What quadrant is $\frac{u}{2}$ in if

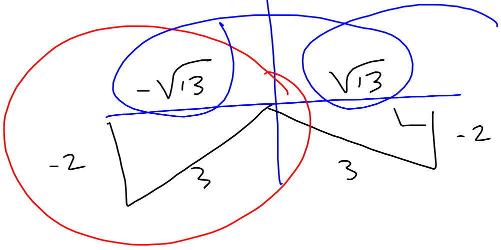
$\boxed{\sin(u) = -\frac{2}{3}}$ and $\cos(u) < 0$?

2 pics

zero in on one of them.

No! $\rightarrow \sqrt{3^2 - 2^2} = \sqrt{9-4} = \sqrt{5}$

Mills missed this!

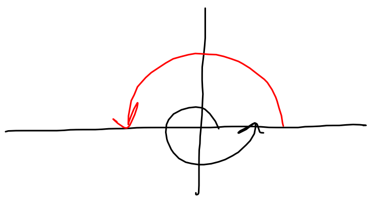


What quadrant is u in?

$\frac{u}{2} \in Q\,\text{III}$

$180^\circ < \frac{u}{2} < 270^\circ$ $\qquad \pi < \frac{u}{2} < \frac{3\pi}{2}$

$360^\circ < u < 540^\circ = 360^\circ + 180^\circ$

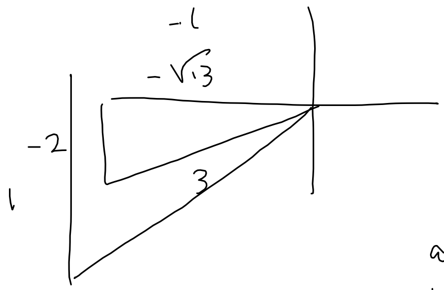


This isn't a quadrant!

It's a half-plane!

We want the quadrant!

To narrow it down, consider



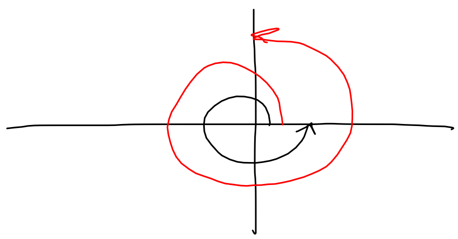
$\sqrt{13} > 2$, so wider than tall.

That says it makes angle of less than 45° with the x-axis.

$180^\circ < \frac{u}{2} < 225^\circ$

$360^\circ < u < 450^\circ = 360^\circ + 90^\circ$

This says we're in Q I !



This comes up in a weird way on bonus.

5. Find all solutions $x \in [0, 2\pi)$ to the trigonometric equation $6\sin^2\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right) - 2 = 0$. Give answers in degrees, rounded to 3 decimal places.

$6u^2 + u - 2$

$= 6u^2 + 4u - 3u - 2$

$= 2u(3u+2) - 1(3u+2)$

$= (3u+2)(2u-1) = 0 \longrightarrow$

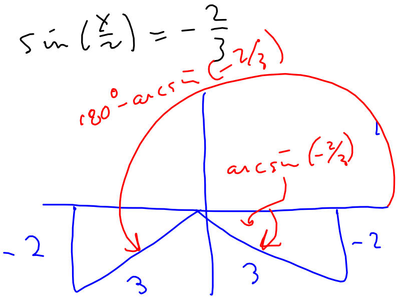
$u = -\frac{2}{3}$ $u = \frac{1}{2}$

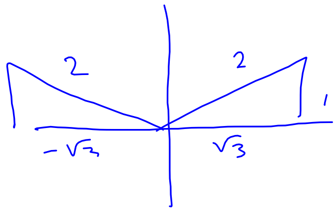
$0 \le x < 2\pi$

$\Rightarrow 0 \le \frac{x}{2} < \pi$

$\sin\left(\frac{x}{2}\right) = -\frac{2}{3}$

$\sin\left(\frac{x}{2}\right) = \frac{1}{2}$





All sol'ns beyond $\frac{x}{2} \in [0, \pi)$!

