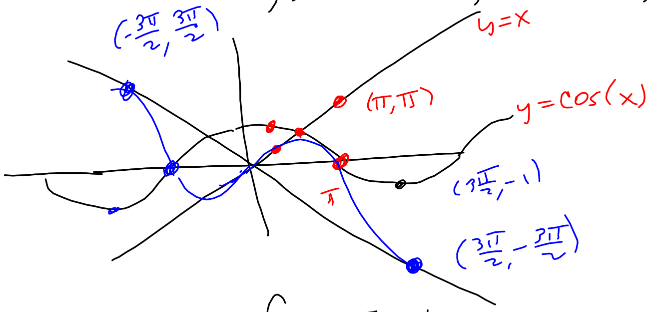
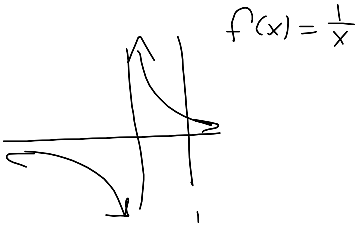
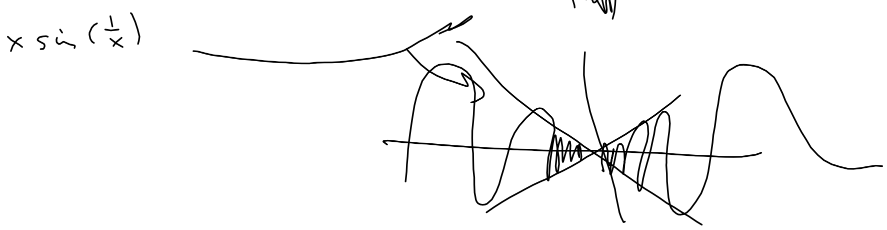
Test 1 Wednesday, tentatively. Probably next Monday





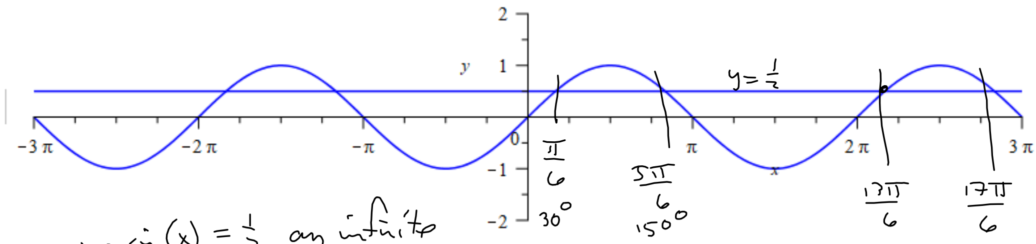
No damped trig funcs Test 1.



Inverse functions

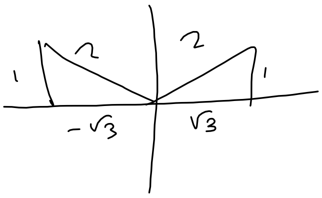
$\arccos(x) = \cos^{-1}(x)$

Go from y-value, back to the x that gave it to us.



$y = \sin(x) = \frac{1}{2}$ an infinite # of times. We want a unique output for $\sin^{-1}(x) = \arcsin(x)$ (not $\frac{1}{\sin(x)}$)

$\sin(x) = \frac{1}{2}$ $\quad \sin(x) = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \ldots$

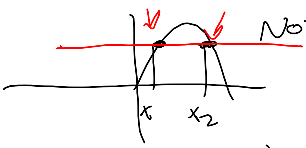


$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$x = \frac{\pi}{6} \pm 2\pi n \quad \forall n \in \mathbb{Z}$

$\frac{5\pi}{6} \pm 2\pi n$

The issue is that we only get one output for the FUNCTION $\arcsin(x)$. For an inverse to be a function, the original function must be 1-to-1.

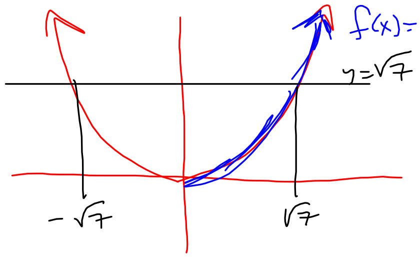


Not 1-to-1

$x_1 \neq x_2$, but $y_1 = y_2$

So the inverse isn't a function.

