$\boxed{\text{I'll upload the cheat sheet I'll be giving you.}}$

REMIND ME TO ADD tan(u+v) formula

For those, we just assume QI

Find an algebraic Expression

From Spring, 2018 #7

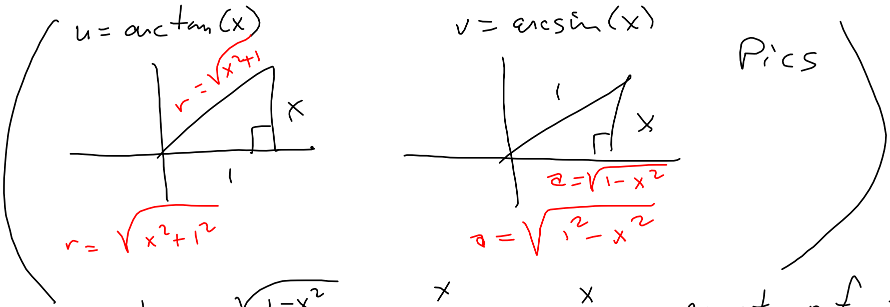
$\cos(u+v) = \cos(u)\cos(v) - \sin(u)\sin(v)$

$\cos(\arctan(x) - \arcsin(x)) = \cos(u-v) = \cos(u+(-v))$

$= \cos(u)\cos(-v) - \sin(u)\sin(-v) = \cos(u)\cos(v) + \sin(u)\sin(v)$

$u = \arctan(x)$ $v = \arcsin(x)$

Pics

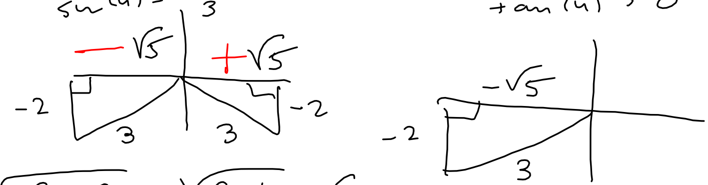


$= \dfrac{1}{\sqrt{x^2+1}} \cdot \dfrac{\sqrt{1-x^2}}{1} + \dfrac{x}{\sqrt{x^2+1}} \cdot \dfrac{x}{1}$ most of the points

$= \boxed{\dfrac{\sqrt{1-x^2}+x^2}{\sqrt{x^2+1}}}$ is fine. You could rationalize the denominator, but I don't think it's necessary.

#8 $\sin(u) = -\frac{2}{3}$ & $\tan(u) > 0$

$\sin(u) = -\frac{2}{3}$ $\tan(u) > 0$



$\sqrt{3^2 - 2^2} = \sqrt{9-4} = \sqrt{5}$

$3^2 - (-2)^2$

$\sin(2u) = 2\sin(u)\cos(u) = 2\left(-\frac{2}{3}\right)\left(-\frac{\sqrt{5}}{3}\right) = \boxed{\dfrac{4\sqrt{5}}{9} = \sin(2u)}$

$\cos(2u) = 1 - 2\sin^2(u) = 1 - \left(-\frac{2}{3}\right)^2 = 1 - \frac{4}{9} = \frac{9-4}{9} = \boxed{\dfrac{5}{9} = \cos(2u)}$

$\tan(2u) = \dfrac{\sin(2u)}{\cos(2u)} = \dfrac{4\sqrt{5}}{9} \cdot \dfrac{9}{5} = \boxed{\dfrac{4\sqrt{5}}{5} = \tan(2u)}$

§2.5 #6 (Laurie's version)

$7\tan(2x) - 7\cot(x) = 0 \Longrightarrow$

$\Longleftrightarrow 7\left(\tan(2x) - \cot(x)\right) = 0$

$\Longleftrightarrow \tan(2x) - \cot(x) = 0$

Scratch:
$\tan(2x) = \dfrac{2\tan(x)}{1-\tan^2(x)}$

$\sec^2(x) - 1 = \tan^2(x)$

$\Longrightarrow \dfrac{2\tan(x)}{1-\tan^2(x)} - \dfrac{\cot(x)}{1} \cdot \dfrac{(1-\tan^2(x))}{(1-\tan^2(x))} = 0$ → in calculus, it'll be >0 or <0

$\Longrightarrow \dfrac{2\tan(x) - \cot(x) + \cot(x)(\tan^2(x))}{LCD} = 0$

$(\cot(x))(\tan^2(x)) = \dfrac{1}{\cancel{\tan(x)}} \cdot \tan^2(x) = \tan(x)$

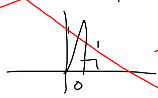
$\Longrightarrow \dfrac{2\tan(x) - \cot(x) + \tan(x)}{LCD} = 0$

$\Longrightarrow 2\tan(x) - \cot(x) + \tan(x) = 0$

$\Longrightarrow 3\tan(x) - \cot(x) = \cancel{3\tan(x) - \tan\left(\frac{\pi}{2} - x\right)}$?!

$\cancel{= 3\tan(x) - \dfrac{\tan(\frac{\pi}{2}) + \tan(-x)}{1 - \tan(\frac{\pi}{2})\tan(-x)}}$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$



No help

$\cancel{\tan\left(\frac{\pi}{2}\right) \nexists}$!

