Questions?

Physical Geology –

§2.1 ? How far?

Done with test? Break trail on new stuff

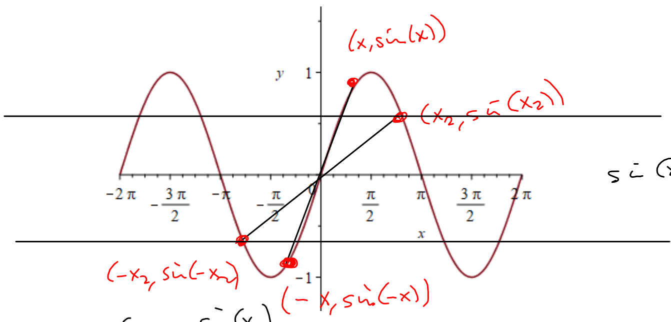
§2.1 Right outta the gate:

Table Reciprocal Identities $\csc\theta = \frac{1}{\sin\theta}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{y}{r}}{\frac{x}{r}} = \frac{y}{r}\cdot\frac{r}{x} = \frac{y}{x} = \tan\theta$$

$+ \quad -$

Even/Odd



$y = \sin(x_2)$

$\sin(x)$ is odd

$y = \sin(-x_2) = -\sin(x_2)$

ODD

$= (-x_2, -\sin(x_2))$

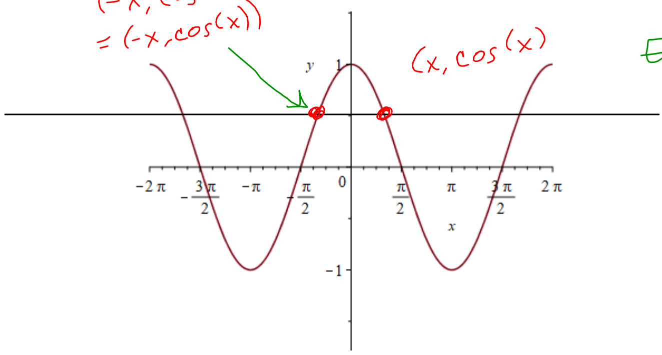
You can't, generally, factor out of a trig function. $\sin(5x) \neq 5\sin(x)$

Nooooo!

But $\sin(-x) = -\sin(x)$

So you can factor out a -1 in an odd trig func.

$\cos(x)$ is even



EVEN!

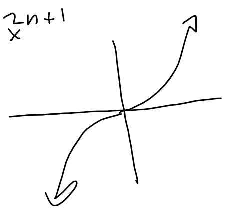
$\cos(-x) = \cos(x)$

Combos

$\sin(x)\cos(x) = (-)(+) = -$ ODD

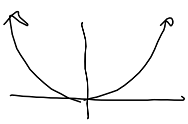
$\tan(x) = \frac{\sin(x)}{\cos(x)} = \frac{-}{+} = -$ is ODD

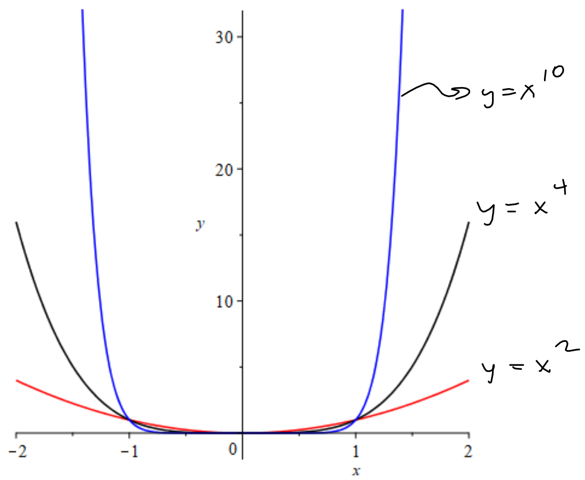
x^{2n+1} is $x^{ODD} = x^{2(3)+1} = x^7$



x^{2n} is x^{EVEN}

x^2, x^8, x^{242}





x^{2n} is even!

$$\frac{x^2\cos(x) + x^3\sin(x)}{x\tan(x)} = \frac{(+)(+) + (-)(-)}{(-)(-)}$$

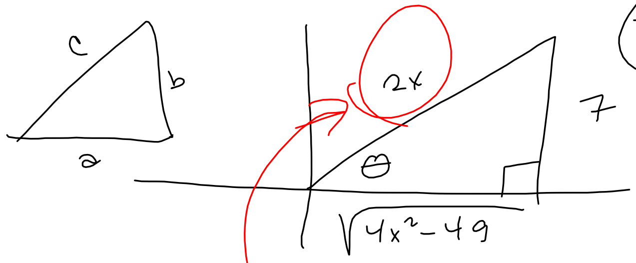
$$= \frac{(+)+(+)}{(+)} = \frac{(+)}{(+)} = +$$

is even.

$$\frac{x^2\cos(x) + x^2\sin(x)}{x\tan(x)} = \frac{(+)(+) + (+)(-)}{(-)(-)} = \frac{(+) + (-)}{(+)}$$