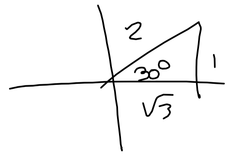
Answer is to restrict the domain of the original



$f(x) = x^2$ not 1-to-1, but $f^{-1}(x) = \sqrt{x}$ is its inverse if you restrict its domain to $x \geq 0$.

$x^2 = 7 \Rightarrow$
$x = \pm\sqrt{7}$

Restrict sine to $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$. That's what your calculator does.

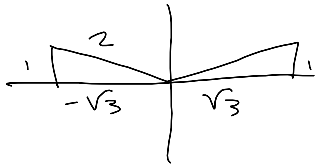


$\boxed{\text{Question}}$ Solve $\sin(x) = \frac{1}{2}$ on $[0, 2\pi]$

$\sin^{-1}(\sin(x)) = \sin^{-1}(\frac{1}{2}) = \frac{\pi}{6} = 30^\circ$

But what if I told you I also want $\cos(x) < 0$?

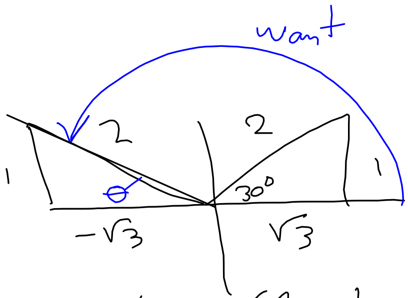
$\sin(x) = \frac{1}{2}$



$x = \frac{\pi}{6}$, $x = \frac{5\pi}{6}$

Your calculator only sees the $x = \frac{\pi}{6}$

That's where reference angles come in:



Similar triangles.

$\theta' = 30^\circ$ (less than 180°)

So we want $\theta = 180^\circ - 30^\circ = 150^\circ$

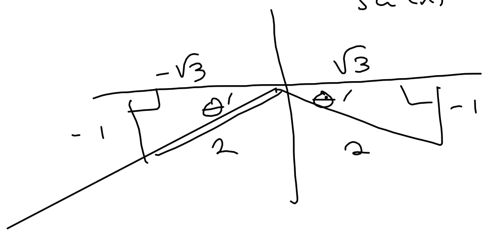
$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

Solve $\sin(x) = -\frac{1}{2}$ $\forall\ x \in [0, 2\pi]$

$\sin^{-1}(-\frac{1}{2}) = -30^\circ$ $(-\frac{\pi}{6})$

But $-\frac{\pi}{6} \notin [0, 2\pi]$

$\sin(x) = -\frac{1}{2}$



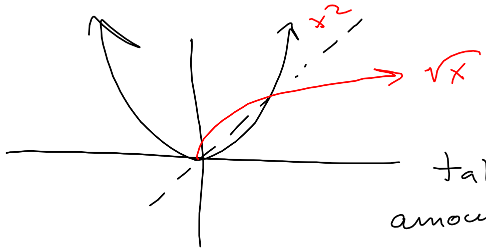
Calc: -30°

so you know $\theta' = 30^\circ$

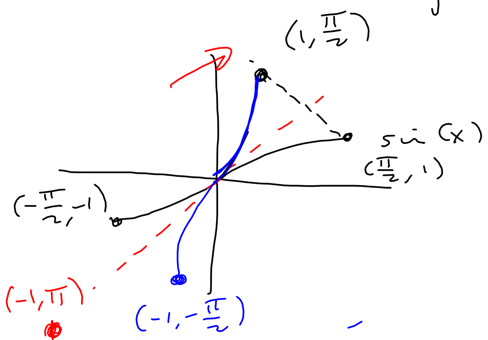
$360^\circ - 30^\circ = 330^\circ = \frac{11\pi}{6}$

$180^\circ + 30^\circ = 210^\circ = \frac{7\pi}{6}$

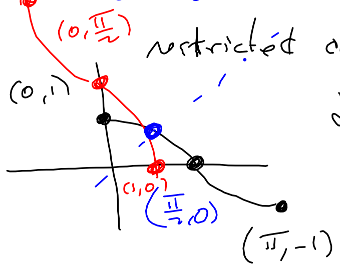
$(210^\circ)(\frac{\pi}{180^\circ}) = \frac{7\pi}{6}$



$f(x)$ & $f^{-1}(x)$ basically take $(x,y) \longmapsto (y,x)$. This amounts to a reflection about $y = x$.