So try this:

$3\tan(x) - \cot(x) = 0$

$3\tan(x) = \cot(x)$

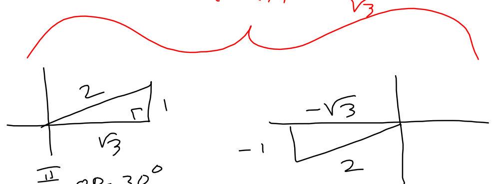
$\dfrac{3\tan(x)}{\cot(x)} = 1$

$3\tan^2(x) = 1$

$\tan^2(x) = \dfrac{1}{3}$

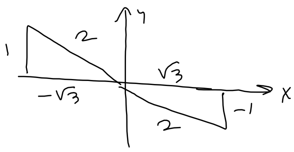
$\tan(x) = \pm \dfrac{1}{\sqrt{3}}$

$\tan(x) = +\dfrac{1}{\sqrt{3}}$



$\frac{\pi}{6}$ or $30°$ $\quad$ $\frac{7\pi}{6}$ or $210°$

$\tan(x) = -\dfrac{1}{\sqrt{3}}$



$\dfrac{5\pi}{6}, \dfrac{11\pi}{6}$

$2\pi - \dfrac{\pi}{6} = \dfrac{12\pi - 1\pi}{6} = \dfrac{11\pi}{6}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ also work.

Algebra techniques sometimes fail

$\boxed{\text{2.5 \# 6 is pretty tough}}$

$\tan(2x) - \cot(x) = 0$

@ $\frac{\pi}{2}, \frac{3\pi}{2}$, but we didn't find it w/ our techniques

This revealed the $\frac{3\pi}{2}$ & $\frac{\pi}{2}$

Step back & ask when

Both are zero & you get $x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\tan(2x) = 0$

$\cot(x) = 0$

#4 from Spr. '18 test 2

$$3\tan^3(x) - 3\tan^2(x) - \tan(x) + 1 = 0$$

$$3u^3 - 3u^2 - u + 1 = 3u^2(u-1) - 1(u-1)$$

$$= (u-1)(3u^2-1) = (u-1)(\sqrt{3}u - 1)(\sqrt{3}u + 1) = 0$$

$3u^2$ is the square of $(\sqrt{3}u)$

$\sqrt{3}u - 1 = 0$ OR $\sqrt{3}u + 1 = 0$

$\sqrt{3}u = 1$ OR $\sqrt{3}u = -1$

$u = \frac{1}{\sqrt{3}}$ OR $u = -\frac{1}{\sqrt{3}}$

$u - 1 = 0$ OR $3u^2 - 1 = 0$

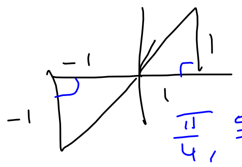
$u = 1$ $\quad 3u^2 = 1$

$u^2 = \frac{1}{3}$

$u = \pm\sqrt{\frac{1}{3}} = \pm\frac{1}{\sqrt{3}}$

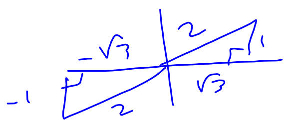
By square root property.

$\tan(x) = 1$

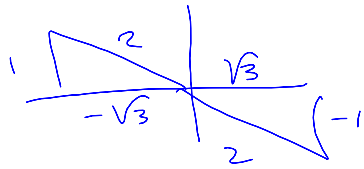


$\frac{\pi}{4}, \frac{5\pi}{4}$

$\tan(u) = \frac{1}{\sqrt{3}}$



$\tan(u) = -\frac{1}{\sqrt{3}}$



(a)

$x \in \left\{\frac{\pi}{6}, \frac{\pi}{4}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{4}, \frac{11\pi}{6}\right\}$

$30^\circ, 45^\circ, 150^\circ, 210^\circ, 135^\circ, 330^\circ$

(b) Find all:

$$x \in \left\{ \frac{\pi}{4} + n\pi, \frac{\pi}{6} + n\pi, \frac{5\pi}{6} + n\pi \;\middle|\; n \in \mathbb{Z} \right\}$$

$$= \left\{ x + n\pi \;\middle|\; x \in \left\{\frac{\pi}{4}, \frac{\pi}{6}, \frac{5\pi}{6}\right\}, n \in \mathbb{Z} \right\}$$

$$= \left\{ \frac{\pi}{6} + 2n\pi, \frac{\pi}{4} + 2n\pi, \frac{5\pi}{6} + 2n\pi, \frac{7\pi}{6} + 2n\pi, \frac{5\pi}{4} + 2n\pi, \frac{11\pi}{6} + 2n\pi \;\middle|\; n \in \mathbb{Z} \right\}$$

$$= \left\{ x + 2n\pi \;\middle|\; x \in \left\{\frac{\pi}{6}, \frac{\pi}{4}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{4}, \frac{11\pi}{6}\right\}, n \in \mathbb{Z} \right\}$$

ANY of the above would be full credit.