2 −3 6 65

$2x^3 - 3x^2 + 6x + 65 = f(x)$. we find $f(3)$:

a

10 pts

$$\begin{array}{r|rrrr} 3 & 2 & -3 & 6 & 65 \\ & & 6 & 9 & 45 \\ \hline & 2 & 3 & 15 & \boxed{110 = P(3)} \end{array}$$

b

10 pts

Check $f(2+3i)$:

$$\begin{array}{r|rrrr} 2+3i & 2 & -3 & 6 & 65 \\ & & 4+6i & -16+15i & -65 \\ \hline & 2 & 1+6i & -10+15i & 0 \end{array}$$ sweet!

$f(2+3i) = 0$, so $f(2-3i)$ does also,

by CPT = Conjugate-pairs theorem.

$(2+3i)(2) = 4+6i$

$(2+3i)(1+6i) = 2 + 12i + 3i + 18i^2 = 2 + 15i - 18 = -16+15i$

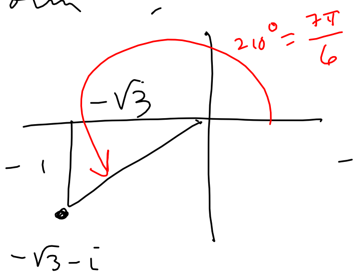
$(2+3i)(-10+15i) = -5(2+3i)(2-3i) = -5(2^2+3^2) = -5(4+9) = -65$!

2a $z = -\sqrt{3} - i \implies \bar{z} = -\sqrt{3} + i$

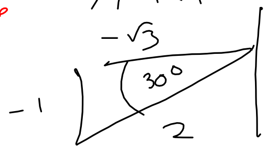
$\boxed{z + \bar{z} = -2\sqrt{3}}$ 10 pts

$z\bar{z} = a^2 + b^2 = (-\sqrt{3})^2 + (1)^2 = 3 + 1 = \boxed{4 = z\bar{z}}$

b Trig form ?



Aha!



so $180^\circ + 30^\circ = 210^\circ = \boxed{\frac{7\pi}{6}}$

$= -\sqrt{3} - 1 \cdot i = a + bi$

and $\|z\| = 2$

$a = -\sqrt{3},\ b = -1$

So, $z = 2\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$

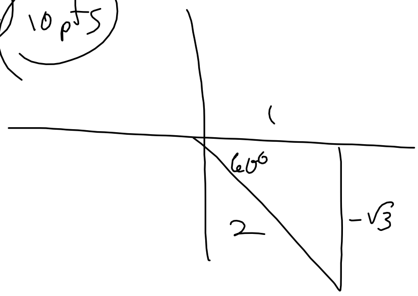
$\|z\| = \sqrt{a^2 + b^2} = \sqrt{(-\sqrt{3})^2 + (-1)^2} = \sqrt{3+1} = \sqrt{4} = 2$

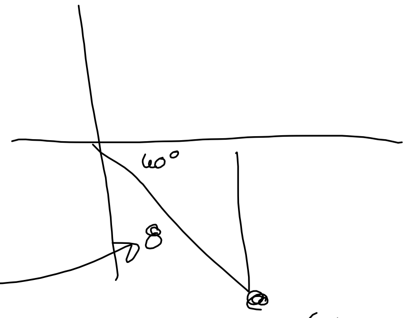
$$\boxed{z = 2\left(\cos\frac{7\pi}{6} + i\sin\left(\frac{7\pi}{6}\right)\right)}$$

3) $z = 8\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$

$\frac{5\pi}{3} \cdot \frac{180}{\pi} = 5 \cdot 60 = 300^\circ$

2) 10 pts





$(4, -4\sqrt{3})$

$4 - 4\sqrt{3}\,i$

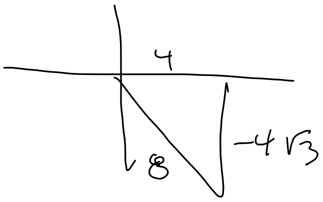
$8\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$

So, similar triangles

$\frac{8}{2} = 4$ times bigger on 2nd one, so,

$1 \cdot 4 = 4$

$-\sqrt{3} \cdot 4 = -4\sqrt{3}$



Standard form:

$\boxed{4 - 4i\sqrt{3} = z}$

or $4 - 4\sqrt{3}\,i$

3b 10 pts $\sqrt[3]{z}$, where $z = 8\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$

$\sqrt[3]{8} = \sqrt[3]{2^3} = 2$

3^{rd} root : $\frac{1}{3}\cdot\frac{5\pi}{3} = \frac{5\pi}{9}$, so

$$\sqrt[3]{z} = 2\left(\cos\frac{5\pi}{9} + i\sin\frac{5\pi}{9}\right)$$

c Find other 2 3^{rd} roots:

increment is $\frac{2\pi}{3}$ b/c $\underline{\underline{3}}^{rd}$ root

So $\frac{5\pi}{9} + \frac{2\pi}{3} = \frac{5\pi}{9} + \frac{6\pi}{9} = \frac{11\pi}{9}$

$\frac{11\pi}{9} + \frac{6\pi}{9} = \frac{17\pi}{9}$, so

The other 2 3^{rd} roots are:

$$2\left(\cos\frac{11\pi}{9} + i\sin\frac{11\pi}{9}\right),\ 2\left(\cos\frac{17\pi}{9} + i\sin\frac{17\pi}{9}\right)$$

3d $z^2 = 8^2\left(\cos\left(2\cdot\frac{5\pi}{3}\right) + i\sin\frac{10\pi}{3}\right)$

