A survey of §4.1 – 4.3

$i = \sqrt{-1}$ $\qquad \mathbb{C} = \{ a+bi = a+ib \mid a,b \in \mathbb{R} \} \supsetneq \mathbb{R}$

$i^2 = -1$

a = Real Part $\qquad$ b = Imaginary Part

$z = a+bi$

$\Rightarrow Im(z) = b, \; Re(z) = a$

$z = 3+2i \qquad a=3, \; b=2$

$\bar{z} = 3-2i$ = complex conjugate of z.

$z + \bar{z} = 6 = 2\,Re(z)$

$z\bar{z} = \bar{z}z = a^2+b^2 = |z|^2$!

$(3+2i)(3-2i) = 9 \underline{- 6i + 6i} - 4i^2 = 9+4 = 3^2+2^2 = 13$!

Write $\frac{3+2i}{1+3i}$ in standard form $(a+bi)$

$= \left(\frac{3+2i}{1+3i}\right)\left(\frac{1-3i}{1-3i}\right) = \frac{9-7i}{1^2+3^2} = \boxed{\frac{9}{10} - \frac{7}{10}i}$

$(3+2i)(1-3i) = 3 - 9i + 2i - 6i^2$

FOIL

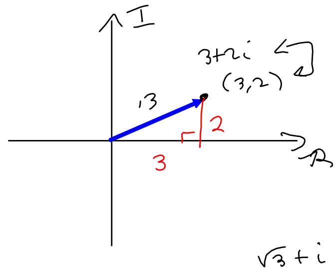
$= 3 - 7i + 6$

$= 9 - 7i$

Similar to rationalizing denominator. $\frac{3}{\sqrt{2}+3} = \left(\frac{3}{3+\sqrt{2}}\right)\left(\frac{3-\sqrt{2}}{3-\sqrt{2}}\right) = \frac{9-3\sqrt{2}}{3^2-\sqrt{2}^2} = \frac{9-3\sqrt{2}}{9-2} =$

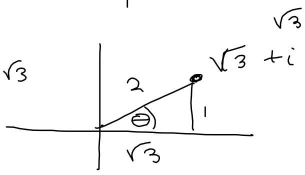
$\frac{9}{7} - \frac{3\sqrt{2}}{7}$

$3+2i$ in complex plane



$3+2i$ is like $\langle 3,2\rangle$

$|z| = \sqrt{3^2+2^2} = \sqrt{13}$



$x = r\cos\theta$

$\frac{\sqrt{3}}{2} = \cos\theta \Rightarrow \sqrt{3} = 2\cos\theta$

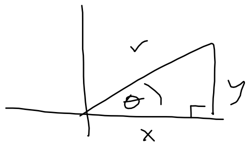
$2 = r$

$a+bi = x+yi$

$= r\cos\theta + (r\sin\theta)i$

$= r\cos\theta + ir\sin\theta$

$= r(\cos\theta + i\sin\theta)$ is the trigonometric form for $x+yi = a+bi$



$r = \sqrt{x^2+y^2}$

$\frac{x}{r} = \cos\theta \Rightarrow x = r\cos\theta$

$\frac{y}{r} = \sin\theta \Rightarrow y = r\sin\theta$

Write $\sqrt{3}+i$ in trig form.

$x = r\cos\theta = 2\cos\theta = 2\cos\left(\frac{\pi}{6}\right)$

$\theta = \arctan\left(\frac{1}{\sqrt{3}}\right) = \arcsin\left(\frac{1}{2}\right) = \arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

$y = r\sin\theta = 2\sin\left(\frac{\pi}{4}\right)$

$z = \underline{\sqrt{3}+i} = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ is trig form for $\sqrt{3}+i$

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Rectangular (coords) form.

DeMoivre says

$z^3 = r^3\left(\cos(3\theta) + i\sin(3\theta)\right)$

$z = 2\left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right) \Rightarrow$

$z^3 = 2^3\left(\cos\left(\frac{3\pi}{6}\right) + i\sin\left(\frac{3\pi}{6}\right)\right)$

$= 8\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right) = 8(0+i) = 8i$!