$\sin(x)$ & $\sin^{-1}(x) = \arcsin(x)$ graphed together.



restricted cosine: Restriction

$\mathcal{D}(\cos(x)) = [0, \pi] = \mathcal{R}(\arccos(x))$

$\mathcal{R}(\cos(x)) = [-1, 1] = \mathcal{D}(\arccos(x))$

$\cos^2\theta + \cos\theta = 0$ on $[0, 2\pi]$

$u^2 + u = 0$

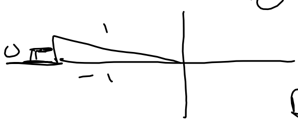
$u(u+1) = 0$

$u = 0$ OR $u + 1 = 0$

$\cos\theta = 0$ $\quad$ $\cos\theta + 1 = 0$

$\cos\theta = -1$

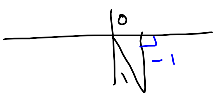
$\theta = 90^\circ = \frac{\pi}{2}$



$\theta = \pi$

$\cos^{-1}(-1) = 180^\circ = \pi$

Degenerate triangles from Quadrant angles.

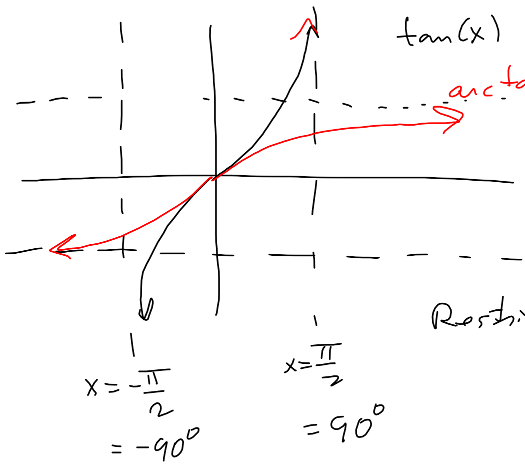


$\theta = 270^\circ = \frac{3\pi}{2}$

$\cos\theta = 0$

$\arccos(0) = 90^\circ = \frac{\pi}{2}$

YOU need to use this to find $\theta = 270^\circ = \frac{3\pi}{2}$



$\tan(x)$ period is $\pi = 180^\circ$

$\arctan(x) = \tan^{-1}(x)$ $\quad y = \frac{\pi}{2}$

$y = -\frac{\pi}{2}$

$= \mathcal{R}(\arctan(x))$

Restricted tangent: $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) = \mathcal{D}$

