Sum and Difference Formulas

Recall S2.4

$\sin(u + v) = \sin u \cos v + \cos u \sin v$

$\sin(u - v) = \sin u \cos v - \cos u \sin v$

$\cos(u + v) = \cos u \cos v - \sin u \sin v$

$\cos(u - v) = \cos u \cos v + \sin u \sin v$

$\tan(u + v) = \dfrac{\tan u + \tan v}{1 - \tan u \tan v}$ $\qquad \tan(u - v) = \dfrac{\tan u - \tan v}{1 + \tan u \tan v}$

S2.4 #22

Write the expression as the sine, cosine, or tangent of an angle.

$\sin 2 \cos 1.4 - \cos 2 \sin 1.4$

$= \sin(2-1.4) = \sin(0.6)$

we'll generally not see these types in our math future.

26. 0/1 points

Find the exact value of the expression.

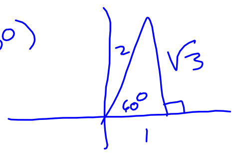
$\sin(100°) \cos(40°) - \cos(100°) \sin(40°)$

$\dfrac{\sqrt{3}}{2}$

Laurie's version

$\sin(100° - 40°) = \sin(60°)$

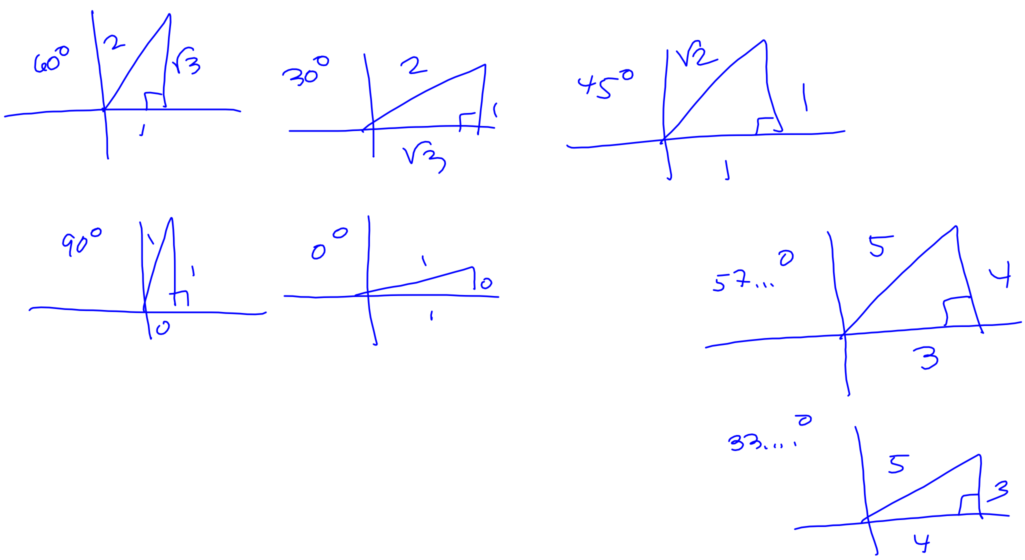
$= \dfrac{\sqrt{3}}{2}$



$\sin(u-v) = \sin(u)\cos(v) - \sin(v)\cos(u)$

$\sin(u-v) = \sin(u + (-v)) = \sin(u)\cos(-v) + \sin(-v)\cos(u)$

$= \sin(u)\cos(v) - \sin(v)\cos(u)$

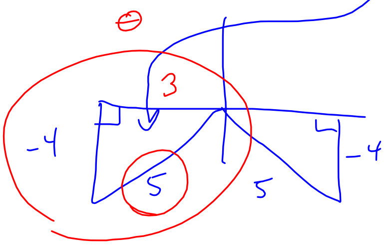


$$\boxed{\sin\left(\frac{u}{2}\right)=\pm\sqrt{\frac{1-\cos(u)}{2}},\ \cos\left(\frac{u}{2}\right)=\pm\sqrt{\frac{1+\cos(u)}{2}},}$$