$= 64\left(\cos\left(\frac{10\pi}{3}\right) + i\sin\left(\frac{10\pi}{3}\right)\right)$

square the length, double the angle

$$z^2 = r^2\left(\cos(2\theta) + i\sin(2\theta)\right)$$

n^{th} roots:

$$z^{\frac{1}{n}} = r^{\frac{1}{n}}\left(\cos\left(\frac{\theta + 2k\pi}{n}\right) i\sin\left(\frac{\theta+2k\pi}{n}\right)\right)$$

$k = 0, 1, 2, \ldots, n-1$

n^{th} power:

$$z^n = r^n\left(\cos(n\theta) + i\sin(n\theta)\right)$$

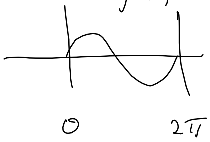
4 10pts $3\csc^3(2\theta) - 6\csc^2(2\theta) - \csc(2\theta) + 2 = 0$

Find All Sol'ns in $[0, 2\pi)$

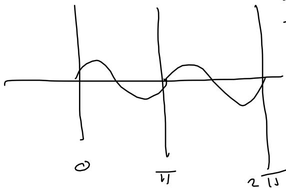
$x \in [0, 2\pi)$ means $0 \le \theta < 2\pi$, so

That means we're looking for $0 \le 2\theta < 4\pi$

trig(x)



trig(2x) Squashes x-axis horizontally Twice as much going on



So when we solve for 2x, we'll have some thinking to do to get to x.

$f(x) = 3x^3 - 6x^2 - x + 2$

$f(2) = 0$

$$\begin{array}{r|rrrr} 2 & 3 & -6 & -1 & 2 \\ & & 6 & 0 & -2 \\ \hline & 3 & 0 & -1 & 0 \end{array}$$

$3x^2 - 1 = 0$

$3x^2 = 1$

$x^2 = \frac{1}{3}$

$x = \pm\sqrt{\frac{1}{3}} = \pm\frac{1}{\sqrt{3}}$

$(x-2)(3x^2 - 1)$

$3(x-2)\left(x - \frac{1}{\sqrt{3}}\right)\left(x + \frac{1}{\sqrt{3}}\right)$

$= 3(x-2)\left(x - \frac{\sqrt{3}}{3}\right)\left(x + \frac{1}{\sqrt{3}}\right)$

$3x^2 - 1 = 0$

$3x^2 + 0x - 1 = 0$

$a = 3,\ b = 0,\ c = -1$

$b^2 - 4ac = 0^2 - 4(3)(-1)$

$= 12$

$\sqrt{12} = 2\sqrt{3}$

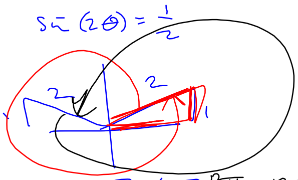
$x = \frac{0 \pm 2\sqrt{3}}{2(3)} = \frac{\pm 2\sqrt{3}}{6}$

$= \pm\frac{\sqrt{3}}{3} = \pm\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$x = 2$

$\csc(2\theta) = 2$

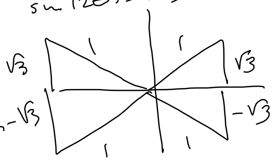
$\sin(2\theta) = \frac{1}{2}$



$x = \pm\frac{1}{\sqrt{3}}$

$\csc(2\theta) = \pm\frac{1}{\sqrt{3}}$

$\sin(2\theta) = \pm\sqrt{3}$



$2\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

un-possible

$\frac{\pi}{6} + 2\pi = \frac{\pi + 12\pi}{6} = \frac{13\pi}{6}$

$\frac{5\pi}{6} + \frac{12\pi}{6} = \frac{17\pi}{6}$

$$\boxed{\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}}$$ is the whole solution!

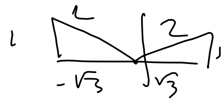
#5 Polar Graph is Bonus this semester.

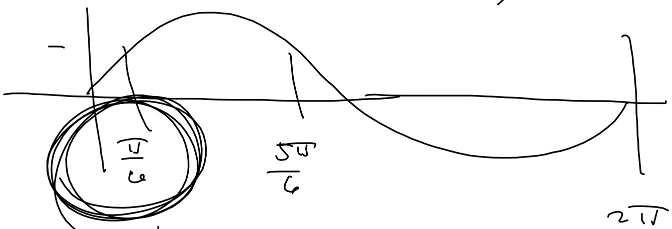
$\sin(2x) = \frac{1}{2}$

$\sin(3x) = \frac{1}{2}$ All solutions $x \in [0, 2\pi)$

Then look for all $3x \in [0, 6\pi)$ that work!

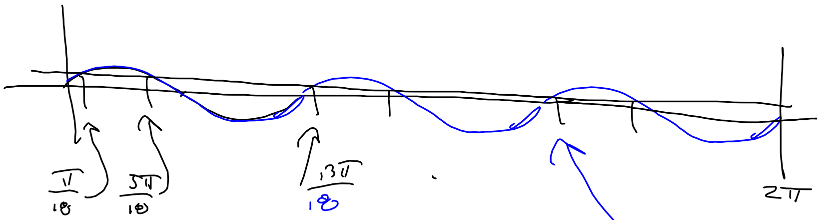
$\sin(x) = \frac{1}{2}$





$\sin 3x = \frac{1}{2}$

$\sin(3x)$



$\frac{\pi}{6} + 2\pi = \frac{\pi}{6} + \frac{12\pi}{6} = \frac{13\pi}{6}$

$\frac{13\pi}{6} + 2\pi = \frac{25\pi}{6} \rightsquigarrow \frac{25\pi}{18}$

Work it, baby!

$\sin(3x) = \frac{1}{2}$

$3x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{25\pi}{6}, \frac{29\pi}{6}$

$\Rightarrow x = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}$