$(\sqrt{3}+i)^3 =$

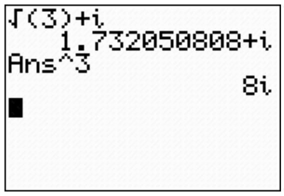
$(\sqrt{3}+i)(\sqrt{3}+i)^2$

$= (\sqrt{3}+i)(3+2i\sqrt{3}+i^2)$

$= (\sqrt{3}+i)(2+2\sqrt{3}\,i)$

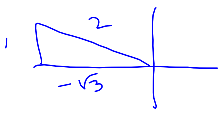
$= 2\sqrt{3}+6i+2i+2\sqrt{3}\,i^2$

$= 2\sqrt{3}+8i-2\sqrt{3} = 8i$



$(\sqrt{3}+i)^5$ is a pain in Rectangular!

$2^5\left(\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right)$ DONE!



$32\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$

$= -16\sqrt{3}+16i$

Roots of Complex #s

$$\sqrt[n]{z} = \sqrt[n]{r}\left(\cos\left(\frac{\theta}{n}\right) + i\sin\left(\frac{\theta}{n}\right)\right)$$

$$\sqrt[3]{\sqrt{3}+i} = \sqrt[3]{2}\left(\cos\left(\frac{\pi}{18}\right) + i\sin\left(\frac{\pi}{18}\right)\right)$$

The PRINCIPAL CUBE ROOT

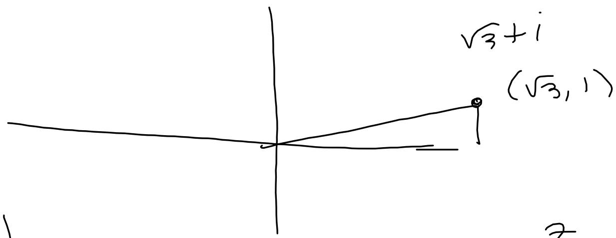
There are ALWAYS 3 cube roots

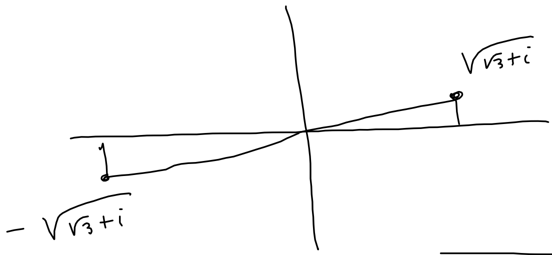
$\sqrt[3]{z}$ is just the 1st one.

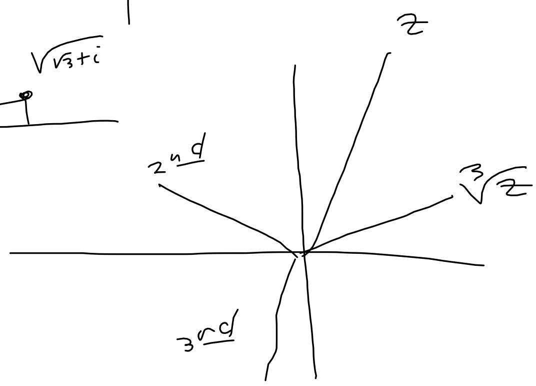
$\sqrt{2}$ is principal square root of 2

The other one is $-\sqrt{2}$

$\sqrt{\sqrt{3}+i}$







When a is a positive real number, the ---Select--- principal square root of $-a$ is defined as

$\sqrt{-a} = \sqrt{a}i$.

The numbers $a + bi$ and $a - bi$ are called ---Select--- conjugate pairs , and their product is a real number $a^2 + b^2$.

$(a+b)(a-b) = a^2 - b^2$

$(a+bi)(a-bi) = a^2 - (bi)^2 = a^2 - b^2 i^2 = a^2 + b^2$

Write the complex number in standard form.

$5 + \sqrt{-16} \quad = 5 + 4i$

The ---Select--- ✗ Fundamental Theorem of ---Select--- ✗ Algebra states that if $f(x)$ is a polynomial of degree n ($n > 0$), then f has at least one zero in the complex number system.

Consider the function.

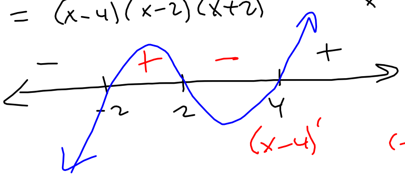
$f(x) = x^3 - 4x^2 - 4x + 16 = x^2(x-4) - 4(x-4) = (x-4)\boxed{(x^2-4)}$

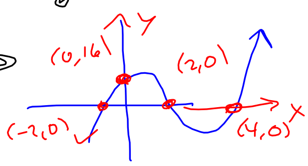
I found $x=4$!

