

$$a^2 = b^2 + c^2 - 2bc \cos \theta$$

$$\|\vec{u} - \vec{v}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\|^2 - 2\|\vec{u}\|\|\vec{v}\|\cos \theta$$

Background $\vec{u} = \langle 1, 2 \rangle$, $\vec{v} = \langle 3, 4 \rangle$,

$$\Rightarrow \vec{u} \cdot \vec{v} = 1 \cdot 3 + 2 \cdot 4 = 3 + 8 = 11$$

$$\text{Note } \vec{u} \cdot \vec{u} = 1 \cdot 1 + 2 \cdot 2 = 1^2 + 2^2 = 5$$

$$\|\vec{u}\| = \sqrt{1^2 + 2^2} = \sqrt{\vec{u} \cdot \vec{u}}$$

$$(\vec{u} - \vec{v}) \cdot (\vec{u} - \vec{v}) = \|\vec{u}\|^2 + \|\vec{v}\|^2 - 2\|\vec{u}\|\|\vec{v}\|\cos \theta$$

$$\vec{u} \cdot \vec{u} - \vec{u} \cdot \vec{v} - \vec{u} \cdot \vec{v} + \vec{v} \cdot \vec{v} = \|\vec{u}\|^2 + \|\vec{v}\|^2 - 2\|\vec{u}\|\|\vec{v}\|\cos \theta$$

$$\|\vec{u}\|^2 - 2\vec{u} \cdot \vec{v} + \|\vec{v}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\|^2 - 2\|\vec{u}\|\|\vec{v}\|\cos \theta$$

$$\Rightarrow \vec{u} \cdot \vec{v} = \|\vec{u}\|\|\vec{v}\|\cos \theta$$

$$\Rightarrow \cos \theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|\|\vec{v}\|}$$

cheat sheet material

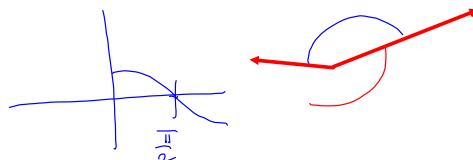
$$\theta = \cos^{-1} \left[\frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|\|\vec{v}\|} \right]$$

Apply this to Projection:

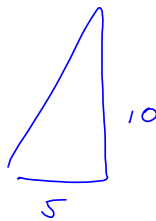
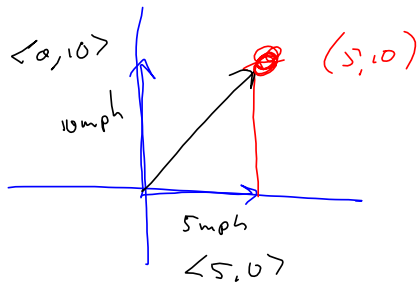
$$\|\text{proj}_{\vec{v}} \vec{u}\| = \|\vec{u}\| \cos \theta = \|\vec{u}\| \cdot \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|\|\vec{v}\|} = \frac{\vec{u} \cdot \vec{v}}{\|\vec{v}\|}$$

And $\text{proj}_{\vec{v}} \vec{u}$

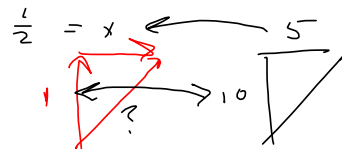
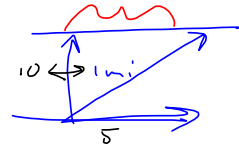
$$= \left(\frac{\vec{u} \cdot \vec{v}}{\|\vec{v}\|} \right) \frac{1}{\|\vec{v}\|} \vec{v} = \frac{\vec{u} \cdot \vec{v}}{\|\vec{v}\|^2} \vec{v} = \text{proj}_{\vec{v}} \vec{u}$$



3b) Resultant.



$$\langle 0, 10 \rangle + \langle 5, 0 \rangle = \langle 5, 10 \rangle$$

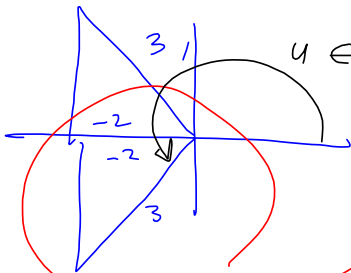


Similar triangles.

2nd do B #9

$$\cos \theta = -\frac{2}{3}$$

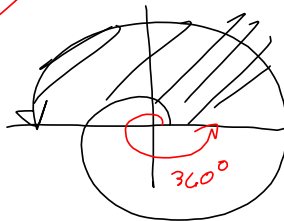
$$\sin \theta < 0$$



$\theta \in \text{Q III}$

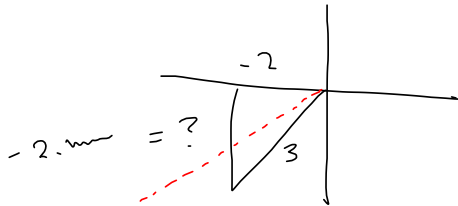
$$180^\circ < \theta < 270^\circ$$

$$360^\circ < 2\theta < 540^\circ$$



Im QII or QI

which?



$$\sqrt{3^2 - (-2)^2}$$

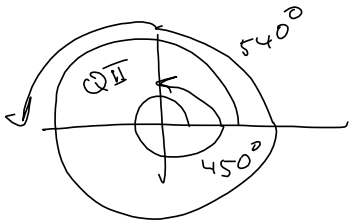
$$\sqrt{9-4} = \sqrt{5} = 2.\text{something}$$

So not only is $180^\circ < \theta < 270^\circ$, but

$$225^\circ < \theta < 270^\circ$$

$$450^\circ < \theta < 540^\circ$$

QII!



$$f(x) = 6x^4 - 35x^3 + 70x^2 + 25x - 26$$

1, 2, 13, 26
1, 2, 3, 6

$$f(3+2i) = 0 \implies f(3-2i) = 0$$

Divide by $x - (3+2i)$

| | | | | | |
|--------|---|-----------|----------|----------|-----|
| $3+2i$ | 6 | -35 | 70 | 25 | -26 |
| | | $18+12i$ | $-75+2i$ | $-19-4i$ | 26 |
| $3-2i$ | 6 | $-17+12i$ | $-5+2i$ | $6-4i$ | 0 |
| | | $18-12i$ | $3-2i$ | $-6+4i$ | |
| | 6 | 1 | -2 | 0 | |

$$(6x^2 + x - 2)(x - (3+2i))(x - (3-2i))$$

$$(3x+2)(2x-1)(x-3-2i)(x-3+2i)$$

| | | |
|----------------------|-------------------|--------------------------|
| $6x^2 + 4x - 3x - 2$ | $(3+2i)(-17+12i)$ | $= -51 + 36i - 34i - 24$ |
| | | $= -75 + 2i$ |
| $2x[3x+2] - 1[3x+2]$ | $(3+2i)(-5+2i)$ | $= -15 + 6i - 10i - 4$ |
| $[3x+2][2x-1]$ | | $= -19 - 4i$ |

$$2(3+2i)(3-2i) = 2(3^2 + 2^2) = 2(13) = 26$$

$$i^2 = -1$$

MAT 121
Test 3
stuff
on homeworks.
 $\sqrt{-1} = i$