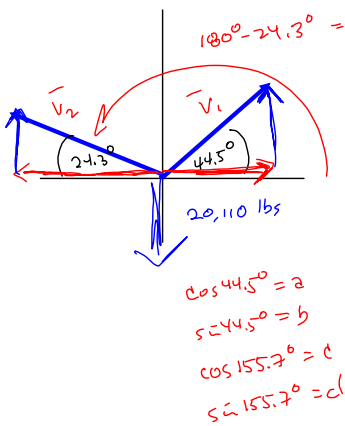


$$\sqrt{\cos^2 \theta + \sin^2 \theta}$$



$$\vec{V}_1 = \|\vec{V}_1\| \langle \cos 44.5^\circ, \sin 44.5^\circ \rangle$$

$$\vec{V}_2 = \|\vec{V}_2\| \langle \cos 155.7^\circ, \sin 155.7^\circ \rangle$$

$$\|\vec{V}_1\| \cos 44.5^\circ + \|\vec{V}_2\| \cos 155.7^\circ = 0$$

$$\|\vec{V}_1\| \sin 44.5^\circ + \|\vec{V}_2\| \sin 155.7^\circ = 20110$$

$$\begin{cases} \|\vec{V}_1\| a + \|\vec{V}_2\| c = 0 \\ \|\vec{V}_1\| b + \|\vec{V}_2\| d = 20110 \end{cases}$$

$$a\|\vec{V}_1\| + c\|\vec{V}_2\| = 0$$

$$\Rightarrow a\|\vec{V}_1\| = -c\|\vec{V}_2\|$$

$$\|\vec{V}_1\| = -\frac{c}{a}\|\vec{V}_2\|$$

Plug it into 2nd eq'n

$$b\|\vec{V}_1\| + d\|\vec{V}_2\| = 20110$$

$$= b\left(-\frac{c}{a}\|\vec{V}_2\|\right) + d\|\vec{V}_2\|$$

```

-sin(44.5)*cos(155.7)/cos(44.5)
50627.13306
-cos(155.7)/cos(44.5)*Ans
64692.19194
    
```

$$\|\vec{V}_2\| = \|\vec{V}_2\| \left[ -\frac{bc}{a} + d \right] = 20110$$

$$\|\vec{V}_1\| \Rightarrow \|\vec{V}_2\| = \frac{20110}{d - \frac{bc}{a}} = \frac{20110}{\sin 155.7^\circ - \frac{(\sin 44.5^\circ)(\cos 155.7^\circ)}{\cos 44.5^\circ}}$$

$$\|\vec{V}_1\| \cos 44.5^\circ + \|\vec{V}_2\| \cos 155.7^\circ = 0$$

$$\|\vec{V}_1\| \sin 44.5^\circ + \|\vec{V}_2\| \sin 155.7^\circ = 20110$$

$$(64692.19194) \cos(44.5^\circ) + (50627.13306) \cos(155.7^\circ) \approx 3 \times 10^{-6} \approx 0$$

$$(64692.19194) \sin(44.5^\circ) + (50627.13306) \sin(155.7^\circ)$$

$$\vec{V}_1 = \langle \|\vec{V}_1\| \cos 44.5^\circ, \|\vec{V}_1\| \sin 44.5^\circ \rangle$$

$a \langle b, c \rangle = \langle ab, ac \rangle$  scalar times vector  
 $3 \langle 2, 1 \rangle = \langle 6, 3 \rangle = \langle 3 \cdot 2, 3 \cdot 1 \rangle = 3 \langle 2, 1 \rangle$

```

627.13306*cos(155.7)
3.296E-6
64692.19194*sin(44.5)+50627.13306*sin(155.7)
66177.14885
    
```

$\approx 0$  so 1st eq'n is happy!

should be 20,110

$$\|\vec{V}_1\| \sin 44.5^\circ + \|\vec{V}_2\| \sin 155.7^\circ \stackrel{\text{Should}}{=} 20,110$$

↓ 20,110

Test 3 Monday, November 2nd  
Covers Chapter 3 and Everything Before.