$\mathcal{R}(\tan(x)) = (-\infty, \infty)$

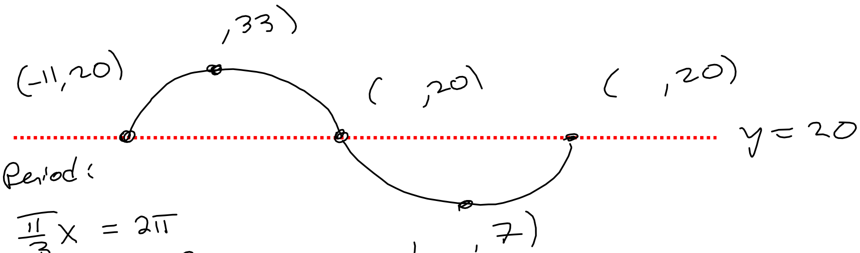
$= \mathcal{D}(\arctan(x))$

$13\sin\left(\frac{\pi}{3}x + \frac{11\pi}{3}\right) + 20$

$= 13\sin\left(\frac{\pi}{3}(x+11)\right) + 20$ → $y=20$ is midline

Amplitude

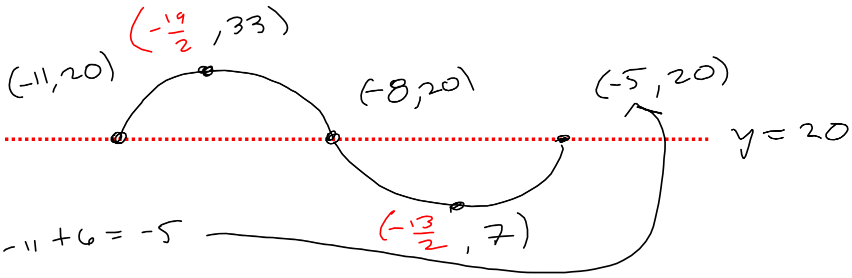
start: $x=-11$



Period:

$\frac{\pi}{3}x = 2\pi$

$x = 2\pi \cdot \frac{3}{\pi} = 6$ = Period!



$-11 + 6 = -5$

$T=6$

$\frac{T}{4} = \frac{6}{4} = \frac{3}{2}$ = step size or increment

$-11 + \frac{3}{2} = \frac{-22+3}{2} = -\frac{19}{2}$

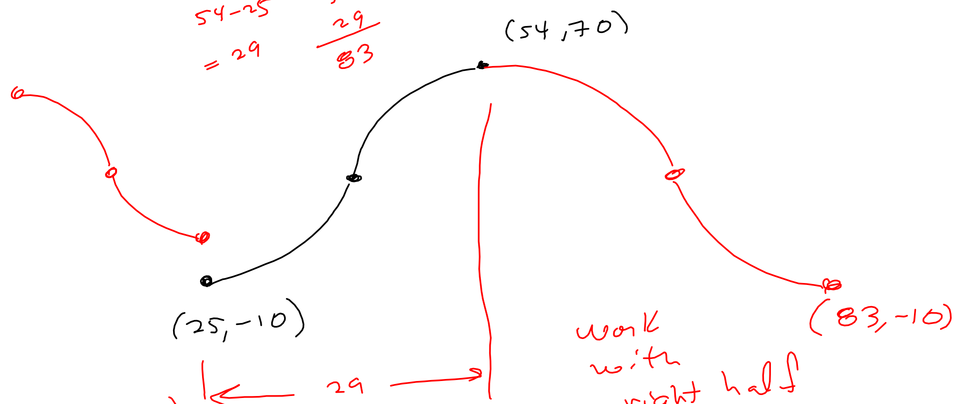
$-\frac{19}{2} + \frac{3}{2} = -\frac{16}{2} = -8$

$-\frac{16}{2} + \frac{3}{2} = -\frac{13}{2}$

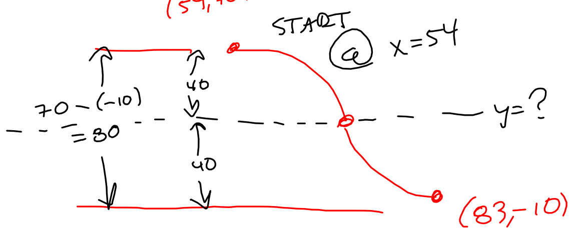
$-\frac{13}{2} + \frac{3}{2} = \frac{-10}{2} = -5$

7. (10 pts) Write the cosine function that achieves its maximum height of $y = 70$ centimeters at time $t = 54$ seconds and its minimum height of $y = -10$ centimeters at $t = 25$ seconds.

$54-25 = 29$ $\quad 54 + 29 = 83$



work with right half



Looking for
$a\cos(b(x-c))+d$
$a\cos(b(x-54))+d$
$40\cos(b(x-54))+d$

d = mid line = $\frac{70 + -10}{2}$ = Average of high & low

$\frac{60}{2} = 30$ $\quad 40\cos(b(x-54))+30$

Now for period:
High to low is $\frac{1}{2}$ period.

$x=54$ to $x=83$
$83-54 = 29 = \frac{1}{2}T$
$58 = T$ = period

Want b so that
$bx = 2\pi$ when $x = 58$
$58b = 2\pi$
$b = \frac{2\pi}{58} = \frac{\pi}{29}$

$$\boxed{40\cos\left(\frac{\pi}{29}(x-54)\right)+30}$$

Test 1 Monday
in person wear mask inside.