Test 3 is Monday, November 2nd @ 8:15-10:00 a.m. at a room to be announced.

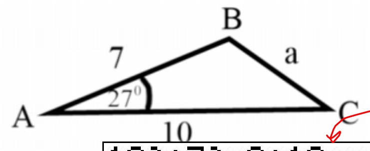
These questions taken from Test 3, Spring, 2018:

https://harryzaims.com/122/122-spring-18/tests-u-took/122-test-3-spring-18.pdf

click to follow link

I think you know the drill on margins and legibility. I can't give points for what I can't read. Take a minute, at the end, to make sure your work is organized and submitted in proper order.

1. Consider the triangle in the figure. Assume lengths are in centimeters.
   a. (10 pts) Use the Law of Cosines to find the length of side a, to 4 decimal places.
   b. (10 pts) Use the Law of Sines to find angle C to 4 decimal places.



where's 7?

```
10²+7²-2*10cos(2
7)
          131.1798695
Ans^.5
          11.45337808
```

(a) $a^2 = b^2 + c^2 - 2bc\cos A$

$= 10^2 + 7^2 - 2(10)(7)\cos(27^\circ) \approx 24.25908661$

$\rightarrow a = \pm\sqrt{131....}$

≈ 4.92535142 (Assume $a > 0$)

$\boxed{a \approx 4.9254}$

```
20110/Ans
          1755.813862
10²+7²-2*10*7cos
(27)
          24.25908661
Ans^.5
           4.92535142
```

$100 + 49 - 140\cos(27^\circ)$

$= 149 - 140\cos(27^\circ)$

```
           4.92535142
cos(27)
          .8910065242
Ans*140
          124.7409134
Ans-149
         -24.25908661
```

How you might do it on another calculator.

I got this by just hitting "$*140$"

" " " " " " "-149"

Then I change sign & take square root.

$\frac{\sin C}{c} = \frac{\sin A}{a} \Rightarrow \sin C = \frac{c\sin A}{a} \approx \frac{7\sin(27^\circ)}{4.92535142}$

$\approx .6452196457$

$\rightarrow C \approx 40.18214574^\circ$

$\boxed{C \approx 40.1821^\circ}$

```
          .6452196457
cos⁻¹(Ans)
          49.81785426
cos(Ans
          .6452196457
sin⁻¹(Ans)
          40.18214574
```

Yes!

```
          24.25908661
Ans^.5
           4.92535142
7*sin(27)/Ans
          .6452196457
cos⁻¹(Ans)
          49.81785426
```

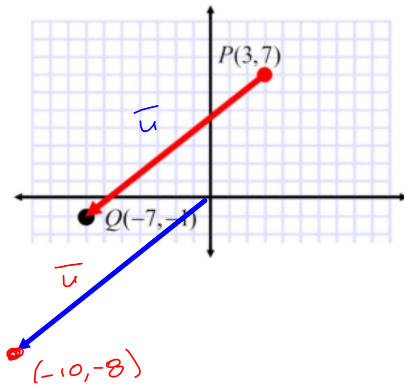
No!

2. Consider the directed line segment $\overrightarrow{PQ}$ in the figure on the right. I want you to provide some basic facts about the vector $\vec{u}$:

   a. (5 pts) Express the vector $\vec{u} = \overrightarrow{PQ}$ in component form.

   b. (5 pts) Compute the magnitude of $\vec{u}$. Leave your answer in simplified radical form.

   c. (10 pts) Find the direction angle of $\vec{u}$. Use degrees, rounded to 4 places.