My Number:  
970-290-0550

$$\text{Let } x = \|\vec{v}_1\|, y = \|\vec{v}_2\|$$

The horizontal forces are balanced (Equal and opposite)

$$x \cdot \cos(44.5^\circ) + y \cdot \cos(155.7^\circ) = 0$$

The vertical force is 20,110 lb

$$x \cdot \sin(44.5^\circ) + y \cdot \sin(155.7^\circ) = 20110$$

$$\begin{aligned} ax + by &= 0 \\ cx + dy &= 20110 \end{aligned}$$

$$\left[ \begin{array}{cc|c} a & b & 0 \\ c & d & 20110 \end{array} \right]$$

$$ax + by = 0$$

$$ax = -by$$

$$x = -\frac{b}{a}y$$

Subst. this to  $cx + dy = 20110$

$$c\left(-\frac{b}{a}y\right) + dy = 20110$$

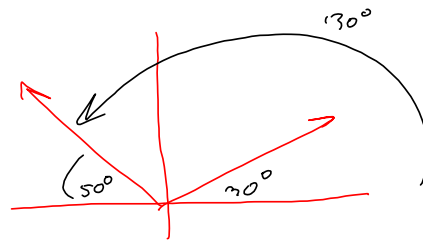
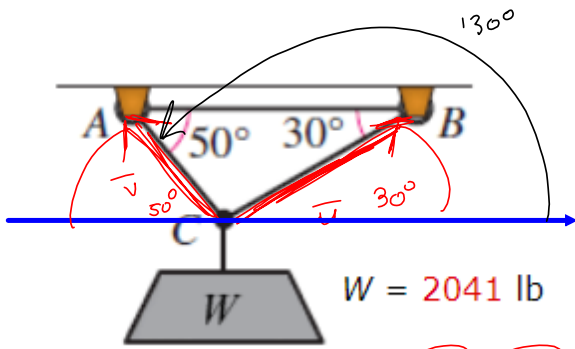
$$-\frac{bc}{a}y + dy = 20110$$

$$y\left(-\frac{bc}{a} + d\right) = 20110$$

$$y = \frac{20110}{d - \frac{bc}{a}}$$

cos(44.5)	1.307147864
20110/Ans	15384.64052
-cos(155.7)/cos(44.5)*Ans	19658.74932

$$y = \|\vec{v}_2\| = \frac{20110}{\sin(155.7^\circ) - \frac{\cos(155.7^\circ) \sin(44.5^\circ)}{\cos(44.5^\circ)}}$$



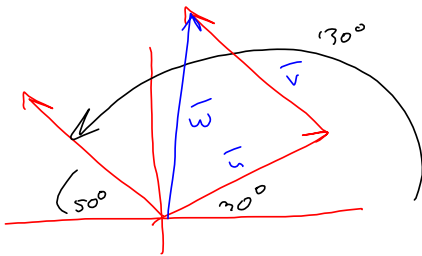
$$\vec{v} = \langle \|\vec{v}\| \cos(130^\circ), \|\vec{v}\| \sin(130^\circ) \rangle$$

$$\vec{u} = \langle \|\vec{u}\| \cos(30^\circ), \|\vec{u}\| \sin(30^\circ) \rangle$$

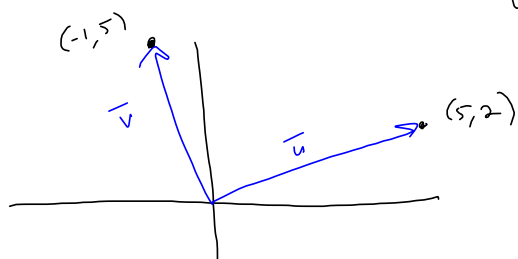
vertical components

horizontal components

$$\vec{w} = \vec{u} + \vec{v} = \text{Resultant}$$



## S'3.4 - DOT PRODUCT

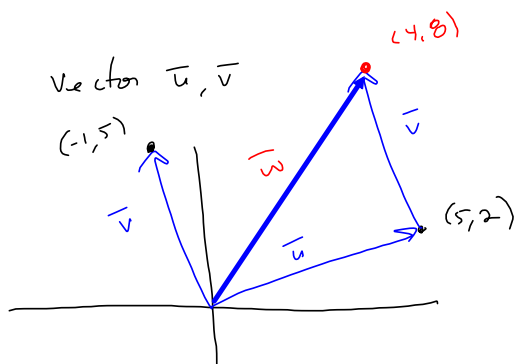
Vector  $\vec{u}, \vec{v}$ 

$$\vec{u} = \langle 5, 2 \rangle$$

$$\vec{v} = \langle -1, 5 \rangle$$

Put a bar over  
vectors, to distinguish  
them from scalars,  
like 5.

