2.1 Using Fundamental Identities ✓

2.2 Verifying Trigonometric Identities ✓

2.3 Solving Trigonometric Equations ✓

2.4 Sum and Difference Formulas

2.5 Multiple-Angle and Product-to-Sum Formulas

Test 1's delivered (graded)

Old Test 2's:

study them!

No surprises!

HOMEWORK PROGRESS

Few have even BEGUN

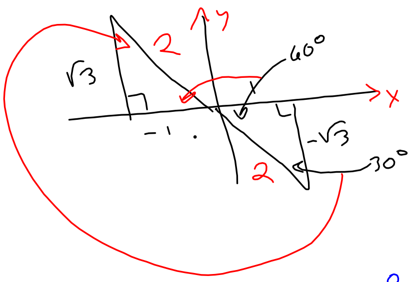
CHAPTER 2.

Test 1 Questions?

§2.3 Questions

$\tan(x) + \sqrt{3} = 0$

$\tan(x) = -\sqrt{3}$



$180^\circ - 60^\circ = 120^\circ$ $\quad$ $360^\circ - 60^\circ = 300^\circ$

$\pi - \frac{\pi}{3} = \frac{2\pi}{3}$ $\quad$ $2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

Solves it in $[0, 360^\circ]$

Rest of 'em

$$\left\{ \frac{2\pi}{3} + 2\pi n \text{ or } \frac{5\pi}{3} + 2\pi n \;\middle|\; n \in \mathbb{Z} \right\}$$

$$= \left\{ \frac{2\pi}{3} + \pi n \;\middle|\; n \in \mathbb{Z} \right\}$$

#11

$12\sec^2(x) - 16 = 0$

$12\sec^2(x) = 16$

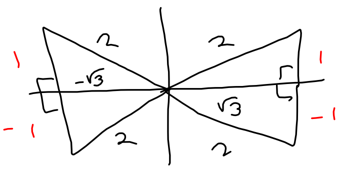
$\sec^2(x) = \frac{16}{12} = \frac{4}{3}$

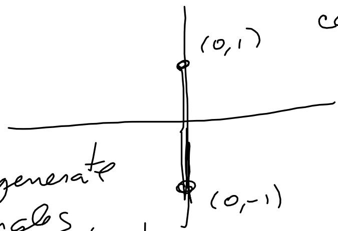
$\sqrt{\sec^2(x)} = \sqrt{\frac{4}{3}}$

$|\sec(x)| = \frac{\sqrt{4}}{\sqrt{3}} = \frac{2}{\sqrt{3}}$

$\sec(x) = \pm\frac{2}{\sqrt{3}}$

$\cos(x) = \pm\frac{\sqrt{3}}{2}$

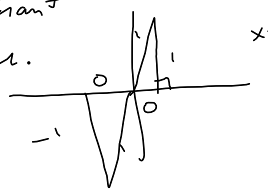




$\cos(\frac{\pi}{2}) = 1$

$\cos(\frac{3\pi}{2}) = -1$

Degenerate triangles make quadrant angles easier.



$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$= 90^\circ, 270^\circ$

$30^\circ, 150^\circ, 210^\circ, 330^\circ$

$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$\left\{ \frac{\pi}{6} + 2\pi n, \frac{5\pi}{6} + 2\pi n, \frac{7\pi}{6} + 2\pi n, \frac{11\pi}{6} + 2\pi n \;\middle|\; n \in \mathbb{Z} \right\}$

$= \left\{ x + 2\pi n \;\middle|\; x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \;\&\; n \in \mathbb{Z} \right\}$

$= \left\{ x + \pi n \;\middle|\; x = \frac{\pi}{6}, \frac{5\pi}{6} \;\&\; n \in \mathbb{Z} \right\}$

writing non-stop > Thinking and not writing.