(a) $\overrightarrow{PQ} = \langle -7-3, -1-7 \rangle = \langle -10, -8 \rangle$
$= \vec{u}$

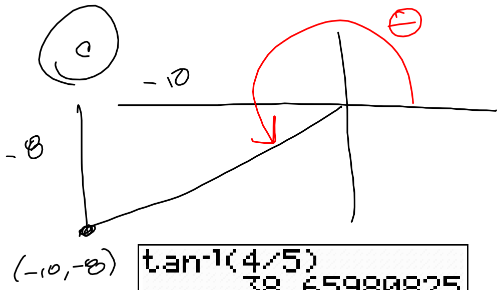
(b) $\|\vec{u}\| = \sqrt{10^2 + 8^2}$
$= \sqrt{164}$
$= 2\sqrt{41}$

$2 \,|\, 164$
$2 \,|\, 82$
41

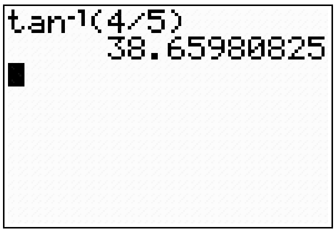
NOTE: $\vec{u} \cdot \vec{u} = \langle -10, -8 \rangle \cdot \langle -10, -8 \rangle$
$= (-10)(-10) + (-8)(-8)$
$= 10^2 + 8^2 = \|\vec{u}\|^2$

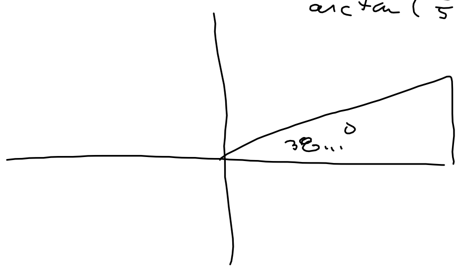
$\|\vec{u}\| = \sqrt{\vec{u} \cdot \vec{u}}$ in terms of dot product

(c)

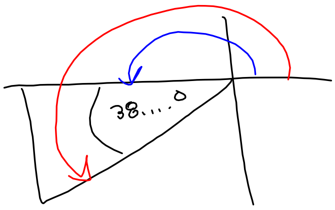


$\arctan\left(\frac{-8}{-10}\right) = \arctan\left(\frac{4}{5}\right) \approx$
$\approx 38.65980825^\circ$
$\arctan\left(\frac{4}{5}\right)$





So our angle has 38....° as reference angle



$\theta = 180 + \arctan\left(\frac{4}{5}\right)$

$\approx 218.65980825^\circ$

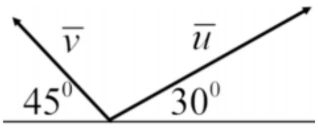
$\approx \boxed{218.6598^\circ \approx \theta}$

3. Let $\bar{u} = \langle -7, 5 \rangle$.

   a. (5 pts) Express $\bar{u}$ as a linear combination of the canonical (standard) unit vectors $\bar{i}$ and $\bar{j}$.

   b. (5 pts) What's another word for the sum of 2 vectors?

4. Dad's out walking his dog and his toddler. The dog pulls with 40 pounds of force in the direction of the vector $\bar{u}$. The toddler pulls with 30 pounds of pressure in the direction of the vector $\bar{v}$.



   a. (10 pts) Express $\bar{u}$ and $\bar{v}$ in component form.

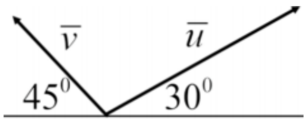
   b. (10 pts) What's the net force, as a vector, on poor Dad?

a $\bar{u} = \langle -7, 5 \rangle = \boxed{-7\bar{i} + 5\bar{j} = \bar{u}}$

$\bar{i} = \langle 1, 0 \rangle$, $\bar{j} = \langle 0, 1 \rangle$

b $\boxed{\text{Resultant}}$ of $\bar{u}$ & $\bar{v}$ is $\bar{u} + \bar{v}$

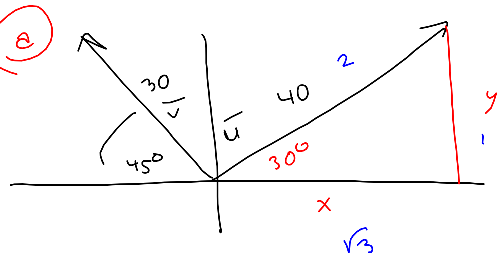
4. Dad's out walking his dog and his toddler. The dog pulls with 40 pounds of force in the direction of the vector $\bar{u}$. The toddler pulls with 30 pounds of pressure in the direction of the vector $\bar{v}$.



