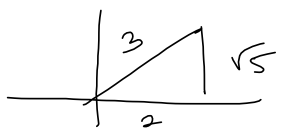
$\Rightarrow \frac{10}{15} = \cos\theta = \frac{2}{3}$

$\theta = \cos^{-1}\left(\frac{2}{3}\right) \approx 48.18968509^\circ$

Bearing (Heading)

$= 90^\circ - \cos^{-1}\left(\frac{2}{3}\right) \approx 41.81031491^\circ$

What's the speed going North.



$\frac{\sqrt{5}}{3} = \sin\theta$

$15\sin\theta = \frac{15\sqrt{5}}{3} = 5\sqrt{5}$ = speed heading due North.

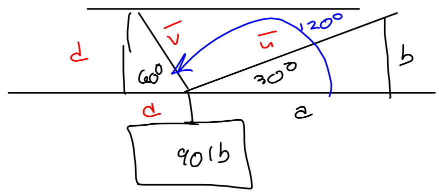
$\boxed{\approx 11.18033988 \text{ knots}}$

$11.18...^2 + 10^2 = 225 = 15^2$ ✓

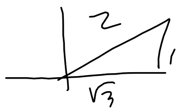
$15\cos\theta = (15)\left(\frac{2}{3}\right) = 5 \cdot 2 = 10$

Find tension in the 2 cables.

$\bar{u} + \bar{v} = \langle 0, 90 \rangle$



$\bar{u} = \langle \|\bar{u}\|\cos 30^\circ, \|\bar{u}\|\sin 30^\circ \rangle$



$\frac{a}{\|\bar{u}\|} = \cos 30^\circ$

$a = \|\bar{u}\|\cos 30^\circ$

$\bar{v} = \langle \|\bar{v}\|\cos 120^\circ, \|\bar{v}\|\sin 120^\circ \rangle$

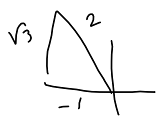
$\bar{u} + \bar{v} = \langle \|\bar{u}\|\cos 30^\circ + \|\bar{v}\|\cos 120^\circ, \|\bar{u}\|\sin 30^\circ + \|\bar{v}\|\sin 120^\circ \rangle$

Let $x = \|\bar{u}\|$

$y = \|\bar{v}\|$

$= \langle \frac{\sqrt{3}}{2}x - \frac{1}{2}y, \frac{1}{2}x + \frac{\sqrt{3}}{2}y \rangle$

$= \langle 0, 90 \rangle$



$\Rightarrow \frac{\sqrt{3}}{2}x - \frac{1}{2}y = 0 \Rightarrow \sqrt{3}x - y = 0$ E1

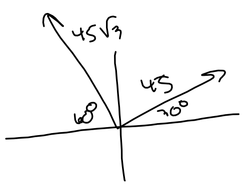
$\frac{1}{2}x + \frac{\sqrt{3}}{2}y = 90$ $\quad x + \sqrt{3}y = 180$ E2

$\Rightarrow y = \sqrt{3}x$ from E1

$\Rightarrow x + \sqrt{3}(\sqrt{3}x) = x + 3x = 4x = 180$ from E2

$x = \frac{180}{4} = \boxed{45 = \|\bar{u}\|}$

$y = \sqrt{3}\ x = \boxed{45\sqrt{3} = y = \|\bar{v}\|}$



$\bar{u} + \bar{v} = \langle 45\frac{\sqrt{3}}{2} + \frac{-45\sqrt{3}}{2}, \frac{45}{2} + \frac{45 \cdot 3}{2} \rangle = \langle 0, \frac{4 \cdot 45}{2} \rangle$

$= \langle 0, 90 \rangle$

$45 \sin 30^\circ = \frac{45}{2}$

$(45\sqrt{3})(\sin 120^\circ) = 45\sqrt{3} \cdot \frac{\sqrt{3}}{2} = \frac{45 \cdot 3}{2} =$