

$$\begin{aligned}
 a^2 &= x^2 + h^2 \\
 &= (c \cos B)^2 + (a \sin B)^2 \\
 &= a^2 \cos^2 B + a^2 \sin^2 B \\
 &= a^2 \left(\frac{x}{c}\right)^2 + a^2 \left(\frac{h}{a}\right)^2
 \end{aligned}$$

$$\begin{aligned}
 \frac{x}{c} &= \cos B \\
 \Rightarrow x &= c \cos B
 \end{aligned}$$

$$\begin{aligned}
 \frac{h}{a} &= \sin B \Rightarrow \\
 h &= a \sin B
 \end{aligned}$$

3.3#27  
 $2\bar{i} - 2\bar{j} = \langle 2, -2 \rangle = \bar{v}$

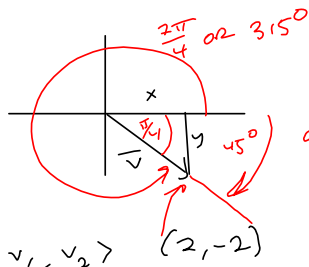
$\bar{i} = \langle 1, 0 \rangle, \bar{j} = \langle 0, 1 \rangle$

$2\bar{i} - 2\bar{j} = 2\langle 1, 0 \rangle - 2\langle 0, 1 \rangle$  } Scalar multiplication  
 $= \langle 2, 0 \rangle + \langle 0, -2 \rangle$  times vector,  
 $= \langle 2+0, 0-2 \rangle = \langle 2, -2 \rangle$  vector addition, entry by entry.

Generally  
 $\arctan(\frac{y}{x})$  &  
 then make sure  
 of quadrant.

Magnitude

$\bar{v} = \langle v_1, v_2 \rangle (2, -2)$



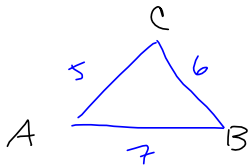
Direction angle is POSITIVE  
 angle measured from positive  
 x-axis.

so  $\theta = 315^\circ$

$\|\bar{v}\| = \sqrt{v_1^2 + v_2^2}$  is Euclidean magnitude ("norm")  
 $= \sqrt{2^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8}$  of  $\bar{v}$ .  
 $= \sqrt{4 \cdot 2} = \sqrt{4} \sqrt{2} = 2\sqrt{2} = \|\bar{v}\|$

$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$  Absolute Value  $|-3| = -(-3) = 3$   
 $|5| = 5$

Law of cosines handles SSS case.



Law of Sines needs at least one angle.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Book gives separate formula for finding angle A

$$\Rightarrow a^2 - b^2 - c^2 = -2bc \cos A$$

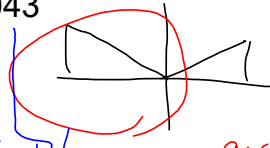
$$\Rightarrow \frac{b^2 + c^2 - a^2}{2bc} = \cos A$$

$$\cos A = \frac{5^2 + 7^2 - 6^2}{2(5)(7)} = \frac{25 + 49 - 36}{70} = \frac{38}{70}$$

$$\arccos(\cos A) \approx \arccos\left(\frac{38}{70}\right) \approx 57.12165043^\circ$$

Hesitant to call this A, but arccosine's range.

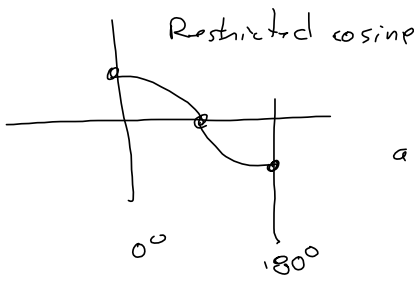
Now THAT WE HAVE A at all 3 sides, use Law of Sines for the rest!



arctangent doesn't see this

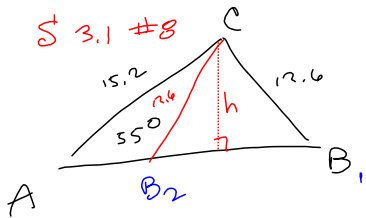
No. Not for arccosine. Not for triangles.

This means  $\arccos(\cos A) = A$  / we don't have to play the  $180^\circ - \arctan(*)$  game.



Because  $0 \leq \theta \leq 180$  in a triangle.

Sine isn't as nice. It can't handle obtuse angles. So you have to be aware of that possibility.



