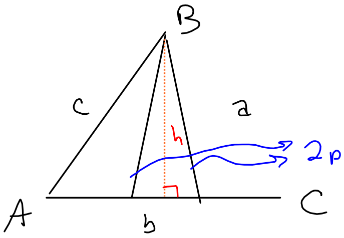
Questions?

§3.1 – LAW OF SINES

§3.2 – LAW OF COSINES.



2 possible triangles when $h < a < c$

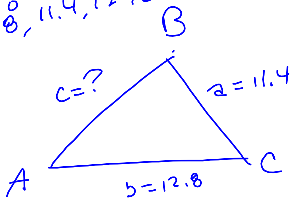
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

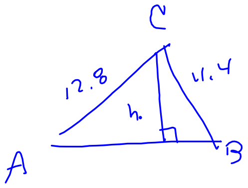
Ambiguous ASS case.
a has to big enough for there to be a solution ($a > h = c \sin A$)

$$\frac{h}{c} = \sin A$$
$$h = c \sin A$$

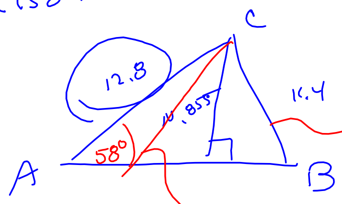
There are 2 possibilities when $h < a < c$ in this picture.

$A = 58°$, $a = 11.4$, $b = 12.8$





$$\frac{h}{12.8} = \sin(A) = \sin(58°) \Longrightarrow$$
$$h = 12.8 \sin(58°) \approx 10.85501563$$



The case where B is acute.

The case where B is obtuse

So there are 2 solutions.
Here's the 1st:

$$\frac{\sin B}{b} = \frac{\sin A}{a} \Longrightarrow \frac{\sin B}{12.8} = \frac{\sin 58°}{11.4} \Longrightarrow \sin B = \frac{12.8 \sin 58°}{11.4}$$

$$\sin B \approx .9521943539 \Longrightarrow$$

$$\boxed{B \approx 72.21217954°} \Longrightarrow$$

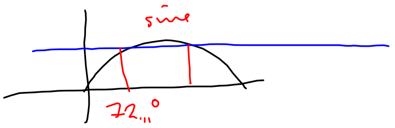
$$A + B + C = 180°$$
$$C = 180° - B - A = 180° - 72.\ldots° - 58° \approx \boxed{49.78782046° \approx C}$$

$$\frac{c}{\sin(C)} = \frac{a}{\sin(A)} \Longrightarrow c = \frac{(\sin(C))(a)}{\sin(A)} \approx \boxed{10.26558565 \approx c}$$

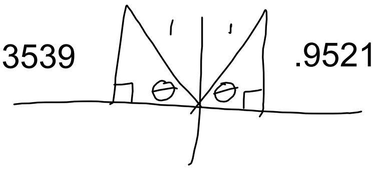
That's one solution. Where's the other one? How do we find it?

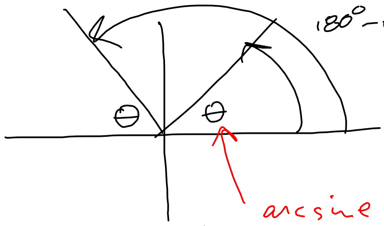
Laurie says subtract B from 180°

Here's why:



$y = .9521943539$





$180° - \theta = 180° - \arcsin\left(\frac{12.8 \sin 58°}{11.4}\right)$

arcsine gives this one

$180° - \arcsin(\text{''}) \approx$ 107.7878205° $\approx$ OBTUSE B

$180° -$ previous $- 58° \approx$ 14.2121795° $\approx$ C corresponding to obtuse B

$\boxed{A = 58°}$

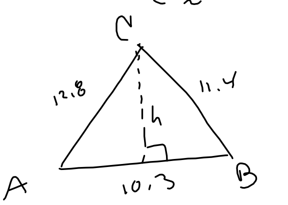
Now to find now b & c:

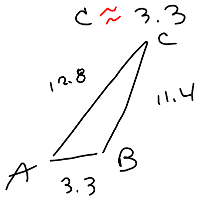
$\frac{c}{\sin C} = \frac{a}{\sin A} \Rightarrow c = \frac{a \sin C}{\sin A} \approx \frac{11.4 \sin(14.2...°)}{\sin(58°)} \approx$ $\boxed{c \approx 3.300347504}$