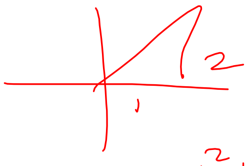
I did this THIS way in class: $\sin(u) = \pm\sqrt{\frac{1-\cos(2u)}{2}}$

$\sin(\theta) = -\frac{4}{5}$ & $\cos(\theta) < 0$ from Spring '17 test 2:

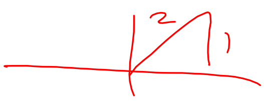
Find $\sin\left(\frac{\theta}{2}\right)$



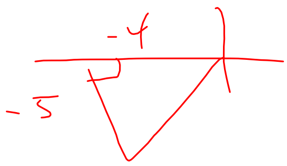
$5^2 - (-4)^2 =$



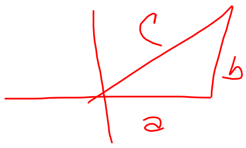
$1^2 + 2^2$

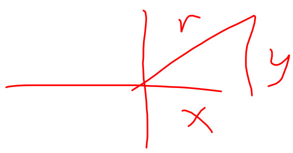


$2^2 - 1^2$



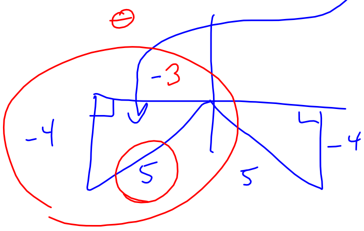
$4^2 + 5^2 =$





$\sin(\theta) = -\frac{4}{5}$ & $\cos(\theta) < 0$ from spring '17 test 2:

Find $\sin\left(\frac{\theta}{2}\right)$



So $\theta \in Q\,III$

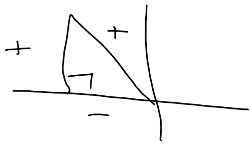
$180^\circ < \theta < 270^\circ$

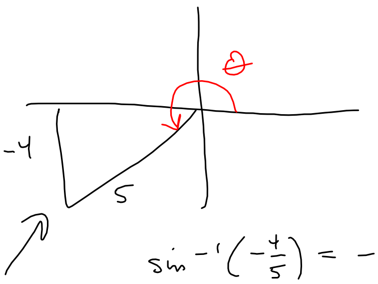
$90^\circ < \frac{\theta}{2} < 135^\circ$

$\frac{\theta}{2} \in Q\,II$

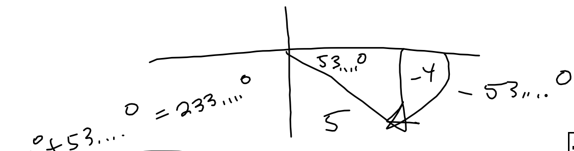
$\sin\left(\frac{\theta}{2}\right) > 0$

$\cos\left(\frac{\theta}{2}\right) < 0$



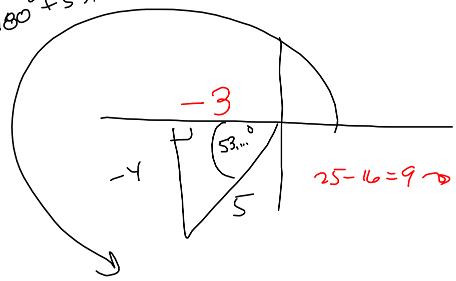


$\sin^{-1}\left(-\frac{4}{5}\right) = -53.\ldots^\circ$




```
sin⁻¹(-4/5)
        -53.13010235
Ans-180
        -233.1301024
```

So $\theta = 233\ldots^\circ$



$25-16=9 \rightsquigarrow \sqrt{9}=3$

```
sin⁻¹(-4/5)
        -53.13010235
-Ans
         53.13010235
Ans+180
         233.1301024
```

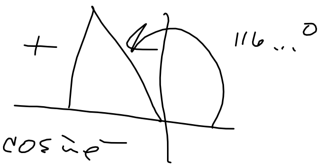
$= $ Reference Angle θ'

$\approx \theta$

```
sin⁻¹(-4/5)
        -53.13010235
Ans-180
        -233.1301024
Ans/2
        -116.5650512
```