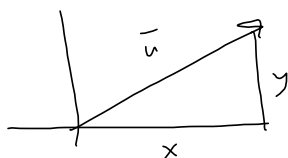
Adding vectors:  $\vec{u} + \vec{v} = \langle 5, 2 \rangle + \langle -1, 5 \rangle$   
 $= \langle 5-1, 2+5 \rangle = \langle 4, 7 \rangle$



$$\vec{w} = \vec{u} + \vec{v} = \langle 4, 7 \rangle$$

Magnitude of a vector:

$$\vec{u} = \langle x, y \rangle$$



$$\|\vec{u}\| = \sqrt{x^2 + y^2}$$

## 3.4 DOT PRODUCT:

$$\vec{u} \cdot \vec{v} = \langle u_1, u_2 \rangle \cdot \langle v_1, v_2 \rangle = u_1 v_1 + u_2 v_2$$

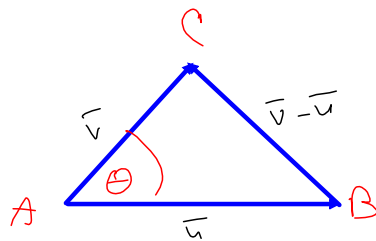
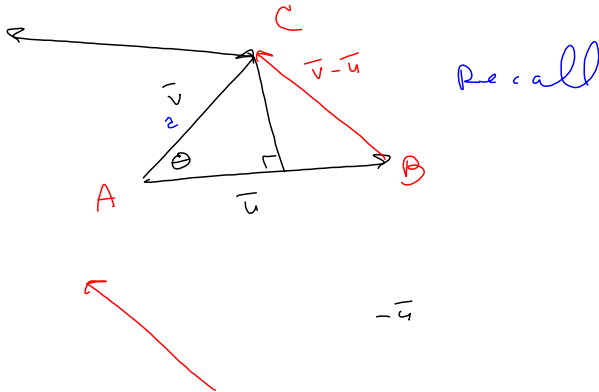
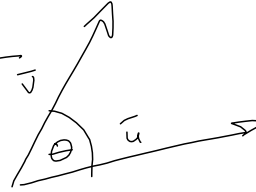
$$\vec{u} = \langle 1, 2 \rangle, \vec{v} = \langle -5, 6 \rangle \Rightarrow$$

$$\vec{u} \cdot \vec{v} = (1)(-5) + (2)(6) = -5 + 12 = \boxed{7 = \vec{u} \cdot \vec{v}}$$

Perpendicular Vectors  $\vec{u}, \vec{v}$ ,  $\Rightarrow$

$$\vec{u} \perp \vec{v} \Leftrightarrow \vec{u} \cdot \vec{v} = 0$$

$$\cos \Theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|}$$



Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos \Theta$$

$$\|\vec{v} - \vec{u}\|^2 = \|\vec{v}\|^2 + \|\vec{u}\|^2 - 2\|\vec{u}\|\|\vec{v}\|\cos \Theta$$

$$\left( \sqrt{(v_1 - u_1)^2 + (v_2 - u_2)^2} \right)^2 = \|\vec{v}\|^2 + \|\vec{u}\|^2 - 2\|\vec{u}\|\|\vec{v}\|\cos \Theta$$

$$(v_1 - u_1)^2 = \underline{v_1^2} - 2u_1 v_1 + \underline{u_1^2} + \underline{v_2^2} - 2u_2 v_2 + \underline{u_2^2}$$

$$= v_1^2 + v_2^2 + u_1^2 + u_2^2 - 2(u_1 v_1 + u_2 v_2)$$

$$\boxed{\|\vec{v}\|^2 + \|\vec{u}\|^2} - 2\vec{u} \cdot \vec{v} = \boxed{\|\vec{u}\|^2 + \|\vec{v}\|^2} - 2\|\vec{u}\|\|\vec{v}\|\cos \Theta$$

$$\cancel{\|\vec{u}\|\|\vec{v}\|\cos \Theta} = \cancel{2\vec{u} \cdot \vec{v}}$$

$$\boxed{\cos \Theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|\|\vec{v}\|}}$$