Neither!

$$\sin\left(\arctan\left(\frac{7}{\sqrt{4x^2-49}}\right)\right) = \sin\theta$$



$$\left(\sqrt{4x^2-49}\right)^2 + 7^2 = c^2$$

$$4x^2-49+49 = 4x^2 = c^2$$

$$4x^2 = c^2$$

$$\sqrt{4x^2} = \sqrt{c^2}$$

$$|2x| = |c|$$

$$2x = \pm c$$

These are in QI

$$2x = c$$

$$\boxed{\sqrt{x^2} = |x| \qquad \left(\sqrt{x}\right)^2 = x}$$

$$\sin^2 x + \cos^2 x = 1 \Rightarrow 1-\cos^2 x = \sin^2 x$$

$$\sqrt{1-\cos^2 x} = \sqrt{\sin^2(x)} = \sqrt{(\sin(x))^2}$$

$$= |\sin(x)|, \text{ NOT } \sin(x)$$

Use Looseleaf, not spiral.

For Trailblazing new material

BOX STUFF

Rest of page blank.

Add stuff to it, later

If more than one page, no problem

GRUNT

Messy, figuring it out

Handed-in

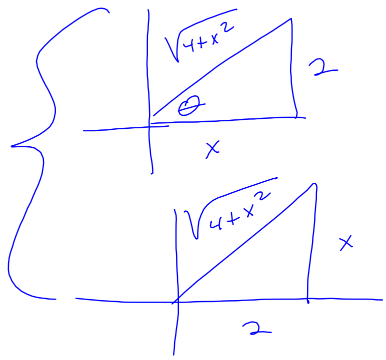
Written-up nicely.

Trig substitution

Draw the doggone triangles!

$\sqrt{4+x^2}$ Let $x = 2\tan\theta$

Both perfectly legit.



$\frac{x}{2} = \cot\theta$

$x = 2\cot\theta$

$\frac{x}{2} = \tan\theta$

$x = 2\tan\theta$

$\int \sqrt{x^2+4}\, dx$ is hard

$\int \sqrt{(2\tan\theta)^2+4}\, dx$

$= \int \sqrt{4\tan^2\theta+4}\, dx = \int \sqrt{4(\tan^2\theta+1)}\, dx$

$= \int \sqrt{4}\sqrt{\tan^2\theta+1}\, dx = \int 2\sqrt{\tan^2\theta+1}\, dx$

$= 2\int \sqrt{\sec^2\theta}\, dx = 2\int |\sec\theta|\, dx$

$= \begin{cases} 2\int \sec\theta\, dx & \text{if } \sec\theta \geq 0 \\ -2\int \sec\theta\, dx & \text{if } \sec\theta < 0 \end{cases}$

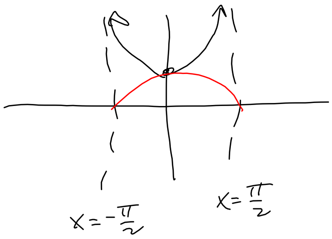
$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

$|3| = 3$

$|-3| = 3 = -(-3) = -x$

The book will restrict the domain to

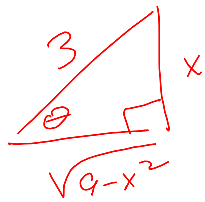
$\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$ $\quad (-\frac{\pi}{2} < \theta < \frac{\pi}{2})$



with this restriction, $|\sec(\theta)| = \sec\theta$, so the book hides the absolute-value considerations for those to get you through trig & set you up for FAIL in calculus.

So I ALWAYS hit the $|\cos(x)| = \sqrt{\cos^2(x)}$

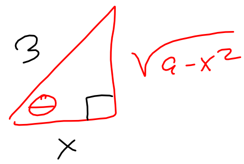
$\sqrt{9-x^2}$



$\frac{x}{3} = \sin\theta$

$x = 3\sin\theta$

or



$\frac{x}{3} = \cos\theta$

$x = 3\cos\theta$

Both legit.

$\sqrt{x^2-16}$

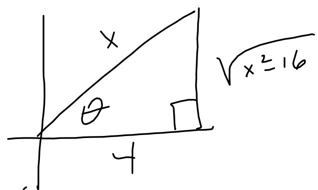
What's a good substitution?

$x = ?$

Buehler?

$x = 4\sin\theta$?

$x = 4\cos\theta$?

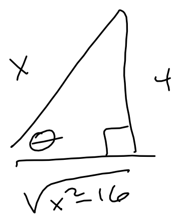


$\frac{4}{x} = \cos\theta$

$\frac{x}{4} = \sec\theta$

$x = 4\sec\theta$

usually easier to work with



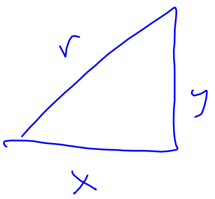
$\frac{4}{x} = \sin\theta$

$\frac{x}{4} = \csc\theta$

$x = 4\csc\theta$

sine, tangent, secant generally better to work with.

