$f(x) = 2x^3 - 7x^2 + 10x - 6$

Remainder Theorem:

To find $f(3)$, divide by $x-3$ & pluck the remainder $= f(3)$

Recall $\frac{28}{3} = \frac{27}{3} + \frac{1}{3} = 9 + \frac{1}{3}$

Means $28 = 3 \cdot 9 + 1$

$$
\begin{array}{r l}
 & 2x^2 - x + 7 \quad r\ 15 \\
x-3 \,\big) & 2x^3 - 7x^2 + 10x - 6 \\
\frac{2x^3}{x} = 2x^2 & -(2x^3 - 6x^2) \\
 & -1x^2 + 10x - 6 \\
\frac{-x^2}{x} = -x & -(-1x^2 + 3x) \\
 & 7x - 6 \\
\frac{7x}{x} = 7 & -(7x - 21) \\
 & 15
\end{array}
$$

This says

$$\frac{2x^3 - 7x^2 + 10x - 6}{x-3} = 2x^2 - x + 7 + \frac{15}{x-3}$$

Also means

$$2x^3 - 7x^2 + 10x - 6 = (x-3)(2x^2 - x + 7) + 15$$

Says $f(3) = 15$!

$$
\begin{array}{r|rrrr}
3 & 2 & -7 & 10 & -6 \\
 & & 6 & -3 & 21 \\
\hline
 & 2 & -1 & 7 & \boxed{15 = f(3)} \\
 & x^2 & x & c & r
\end{array}
$$

$f(x) = (x-3)(2x^2 - x + 7) + 15$

Remainder $= 0$ is our fave!

Show that $1+i$ is a zero of $f(x)$, i.e., $f(1+i) = 0$. Divide by $x - (1+i)$

$$
\begin{array}{r|rrrr}
1+i & 2 & -7 & 10 & -6 \\
 & & 2+2i & -7-3i & 6 \\
\hline
1-i & 2 & -5+2i & 3-3i & 0 = f(1+i) \\
 & & 2-2i & -3+3i & \\
\hline
 & 2 & -3 & 0 &
\end{array}
$$

& $x-(1+i)$ is a factor

This says $\boxed{f(x) = (x-(1+i))(x-(1-i))(2x-3)}$

$2(1+i) = 2+2i$

$(1+i)(-5+2i) = -5+2i-5i+2i^2$

$= -5-3i-2 = -7-3i$

$(1+i)(3-3i) = 3-3i+3i-3i^2 = 3+3 = 6$

$$\boxed{\begin{array}{l} z\bar{z} = a^2+b^2 \\ z+\bar{z} = 2a \end{array}}$$

$z = a+bi$

$\bar{z} = a-bi$

NOTE $z = 1+i \Longrightarrow \bar{z} = 1-i$

$(1+i)(1-i) = 1^2 - i^2 = 1^2+1$

$(a+bi)(a-bi) = a^2+b^2$

$(1+i)(3-3i) = (1+i)(1-i)(3) = (1^2+1^2)(3)$

$= 6$

§4.1 #39

$(4+8i)^2$

$(4+8i)^2 = 4^2 + 2(4)(8i) + (8i)^2$

$= 16 + 64i - 64$

$(8i)^2 = 8^2 i^2$

$= 64 i^2$

$= -64$

$(a+b)^2 = (a+b)(a+b) = a^2 + ab + ba + b^2$

$= a^2 + ab + ab + b^2$

$= a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

$(ab)^2 = a^2b^2$

$3(a+b) = 3a + 3b$

$\sqrt{-12} = i\sqrt{12}$

$= i \cdot 2\sqrt{3}$

$= 2i\sqrt{3}$

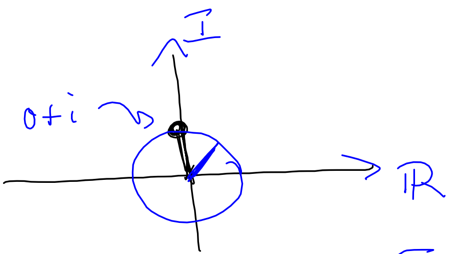
$2\sqrt{3}\,i$

$2\sqrt{3i}$ No!

Keep the i out from under the radical.

Why?