Grades will be sent to you tomorrow

§2.4

Sum and Difference Formulas

sin(x) is odd

cos(x) is even

$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$

$$\cancel{\sin(u-v) = \sin u \cos v - \cos u \sin v}$$

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

$$\cancel{\cos(u-v) = \cos u \cos v + \sin u \sin v}$$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} \qquad \tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

Have on cheat sheet

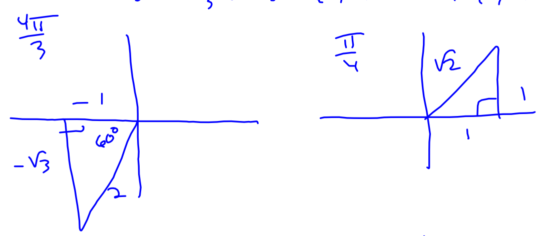
But $\tan\theta = \frac{\sin\theta}{\cos\theta}$, so if you've done $\sin\theta$ & $\cos\theta$, already, just plug them into this & get $\tan\theta$ for free!

$$\sin(u+v) = \sin(u)\cos(v) + \sin(v)\cos(u)$$

$$\sin\left(\frac{19\pi}{12}\right) = \sin\left(\frac{16\pi}{12} + \frac{3\pi}{12}\right) = \sin\left(\frac{4\pi}{3} + \frac{\pi}{4}\right)$$

$19 = 18+1$
$= 17+2$
$= 16+3$

$$= \sin\left(\frac{4\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{4\pi}{3}\right)$$



Pics

$$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{2}\right)$$

$$= \frac{-\sqrt{3}-1}{2\sqrt{2}}$$

If I ask "in simplified form"...

$$= \left(\frac{-\sqrt{3}-1}{2\sqrt{2}}\right)\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{-\sqrt{3}\sqrt{2} - \sqrt{2}}{2\underbrace{\sqrt{2}\sqrt{2}}_{2}}$$

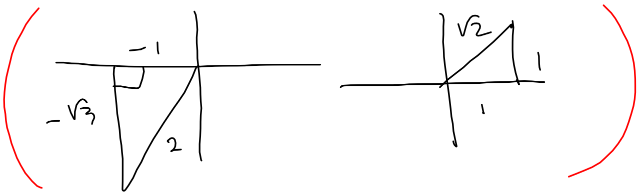
$$\boxed{= \frac{-\sqrt{6}-\sqrt{2}}{4} \overset{\text{OR}}{=} -\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) = \sin(u+v)}$$

Change-up $\sin(u+v) = \sin(u)\cos(v) + \sin(v)\cos(u)$

$$\sin\left(\frac{4\pi}{3}-\frac{\pi}{4}\right) = \sin\left(\frac{4\pi}{3}+\left(-\frac{\pi}{4}\right)\right)$$

$$= \sin\left(\frac{4\pi}{3}\right)\cos\left(-\frac{\pi}{4}\right) + \sin\left(-\frac{\pi}{4}\right)\cos\left(\frac{4\pi}{3}\right)$$

$$= \sin\left(\frac{4\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{4\pi}{3}\right)$$



$$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{2}\right) = \boxed{\frac{-\sqrt{3}+1}{2\sqrt{2}}} = \boxed{\text{simplified } \frac{-\sqrt{6}+\sqrt{2}}{4}}$$

$\frac{4}{5}$ or $\frac{8}{10}$

$$\cos\left(\frac{4\pi}{3}+\frac{\pi}{4}\right)$$

$$= \cos\left(\frac{4\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{4\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$$

$$= \left(-\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) = \frac{-1+\sqrt{3}}{2\sqrt{2}} = \boxed{\frac{-\sqrt{2}+\sqrt{6}}{4} = \cos(u+v)}$$

$$\boxed{\frac{-\sqrt{6}-\sqrt{2}}{4} = \sin(u+v)}$$

$$\text{So, } \tan\left(\frac{4\pi}{3}+\frac{\pi}{4}\right) = \frac{\frac{-\sqrt{6}-\sqrt{2}}{4}}{\frac{-\sqrt{2}+\sqrt{6}}{4}} = \frac{-\sqrt{6}-\sqrt{2}}{4}\cdot\frac{4}{-\sqrt{2}+\sqrt{6}}$$

$$= \frac{-\sqrt{6}-\sqrt{2}}{-\sqrt{2}+\sqrt{6}} = \frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}} = \left(\frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\left(\frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)$$

$(a-b)(a+b) = a^2-b^2$

$(a+b)^2 = a^2+2ab+b^2$

$$= \frac{(\sqrt{6}+\sqrt{2})^2}{6-2}$$

$$= \frac{\sqrt{6}^2+2\sqrt{6}\sqrt{2}+\sqrt{2}^2}{4}$$

$$= \frac{6+2\sqrt{12}+2}{4} = \frac{8+2\sqrt{12}}{4} = \boxed{\frac{4+\sqrt{12}}{2} = \tan(u+v)}$$