(a) Use a graphing utility to graph the function.

We're left with a depressed polynomial of degree $n-1 = 3-1 = 2$

$= (x-4)(x-2)(x+2)$ $\quad x^3$





The ---Select--- ✗ Linear Factorization Theorem states that if $f(x)$ is a polynomial of degree n ($n > 0$), then f has precisely n linear factors, $f(x) = a_n(x - c_1)(x - c_2) \cdots (x - c_n)$, where $c_1, c_2, \ldots, c_n$ are complex numbers.

1st degree ⟷ linear

The quantity under the radical sign of the Quadratic Formula, $b^2 - 4ac$, is the ---Select--- ✗ discriminant $:= b^2 - 4ac$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$b^2 - 4ac > 0$ 2 real sol'ns

$b^2 - 4ac = 0$ 1 real sol'n

$b^2 - 4ac < 0$ 2 nonreal sol'ns

Determine the number of solutions of the equation in the complex number system.

$x^8 + 3x^2 + 12 = 0$

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Use the discriminant to find the number of real and imaginary solutions of the quadratic equation.

$\frac{1}{4}x^2 - 7x + 49 = 0$ $\quad a = \frac{1}{4}, \; b = -7, \; c = 49$

$\Longrightarrow b^2 - 4ac = (-7)^2 - 4(\frac{1}{4})(49)$

$= 49 - 49 = 0$

One real solution of multiplicity $m = 2$. ONE REPEATED ROOT

$x = \frac{7 \pm 0}{2(\frac{1}{4})} = \frac{7}{\frac{1}{2}} = 14$ $\quad \frac{1}{4}(x-14)(x-14)$

$4f(x) = x^2 - 28x + 196$

$= x^2 - 28x + (14)^2 - 14^2 + 196$

$= (x-14)^2 - 196 + 196 = (x-14)^2 \Rightarrow f(x) = \frac{1}{4}(x-14)^2$

Solve the polynomial equation. (Enter your answers as a comma-separated list.)

$x^4 - 6x^2 - 7 = 0$ Let $u = x^2$

$\Rightarrow u^2 - 6u - 7 = (u-7)(u+1) = 0 \Rightarrow u = -1, 7$

$x^2 = -1$ $x^2 = 7$

$x = \pm i$ $x = \pm\sqrt{7}$

Factored Form of f: "Split f into linear factors"

$\boxed{(x-i)(x+i)(x-\sqrt{7})(x+\sqrt{7}) = f(x)}$

Use the given zero to find all the zeros of the function. (Enter your answers as a comma-separated list. Include the given zero in your answer.)

Function	Zero
$g(x) = 4x^3 + 15x^2 + 16x - 5$	$-2 + i$

Divide by $x - (-2+i)$

Refresher on synthetic division: Divide $g(x)$ by $x - \frac{1}{4}$

$$\begin{array}{r|rrrr} \frac{1}{4} & 4 & 15 & 16 & -5 \\ & & 1 & 4 & 5 \\ \hline & 4 & 16 & 20 & 0 \end{array}$$

sweet!

Says $g(x)$
$= (x-\frac{1}{4})(4x^2+16x+20)$

They want:

$$\begin{array}{r|rrrr} -2+i & 4 & 15 & 16 & -5 \\ & & -8+4i & -18-i & 5 \\ \hline -2-i & 4 & 7+4i & -2-i & 0 \\ & & -8-4i & 2+i & \\ \hline & 4 & -1 & 0 & \end{array}$$

Nice!

$4x - 1 = 0$
$4x = 1$
$x = \frac{1}{4}$
$x - \frac{1}{4}$ is factor!

$(-2+i)(7+4i) = -14 - 8i + 7i + 4i^2 = -18 - i$

$(-2+i)(-2-i) = 4 + 2i - 2i - i^2 = 4+1 = 5$

$-2+i$ is a root $\Rightarrow$
$-2-i$ " " " by Conjugate Pairs Theorem
& fact that coefficients of $g(x)$ are real.

This work says

$$g(x) = 4(x-\tfrac{1}{4})(x-(-2+i))(x-(-2-i))$$

Get Rollin'!