$\sqrt{i}$



$\sqrt{0^2+1^2}$

$\|a+bi\| = \sqrt{a^2+b^2}$

$\|i\| = 1$

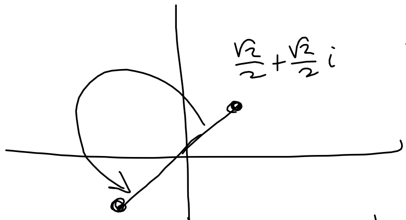
$\sqrt{i} = \sqrt[2]{i}$

θ for i is $\frac{\pi}{2}$

θ for $\sqrt{i}$ is $\frac{\frac{\pi}{2}}{2}$

$= \frac{\pi}{4}$

Increment $\frac{2\pi}{2} = \pi$

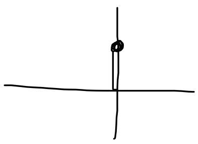


$-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$ is the 2nd 2nd root.

Easier to do this for trig. form

$i = 0 + 0i$ $\quad \|i\| = 1$

$\theta = \frac{\pi}{2}$



$i = 1\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$\sqrt{i} = \sqrt{1}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

& increment $\frac{2\pi}{2} = \pi$

$\frac{\pi}{4} + \pi$
$= \frac{\pi}{4} + \frac{4\pi}{4} = \frac{5\pi}{4}$

& the other 2nd root of i is $1\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$

Find all the 4th roots of $2i$

Length $= r = 2$

$2i$: $\theta = \frac{\pi}{2}$

Increment $\frac{2\pi}{4} = \frac{\pi}{2} = \frac{2\pi}{4}$ (for calc)

$\sqrt[4]{2i}$ & its magnitude is $\sqrt[4]{2}$

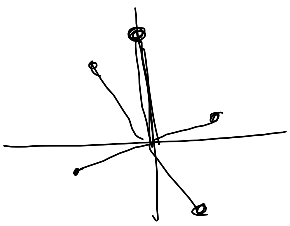
$\sqrt[4]{2i} = \sqrt[4]{2}\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right)$

$\frac{\pi}{8} + \frac{\pi}{2}\cdot\frac{4}{4} = \frac{\pi}{8} + \frac{4\pi}{8} = \frac{5\pi}{8}$

$\sqrt[4]{2}\left(\cos\frac{5\pi}{8} + i\sin\frac{5\pi}{8}\right)$

$\sqrt[4]{2}\left(\cos\frac{9\pi}{8} + i\sin\frac{9\pi}{8}\right)$

$\sqrt[4]{2}\left(\cos\frac{13\pi}{8} + i\sin\frac{13\pi}{8}\right)$



$\frac{\frac{\pi}{2}}{4} = \frac{\pi}{8}$

$\frac{13\pi}{8} + \frac{4\pi}{8} = \frac{17\pi}{8} \Leftrightarrow \frac{\pi}{8}$

Full circle.

n^{th} roots

$r^{\frac{1}{n}}\left(\cos\left(\frac{\theta+2k\pi}{n}\right) + i\sin\left(\frac{\theta+2k\pi}{n}\right)\right)$

$k = 0, 1, 2, \ldots, n-1$

$z = 3\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$w = 2\left(\cos\frac{2\pi}{5} + i\sin\left(\frac{2\pi}{5}\right)\right)$

$\Rightarrow zw = wz = 6\left(\cos\left(\frac{11\pi}{15}\right) + i\sin\left(\frac{11\pi}{15}\right)\right)$

$\frac{\pi}{3} + \frac{2\pi}{5} = \frac{5\pi + 6\pi}{15} = \frac{11\pi}{15}$

Multiply moduli (lengths) & add arguments (angles)

$-2+\sqrt{-8} \quad + \quad (5-\sqrt{-50})$

$2 \lfloor 8 \quad 2 \lfloor 4 \quad 2$

$2 \lfloor 50 \quad 5 \lfloor 25 \quad 5$

$= -2 + 2i\sqrt{2} + 5 - 5i\sqrt{2}$

$= 3 + 2i\sqrt{2} - 5i\sqrt{2}$

$= 3 + i\sqrt{2}[2-5]$

$= 3 + i\sqrt{2}[-3] = 3 - 3i\sqrt{2}$

$= 3 - 3\sqrt{2}i$

$2x - 5x = x[2-5] = x[-3]$

$= -3x$

Simplify: $\sqrt{6773526760}$

2	6773526760
2	3386763380
2	1693381690
5	846690845
7	169338169
7	24191167
11	3455881
11	314171
13	28561
13	2197
13	169
	13

$2 \cdot 7 \cdot 11 \cdot 13^2 \sqrt{10}$

=

$2 \lfloor 106$

53

$\cancel{3}, \cancel{5}, \cancel{7}, 11 > \sqrt{53}$

7

$\sqrt{106}$

$\sqrt{212}$

2	212
2	106
	53

$= 2\sqrt{53}$