12.45111108

$$\frac{\sin A}{a} = \frac{\sin B}{b} \Rightarrow$$

$$\sin B = \frac{b \sin A}{a} = \frac{12.6 \sin(55^\circ)}{15.2} \approx .9881834185 \Rightarrow$$

$$B_1 = \text{smaller "B"} \approx \boxed{81.18317653^\circ \approx B_1}, \text{ via arcsin (previous)}$$

wait! Didn't even check to see if or how many solutions exist.

$$\frac{h}{15.2} = \sin(55^\circ) \Rightarrow h = 15.2 \sin(55^\circ) \approx 12.45111108 < 12.6$$

$\Rightarrow$  at least one solution  $\nexists$   $12.6 > 15.2 \Rightarrow \exists 2!$

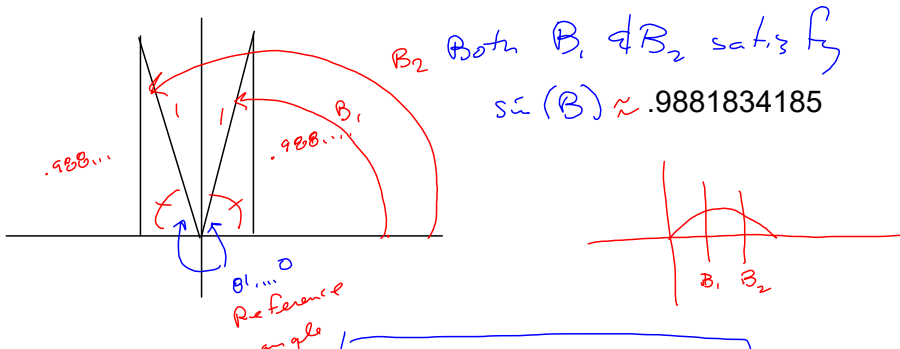
$$C = 180^\circ - A - B \approx 180^\circ - 55^\circ - 81.18317653$$

$$\approx \boxed{43.81682347^\circ \approx C_1}$$

$$\frac{c}{\sin C} = \frac{a}{\sin A} \Rightarrow c = \frac{a \sin C}{\sin A} \approx \frac{12.6 \sin(43.81682347^\circ)}{\sin(55^\circ)}$$

$$\approx \boxed{10.64963940 \approx c_1}$$

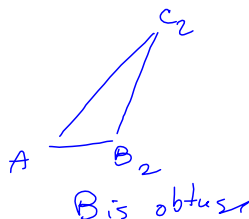
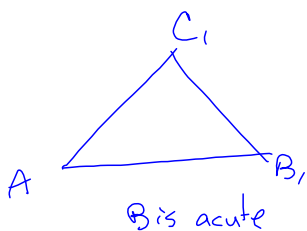
Now find the triangle for the BIG  $B_2$  ( $B_2$  is obtuse!)

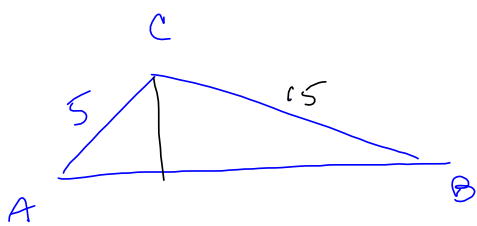


$$\sin(B) \approx .9881834185$$

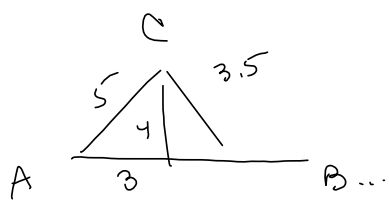
$$B_2 = 180 - B_1 \approx \boxed{98.81682347^\circ \approx B_2}$$

$$\text{Now, } C_2 = 180^\circ - B_2 - 55^\circ \approx \boxed{26.18317653^\circ \approx C_2}$$

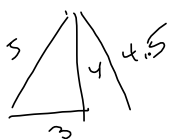




Clearly only one solution



No Solution!

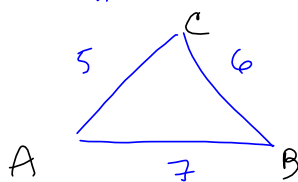


2 solutions

A bit of a cheat to get around Law of Cosines using Heron's Formula! SSS case!

Area =  $\frac{1}{2}bc \sin A$ , so if we could just get at  $\sin A$ ...

Heron's says. Area =  $\sqrt{s(s-a)(s-b)(s-c)}$



$$s = \frac{a+b+c}{2} = \frac{5+6+7}{2} = \frac{18}{2} = 9$$

$$\text{Area} = \sqrt{9(9-5)(9-6)(9-7)} = \sqrt{9(4)(3)(2)} \\ = 6\sqrt{6} \rightarrow$$

$$\frac{1}{2}bc \sin A = \frac{1}{2}(5)(6) \sin A = 15 \sin A = 6\sqrt{6}$$

$\sin A = \frac{6\sqrt{6}}{15} \rightarrow$  you can find  $A$ ! This works perfectly for  $A$  acute.

From this on, you can use law of sines, bypassing law of cosines. Just understand that I will REQUIRE Law of Cosines on the test,