So $\frac{\theta}{2} = 116.56\ldots^\circ$

$\cancel{\theta}$ $\theta \in Q\,II$



$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{2}}$

$= +\sqrt{\frac{1-\cos(\theta)}{2}}$

Deciding "+" or "−" is ½ the problem,

B/C $\frac{\theta}{2} \in Q\,II$ by our analysis

$= \sqrt{\frac{1-\left(-\frac{3}{5}\right)}{2}} = \sqrt{\frac{\frac{5+3}{5}}{2}} = \sqrt{\frac{\frac{8}{5}}{2}} = \sqrt{\frac{8}{10}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$

$\boxed{\sin\left(\frac{\theta}{2}\right) = \frac{2}{\sqrt{5}}}$ is fine

$= \frac{2\sqrt{5}}{5}$ is rationalized.

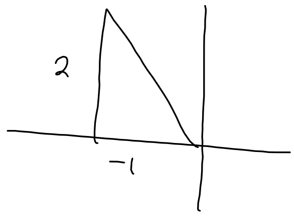
$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos(\theta)}{2}} = -\sqrt{\frac{1+\cos(\theta)}{2}} = -\sqrt{\frac{1+\left(-\frac{3}{5}\right)}{2}}$$

$\frac{\theta}{2} \in Q\,II$

$\boxed{\cos\left(\frac{\theta}{2}\right) =}$

$$= -\sqrt{\frac{\frac{5-3}{5}}{2}} = -\sqrt{\frac{\frac{2}{5}}{2}} = -\sqrt{\frac{2}{10}} = -\sqrt{\frac{1}{5}} = \boxed{-\frac{1}{\sqrt{5}}}$$ is fine

$$\tan\left(\frac{\theta}{2}\right) = \frac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)} = \frac{\frac{2}{\sqrt{5}}}{-\frac{1}{\sqrt{5}}} = -\frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{1} = -2$$



$\frac{\theta}{2}$ pic from $\tan\left(\frac{\theta}{2}\right)$

$\sin\left(\frac{\theta}{2}\right) = \frac{2}{\sqrt{5}}$

$(\sqrt{5})^2 - 2^2 = 5 - 4 = 1 \rightsquigarrow \sqrt{1} = 1$

$\cos\left(\frac{\theta}{2}\right) = -\frac{1}{\sqrt{5}}$

$2^2 + 1^2 = 4 + 1 = 5 \rightsquigarrow \sqrt{5}$

Section 2.5:

Double-Angle Formulas

$$\sin 2u = 2\sin u\cos u \qquad \cos 2u = \cos^2 u - \sin^2 u$$
$$\tan 2u = \frac{2\tan u}{1-\tan^2 u} \qquad = 2\cos^2 u - 1$$
$$= 1 - 2\sin^2 u$$

$\cos(2x) + \sin(x) = 0$

$1 - 2\sin^2(x) + \sin(x) = 0$

$-2\sin^2(x) + \sin(x) + 1 = 0$

$2\sin^2(x) - \sin(x) - 1 = 0$

$2u^2 - u - 1 = 0$

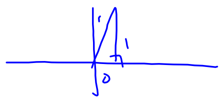
$(2u+1)(u-1) = 2u^2 - u - 1$ ✓

$2u + 1 = 0$ OR $u - 1 = 0$

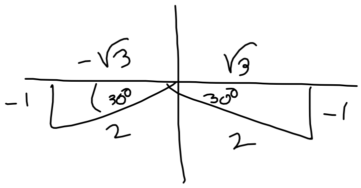
$2u = -1$ $\qquad u = 1$

$u = -\frac{1}{2}$ $\qquad \sin(x) = 1 \Longrightarrow x = 90^\circ = \frac{\pi}{2}$

$\sin(x) = -\frac{1}{2}$



$$\boxed{x \in B = \left\{ \frac{\pi}{2} + 2\pi n \;\middle|\; n \in \mathbb{Z} \right\}}$$
$$= \left\{ 90^\circ + 360^\circ n \;\middle|\; n \in \mathbb{Z} \right\}$$

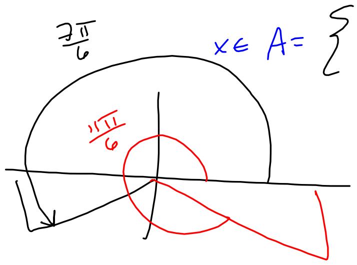


sol'ns $\in [0, 360^\circ]$

$210^\circ, 330^\circ$

ALL solutions!