Double Angle

$\sin(2x) = 2\sin(x)\cos(x)$

Proof: $\sin(x+x) = \sin(x)\cos(x) + \sin(x)\cos(x) = 2\sin(x)\cos(x)$ ∎

$\cos(2x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x) = \cos^2(x) - \sin^2(x)$

One may be better than another in a given situation

Power-Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

$\cos(2u) = 1 - 2\sin^2(u)$

$\Rightarrow \cos(2u) - 1 = -2\sin^2(u)$

$\Rightarrow \dfrac{\cos(2u) - 1}{-2} = \sin^2(u)$

$\boxed{\dfrac{1 - \cos(2u)}{2} = \sin^2(u)}$

This leads to HALF-ANGLE Formulas:

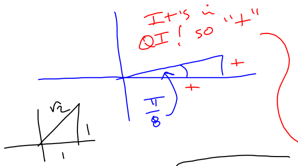
$\sin^2(u) = \dfrac{1 - \cos(2u)}{2} \Rightarrow$

$\sin(u) = \pm\sqrt{\dfrac{1 - \cos(2u)}{2}}$ whether it's "+" or "−" will depend on the picture for u.

This is used for getting exact answers for this sort of situation:

$\sin\left(\frac{\pi}{8}\right) = \sin(22.5^\circ)$ Don't know.

But $2\left(\frac{\pi}{8}\right) = \frac{\pi}{4}$, $2(22.5^\circ) = 45^\circ$ Do know that.



$u = \frac{\pi}{8} \Rightarrow 2u = \frac{\pi}{4}$

$\sin\left(\frac{\pi}{8}\right) = \pm\sqrt{\dfrac{1 - \cos\left(\frac{\pi}{4}\right)}{2}}$ which? POSITIVE

$\sin\left(\frac{\pi}{8}\right) = \sqrt{\dfrac{1 - \cos\left(\frac{\pi}{4}\right)}{2}}$

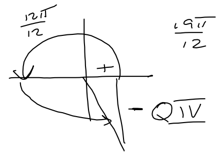
$= \sqrt{\dfrac{1 - \frac{1}{\sqrt{2}}}{2}} = \sqrt{\dfrac{\frac{\sqrt{2} - 1}{\sqrt{2}}}{2}} = \sqrt{\dfrac{\sqrt{2} - 1}{\sqrt{2}} \cdot \dfrac{1}{2}}$

$= \sqrt{\dfrac{\sqrt{2} - 1}{2\sqrt{2}}} = \sqrt{\left(\dfrac{\sqrt{2} - 1}{2\sqrt{2}}\right) \cdot \dfrac{\sqrt{2}}{\sqrt{2}}} = \sqrt{\dfrac{2 - \sqrt{2}}{4}} = \dfrac{\sqrt{2 - \sqrt{2}}}{\sqrt{4}}$

$= \dfrac{\sqrt{2 - \sqrt{2}}}{2}$

$\sin\left(\frac{19\pi}{12}\right) = \sin\left(\frac{1}{2} \cdot \frac{19\pi}{6}\right)$ $u = \frac{19\pi}{12}$, $2u = \frac{19\pi}{6}$

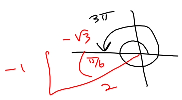
$= -\sqrt{\dfrac{1 - \cos\left(\frac{19\pi}{6}\right)}{2}}$



$= -\sqrt{\dfrac{1 + \frac{\sqrt{3}}{2}}{2}} = -\sqrt{\dfrac{\frac{2 + \sqrt{3}}{2}}{2}} = -\dfrac{\sqrt{2 + \sqrt{3}}}{\sqrt{4}} = -\dfrac{\sqrt{2 + \sqrt{3}}}{2}$ from $\frac{1}{2}$-angle

$\frac{19\pi}{6} = \frac{18\pi}{6} + \frac{\pi}{6}$

$= 3\pi + \frac{\pi}{6}$



$= \dfrac{-\sqrt{3} - 1}{2\sqrt{2}}$?! from $\sin(u+v)$!

Book formula:

$\sin\left(\frac{u}{2}\right) = \pm\sqrt{\dfrac{1 - \cos(u)}{2}}$

I did this:

$\sin(u) = \pm\sqrt{\dfrac{1 - \cos(2u)}{2}}$