$x^2+y^2=r^2$

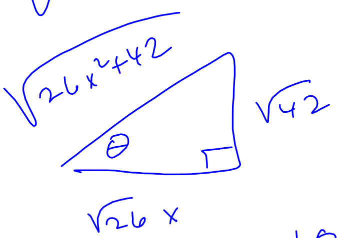


$\sqrt{x^2+y^2} = \sqrt{r^2}$

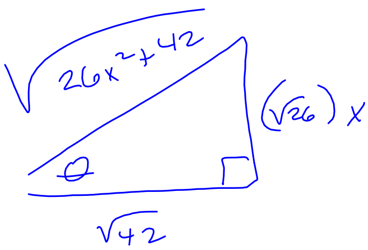
$\sqrt{r^2-x^2} = \sqrt{y^2}$

$\sqrt{r^2-y^2} = \sqrt{x^2}$

$\sqrt{26x^2+42}$



... $x = \sqrt{7}\cot\theta$



$\frac{\sqrt{26}\,x}{\sqrt{42}} = \tan\theta$

$x = \sqrt{\frac{42}{26}}\tan\theta$

$= \sqrt{\frac{21}{3}}\tan\theta$

$= \frac{\sqrt{63}}{3}\tan\theta$

$= \frac{3\sqrt{7}}{3}\tan\theta$

$= \sqrt{7}\tan\theta = x$

$$\frac{\sec^2(x)-1}{\sin^2(x)} = \frac{\tan^2(x)}{\sin^2(x)} = \frac{\frac{\sin^2(x)}{\cos^2(x)}}{\sin^2(x)} = \frac{\sin^2(x)}{\cos^2(x)}\cdot\frac{1}{\sin^2(x)}$$

$$= \frac{1}{\cos^2(x)} = \sec^2(x)$$

Recall:
Rationalizing Denominators

$$\left(\frac{3}{1+\sqrt{2}}\right)\left(\frac{1-\sqrt{2}}{1-\sqrt{2}}\right) = \frac{3-3\sqrt{2}}{1^2-\sqrt{2}^2} = \frac{3-3\sqrt{2}}{1-2} =$$

$$\frac{3-3\sqrt{2}}{-1} = 3\sqrt{2}-3$$

Conjugates
$z = a+bi \Rightarrow \bar{z} = a-bi$ is conjugate
Generally:

$1+\sqrt{2} \longrightarrow 1-\sqrt{2}$

Trig:

$1+\sin(x) \longrightarrow 1-\sin(x)$

$$\left(\frac{1}{1+\sin(x)}\right)\left(\frac{1-\sin(x)}{1-\sin(x)}\right) = \frac{1-\sin(x)}{1-\sin^2(x)} = \frac{1-\sin(x)}{\cos^2(x)}$$

$$= \frac{1}{\cos^2(x)} - \frac{\sin(x)}{\cos^2(x)} = \sec^2 x - \sin(x)\sec^2(x)$$

$$\tan(x) - \frac{\sec^2(x)}{\tan(x)}$$

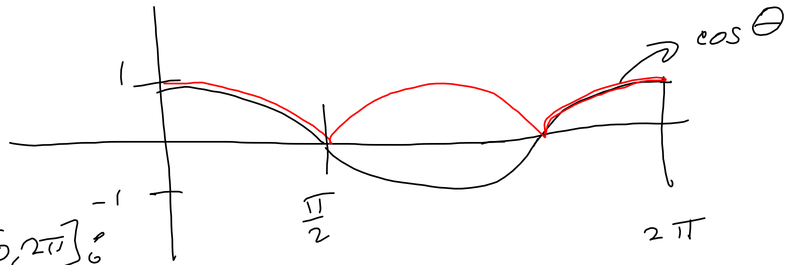
$$= \frac{\sin(x)}{\cos(x)} - \frac{\frac{1}{\cos^2(x)}}{\frac{\sin(x)}{\cos(x)}} = \frac{\sin(x)}{\cos(x)} - \frac{1}{\cos^2(x)}\cdot\frac{\cancel{\cos(x)}}{\sin(x)}$$

$$= \frac{\sin(x)}{\cos(x)}\cdot\frac{\sin(x)}{\sin(x)} - \frac{1}{\sin(x)}\cdot\frac{\cos(x)}{\cos(x)} = \frac{\sin^2(x)-\cos(x)}{\sin(x)\cos(x)}$$

$LCD = \sin(x)\cos(x)$

$$= \frac{1-\cos^2(x)-\cos(x)}{\sin(x)\cos(x)}$$

$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{\cos^2\theta} = |\cos\theta|$



on $[0, 2\pi]$:

$\cos\theta = |\cos\theta| \Longrightarrow \boxed{\left[0, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2}, 2\pi\right]}$

$\cos\theta = \begin{cases} \cos\theta & \text{if } \cos\theta \geq 0 \\ -\cos\theta & \text{if } \cos\theta < 0 \end{cases}$ $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

Answer to §2.1 #64

Be ready w/ §2.2 & maybe §2.3 questions next time.

I'll start archiving videos on harryzaims.com. Look for announcement.