$\frac{b}{\sin B} = \frac{a}{\sin A} \Rightarrow b = \frac{a \sin B}{\sin A} \approx \frac{11.4 \sin(107...°)}{\sin(58°)} \approx$ $\boxed{12.80004412 \approx b}$

```
95)/sin(58)
      3.300347504
tan⁻¹(-√(5)/3)
     -36.6992252
11.4sin(107.7872
05)/sin(58)
      12.80004412
```

Soln 1	Soln 2
$A = 58°$	$A = 58°$
$B \approx 71.2°$	$B \approx 107.8°$
$C \approx 49.8°$	$C \approx 14.2°$
$a = 11.4$	$a = 11.4$
$b = 12.8$	$b = 12.8$
$c \approx 10.3$	$c \approx 3.3$





For a nice interactive thingie on Example 3 from 3.1:

https://www.larsonprecalculus.com/trig10e/content/interactive-activities/chapter-3/

click to follow link. If you wanna play with it.

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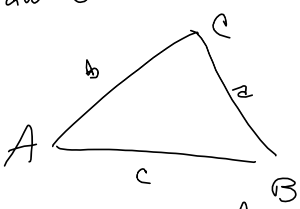
§3.2 Law of cosines



$a^2 = b^2+c^2-2bc\cos A$

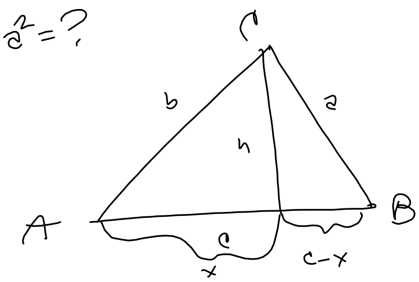
$c^2 = a^2+b^2-2ab\cos C$

$b^2 = a^2+c^2-2ac\cos B$

Between Law of Sines & Law of Cosines we have as big of muscles as anybody's when it comes to solving triangles.

Proof (Sketch)

$a^2 = ?$



$h^2 + x^2 = b^2$

$(c-x)^2 + h^2 = a^2$

$\frac{h}{b} = \sin A \Rightarrow h = b\sin A$

$\frac{x}{b} = \cos A \Rightarrow x = b\cos A$

$a^2 = b^2+c^2-2bc\cos A$ will prove (efficiently) next time.

$a^2 = h^2 + (c-x)^2$

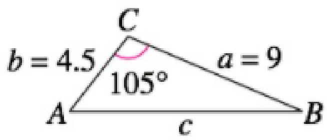
$= h^2 + c^2 - 2cx + x^2$

$= b^2\sin^2 A - 2cb\cos A + b^2\cos^2 A + c^2$

$= b^2(\sin^2 A + \cos^2 A) - 2bc\cos A + c^2$

$= b^2+c^2-2bc\cos A$

10.



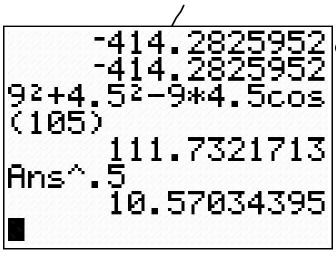
SAS sitch

$$\frac{\sin B}{b} = \frac{\sin A}{a} \Rightarrow \sin B = \frac{b \sin A}{a}$$

$$= \frac{4.5 \sin(105^\circ)}{9}$$ can be done,

$c^2 = a^2 + b^2 - 2ab\sin C$ Don't have ~~clo,~~

idiot!

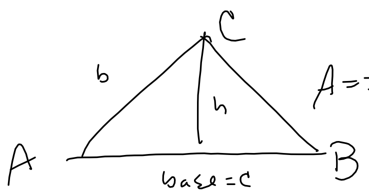


$c^2 = 9^2 + 4.5^2 - 2(9)(4.5)\cos(105^\circ) \approx$

$\rightarrow \boxed{c \approx 10.57034395}$

Area of triangle

$A = \frac{1}{2} bc \sin A = \frac{1}{2} ab \sin C = \frac{1}{2} ac \sin B$



$A = \frac{1}{2}$ base · height

$= \frac{1}{2} c \cdot b \sin A$

$\frac{h}{b} = \sin A$

$h = b \sin A$

§3.2 Heron's: $A = \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \frac{a+b+c}{2}$

GET ROLLING ON §3.1, 3.2.