$\frac{x}{2} = 30^\circ, 150^\circ$

or $\frac{\pi}{6}, \frac{5\pi}{6}$

$\Rightarrow \boxed{x = \frac{\pi}{3}, \frac{5\pi}{3} \in [0, 2\pi)}$ ✓

See: $\frac{x}{2} = 360^\circ + \arcsin(-2/3) \Rightarrow x = 2(360^\circ) + 2(\arcsin(-2/3))$

or

$\frac{x}{2} = 180^\circ - \arcsin(-2/3) \Rightarrow x = 360^\circ - (\arcsin(-2/3))(2)$

Same Eq'n but w/ 2: $6\sin^2(2x) + \sin(2x) - 2 = 0$

want $x \in [0, 2\pi)$, so

$0 \le x < 2\pi \longrightarrow$

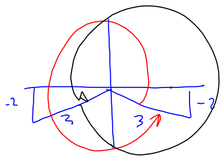
$0 \le 2x < 4\pi$, so we have to look for ALL 2x's in $[0, 4\pi)$!

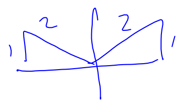
$(0 \le 2x \le 720^\circ)$

Same algebra:

$u = -\frac{2}{3}$ OR $u = \frac{1}{2}$

$\sin(2x) = -\frac{2}{3}$ OR $\sin(2x) = \frac{1}{2}$





So, $2x = 180 - \arcsin(-2/3)$ OR $2x = 360 + \arcsin(-2/3)$

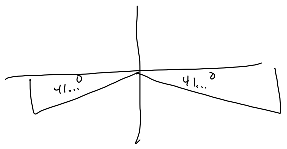
$x = 90^\circ - \frac{1}{2}\arcsin(-2/3)$ $x = 180 + \frac{1}{2}(\arcsin(-2/3))$

Add 360° to BOTH to capture all $x \in [0, 2\pi)$

Degrees $360^\circ +$ previous $360^\circ +$ previous

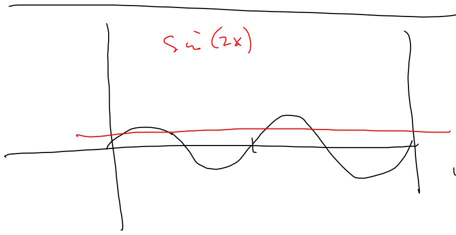
Radians $2\pi +$ previous $2\pi +$ previous.

-41.81031490



$360^\circ - 41....^\circ = 2x$ & any $2x + 360^\circ$

$180^\circ + 41....^\circ = 2x$



4 sol'ns when we only had 2, before because $\sin(2x)$ is twice as nervous

$\sin^2(3x) - \cos(3x)$

$1 - \cos^2(3x) - \cos(3x) = 0$

$-\cos^2(3x) - \cos(3x) + 1 = 0$

$\cos^2(3x) + \cos(3x) - 1 = 0$

$u^2 + u - 1 = 0$

$u^2 + u + \left(\frac{1}{2}\right)^2 = 1 + \frac{1}{4}$

$\left(u + \frac{1}{2}\right)^2 = \frac{5}{4}$

$u = -\frac{1}{2} \pm \frac{\sqrt{5}}{2} = \frac{-1 \pm \sqrt{5}}{2} = \cos(3x)$?!

Look for $3x \in [0, 6\pi)$

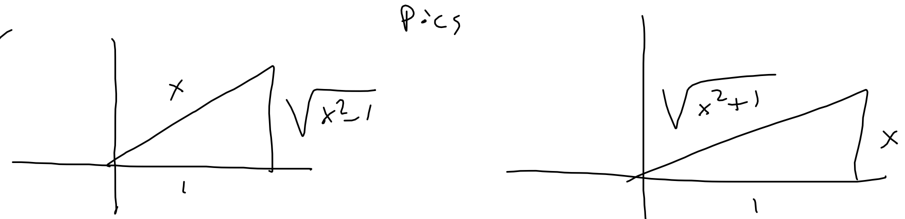
$$\sin(\operatorname{arcsec} x + \arctan x)$$

$$= \sin(u+v)$$

$$= \sin u \cos v + \sin v \cos u.$$

$$= \sin(\operatorname{arcsec}(x))\cos(\arctan(x)) + \sin(\arctan(x))\cos(\arcsin(x))$$

Pics



$\operatorname{arcsec}(x) = \arccos\left(\frac{1}{x}\right)$ $\arctan(x)$

$$= \left(\frac{\sqrt{x^2-1}}{x}\right)\left(\frac{1}{\sqrt{x^2+1}}\right) + \left(\frac{x}{\sqrt{x^2+1}}\right)\left(\frac{1}{x}\right)$$ is OK.

$$\boxed{= \frac{\sqrt{x^2-1} + x}{x\sqrt{x^2+1}}}$$ is nicer, but please stop here!