$\boxed{\text{SOL'N}} \quad \boxed{x \in A \cup B}$



$x \in A = \left\{ 210^\circ + 360^\circ n,\ 330^\circ + 360^\circ n \;\middle|\; n \in \mathbb{Z} \right\}$

$(210^\circ)\left(\frac{\pi}{180^\circ}\right) = \frac{7\pi}{6}$

$(330^\circ)\left(\frac{\pi}{180^\circ}\right) = \frac{11\pi}{6}$

$= \left\{ \frac{7\pi}{6} + 2\pi n,\ \frac{11\pi}{6} + 2\pi n \;\middle|\; n \in \mathbb{Z} \right\}$

$= \left\{ x + 2\pi n \;\middle|\; x = \frac{7\pi}{6}, \frac{11\pi}{6} \text{ \& } n \in \mathbb{Z} \right\}$

$$\boxed{= \left\{ x + 2\pi n \;\middle|\; x \in \left\{ \frac{7\pi}{6}, \frac{11\pi}{6} \right\},\ n \in \mathbb{Z} \right\} = A}$$

Power-Reducing Formulas

$\sin^2 u = \dfrac{1 - \cos 2u}{2}$ from $\cos(2u) = 1 - 2\sin^2(u)$

$\cos^2 u = \dfrac{1 + \cos 2u}{2}$ from $\cos(2u) = 2\cos^2(u) - 1$ } Double-angle formulas

$(a-b)^2 = a^2 - 2ab + b^2$

$\tan^2 u = \dfrac{1 - \cos 2u}{1 + \cos 2u}$

Reducing Powers **In Exercises 27–34, use the power-reducing formulas to rewrite the expression in terms of first powers of the cosines of multiple angles.**

27. $\cos^4 x$	**28.** $\sin^8 x$
29. $\sin^4 2x$	**30.** $\cos^4 2x$
31. $\tan^4 2x$	**32.** $\tan^2 2x \cos^4 2x$
33. $\sin^2 2x \cos^2 2x$	**34.** $\sin^4 x \cos^2 x$

#29ish

$$\sin^4(x) = \left(\sin^2(x)\right)^2 = \left(\frac{1-\cos(2x)}{2}\right)^2$$

$$= \frac{1^2 - 2\cos(2x) + \cos^2(2x)}{4} = \frac{1}{4}\left[1 - 2\cos(2x) + \cos^2(2x)\right]$$

(Scratch $\cos^2(2x) = \dfrac{1+\cos(4x)}{2}$)

$$= \frac{1}{4}\left[1 - 2\cos(2x) + \frac{1+\cos(4x)}{2}\right]$$ is fine by me.

$$= \frac{1}{8}\left[2 - 4\cos(2x) + 1 + \cos(4x)\right]$$

$$= \frac{1}{8}\left[\cos(4x) - 4\cos(2x) + 3\right]$$

Half-Angle Formulas

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}} \qquad \cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$$

$$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$$

The signs of $\sin\frac{u}{2}$ and $\cos\frac{u}{2}$ depend on the quadrant in which $\frac{u}{2}$ lies.

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$$

$$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$

$$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$$

Can be useful in Calc II

$\int \sin(x)\cos(2x)\,dx$ is painful, but

$u = x$
$v = 2x$

$$= \frac{1}{2}\int\left(\cos(x-2x) - \cos(x+2x)\right)dx$$

$$= \frac{1}{2}\int\left(\cos(-x) - \cos(3x)\right)dx$$

$$= \frac{1}{2}\int\left(\cos(x) - \cos(3x)\right)dx$$

Sum-to-Product Formulas

$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$