   a. (10 pts) Express $\bar{u}$ and $\bar{v}$ in component form.

   b. (10 pts) What's the net force, as a vector, on poor Dad?

a



$\bar{u} = \langle x, y \rangle$

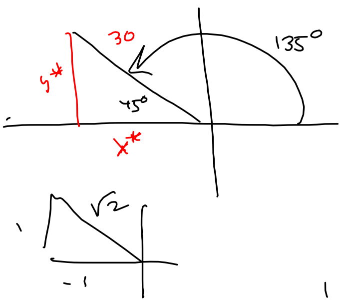
$= \langle 40\cos(30^\circ), 40\sin(30^\circ) \rangle$

$\frac{x}{40} = \cos 30^\circ \Rightarrow$

$x = 40\cos 30^\circ$

$= \langle 40 \cdot \frac{\sqrt{3}}{2}, 40 \cdot \frac{1}{2} \rangle$

$= \boxed{\langle 20\sqrt{3}, 20 \rangle = \bar{u}}$



$\bar{v} = \langle 30\cos(135^\circ), 30\sin(135^\circ) \rangle$

$\frac{x^*}{30} = \cos(135^\circ)$

$= \langle 30(-\frac{1}{\sqrt{2}}), 30(\frac{1}{\sqrt{2}}) \rangle$

$= \langle -\frac{30\sqrt{2}}{2}, \frac{30\sqrt{2}}{2} \rangle$

$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

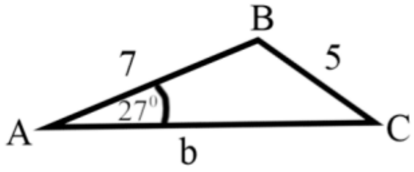
$= \boxed{\langle -15\sqrt{2}, 15\sqrt{2} \rangle = \bar{v}}$

b $\boxed{\bar{u} + \bar{v} = \langle 20\sqrt{3} - 15\sqrt{2}, 20 + 15\sqrt{2} \rangle}$ is net force on evil stepfather.

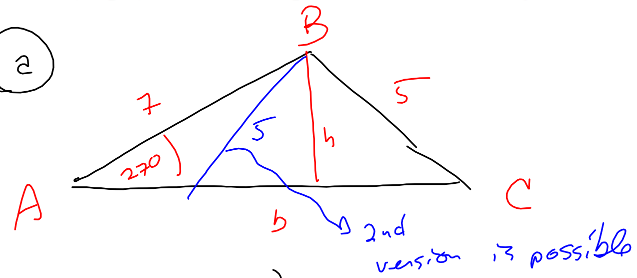
5. Consider the triangle in the figure on the right.

   a. (10 pts) Prove there are 2 triangles that are possible from this ambiguous information.

   b. (10 pts) Find the two possible values for Angle C.



(a)




```
7sin(27)
          3.177933498
```
$\approx h$

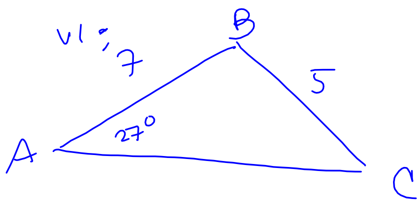
$\frac{h}{7} = \sin(27^\circ)$

$h = 7\sin(27^\circ) \approx 3.1779$ b/c

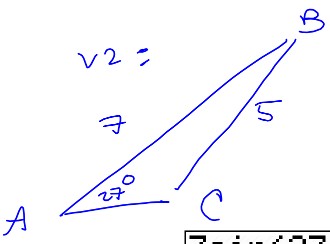
$\boxed{h \approx 3.18 < 5 < 7}$

(b)

v1:



v2:




```
7sin(27)
          3.177933498
Ans/5
          .6355866996
sin⁻¹(Ans)
          39.46351248
```

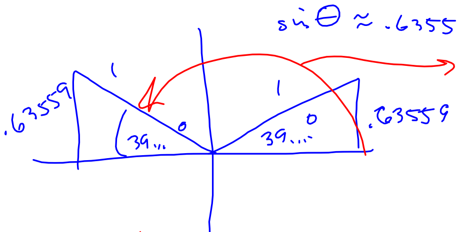
v1

$\frac{\sin C}{7} = \frac{\sin A}{5}$

$\sin(C) = \frac{7\sin(27^\circ)}{5} \approx .6355866996$

$\Rightarrow \boxed{C \approx 39.46351248^\circ \text{ V1}}$ (Didn't specify a precision)

$\sin\theta \approx .63559$



2^{nd} (obtuse) C is $180^\circ - \arcsin(.63559...)$

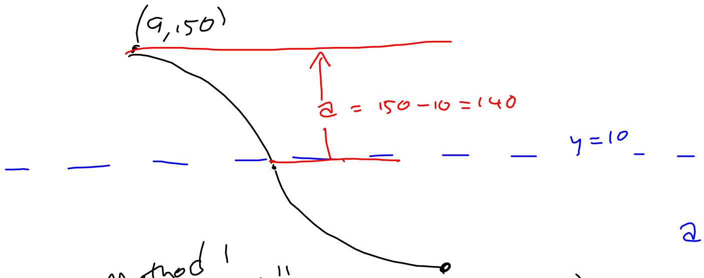
$\Rightarrow \boxed{C \approx 140.5364875^\circ}$ OBTUSE case (V2)


```
          3.177933498
Ans/5
          .6355866996
sin⁻¹(Ans)
          39.46351248
Ans-180
          -140.5364875
```

6. (10 pts) Find $\sin\left(\frac{u}{2}\right)$, $\cos\left(\frac{u}{2}\right)$ and $\tan\left(\frac{u}{2}\right)$, given that $\cos(u) = -\frac{7}{11}$ and $\sin(u) > 0$.

BONUS SECTION: Answer up to 3 questions for up to 15 bonus points.

B1 (5 pts) Build a cosine function that achieves its maximum height of $y = 150$ meters at time $x = 9$ seconds and its minimum height of $y = -130$ meters at $x = 17$ seconds.



$y = \frac{150 + (-130)}{2} = 10$

$a\cos(b(x-c)) + d$

$= a\cos(b(x-9)) + 10$

$= 140\cos(b(x-9)) + 10$

$\boxed{= 140\cos\left(\frac{\pi}{8}(x-9)\right) + 10}$

Method 1 for "b"

9 ← 8 → 17

$8 = \frac{1}{2}$ Period

Method 2 is less cook-bookie.

$bx = 2\pi$, when $x = 16$

$16b = 2\pi$

$b = \frac{2\pi}{16} = \frac{\pi}{8} = b$

Need to specify degrees & or radians.

B2 (5 pts) Find all solutions of the equation $4\cos^2(2x)-1=0$ in the interval $[0,2\pi)$.

want $x \in [0, 2\pi)$ $\qquad 0 \le x < 2\pi$

Let $u = \cos(2x)$ $\qquad 0 \le 2x < 4\pi$

$4u^2 - 1 = 0$

$4u^2 = 1$

$u^2 = \frac{1}{4}$

$u = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{\sqrt{2}}$

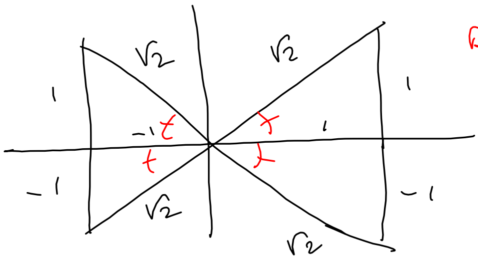
$\cos(2x) = \pm\frac{1}{\sqrt{2}}$

Alexander says: $\sqrt{\frac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}} = \frac{1}{2}$, not $\frac{1}{\sqrt{2}}$, Mills!

"Correct, Sir," says the Dr.

for all

Ref angle 45° $\forall$



$2x = 45^\circ, 135^\circ, 225^\circ, 315^\circ, 405^\circ, 495^\circ, 585^\circ, 675^\circ$

$+360 \quad +360 \quad +360 \quad +360$

$405 \quad 495$

$2x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4}, \frac{15\pi}{4}$

2π

~~$\Rightarrow x \in \{45^\circ, 135^\circ, 225^\circ, 315^\circ, 405^\circ, 495^\circ, 585^\circ, 675^\circ\}$~~

No $x =$ Divide $2x$ by 2, idiot! π-radians are nicer for that;

$$\boxed{x \in \left\{\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}\right\}}$$

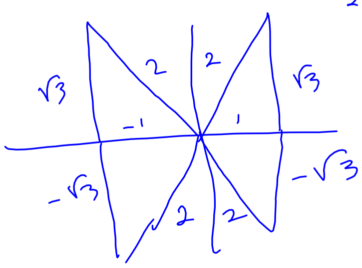
$u^2 = \frac{1}{4}$

$\pm\sqrt{\frac{1}{4}} = \pm\frac{1}{\sqrt{2}}$ Fix it!

$\pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2} = \cos(2x)$

$\cos(2x) = \pm\frac{1}{2}$

$2x \in \left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3} \right\}$



$x \in \left\{ \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{11\pi}{6} \right\}$

Thanks, Alexander!

B3 (5 pts) Sketch the graph of $-20\sin\left(\frac{7\pi}{22}x-\frac{14\pi}{11}\right)+53$.

$a = -20$

$a\sin(b(x-c))+d$

$d = y = 53$ is midline

$-20\sin\left(\frac{7\pi}{22}(x-4)\right)+53$

$\frac{7\pi}{22}x - \frac{14\pi}{11} = \frac{7\pi}{22}(x-4)$

$\frac{\cancel{14\pi}}{\cancel{11}} \cdot \frac{\cancel{22}}{\cancel{7\pi}} = 4$

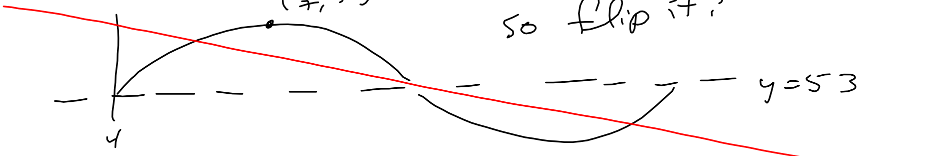
https://harryzaims.com/122/videos/chapter-03/

Period: $\frac{7\pi}{22}x = 2\pi$

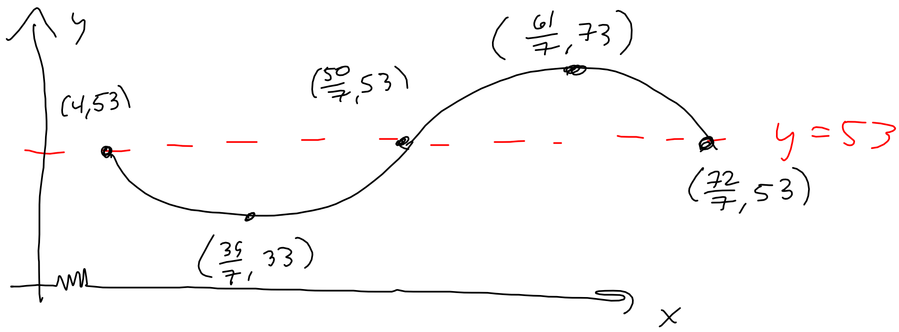
$x = \frac{(2\pi)(22)}{7\pi} = \frac{44}{7} = T = $ Period.

Increment: $\frac{\frac{44}{7}}{4} = \frac{11}{7}$

$(\frac{39}{7}, 73$ ... oops! -20, not $+20$, so flip it!



$4 + \frac{11}{7} = \frac{28+11}{7} = \frac{39}{7}$



$\begin{array}{r} 53 \\ -20 \\ \hline \end{array}$

$\frac{39}{7} + \frac{11}{7} = \frac{50}{7}$

$\frac{50+11}{7} =$

$\frac{61}{7} + \frac{11}{7}$