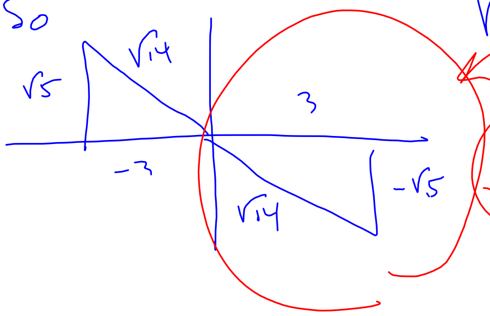
$\sin(2u) = 2\sin u \cos u = 2\left(-\sqrt{\frac{5}{14}}\right)\left(-\frac{3}{\sqrt{5}}\right)$

8. (10 pts) Find $\sin(2u), \cos(2u)$ and $\tan(2u)$, given that $\tan(u) = -\frac{\sqrt{5}}{3}$ and $\cos(u) > 0$. Give *exact* answers, in simplified radical form.

$\tan(u) = -\frac{\sqrt{5}}{3}$ & $\cos(u) > 0$

$\sqrt{\sqrt{5}^2 + (-3)^2} = \sqrt{5+9} = \sqrt{14}$

So

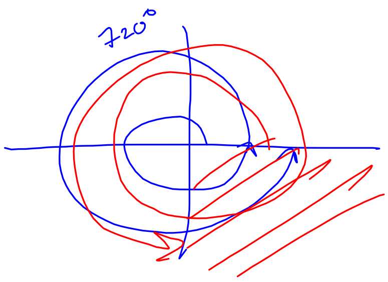


cos u > 0

We have $270^\circ < u < 360^\circ$

$315^\circ < u < 360$

$\Rightarrow \quad 630^\circ < 2u < 720^\circ$



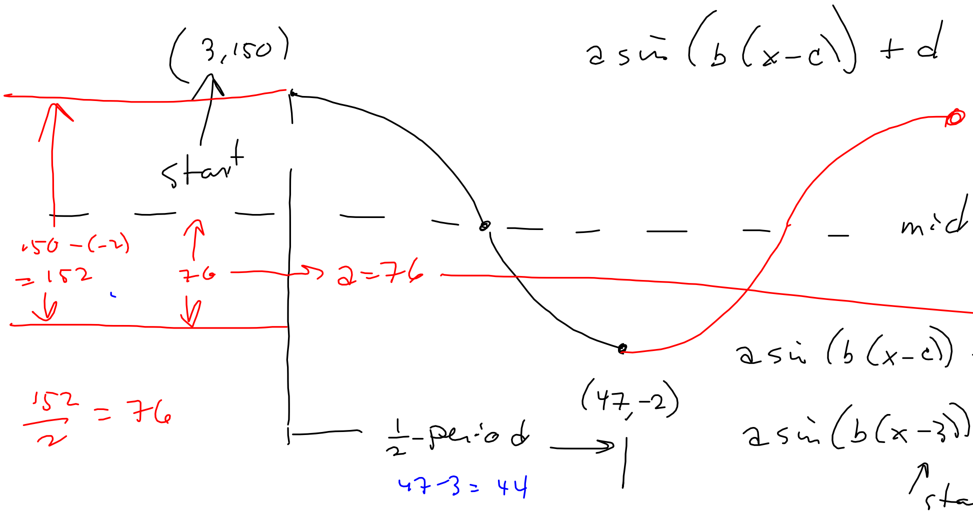
$630^\circ - 360^\circ = 270^\circ$

Q IV !

is where 2u lives!

Build a cosine function that achieves its maximum height of y = 150 m at time x = 3 seconds and its minimum height of y = _2 at x = 47 seconds.

$a\sin(b(x-c)) + d$



$\frac{150 + -2}{2} = \frac{148}{2}$

$\boxed{d = 74}$

$a\sin(b(x-c)) + 74$

$a\sin(b(x-3)) + 74$

start @ $x = 3$

Period:

Trick $b = \frac{\pi}{\frac{1}{2}\text{period}} = \frac{\pi}{44} \rightsquigarrow a\sin\left(\frac{\pi}{44}(x-3)\right) + 74$

Prefer: $\frac{1}{2}T = 44 \Rightarrow T = 88$

$\Rightarrow$ $bx = 2\pi$, when $x = 88$

$88b = 2\pi$

$b = \frac{\pi}{44}$

$\boxed{f(x) = 76\sin\left(\frac{\pi}{44}(x-3)\right) + 74}$

Amp: