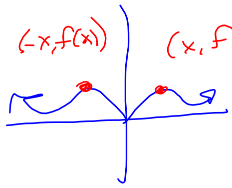
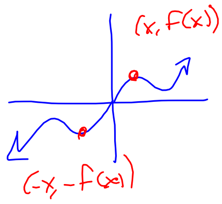
Best exercises for maximum help are the ones on harryzaims.com in the videos section.

§2.4 : Basically 2 formulas, once you understand odd & even.

Even: constant, x^2, x^4, ..., x^{2n}, cosine



Odd : $x, x^3, \dots, x^{2n+1}$, sine.



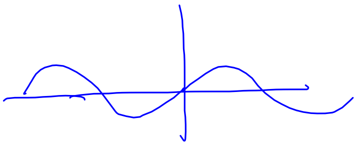
Recall $(-1)(+1)(-1)(-1)(-1) = +1$

$(-)(-)(-)(-) = +$

$(-)(+) = -$

$\sin(-x) = -\sin(x)$

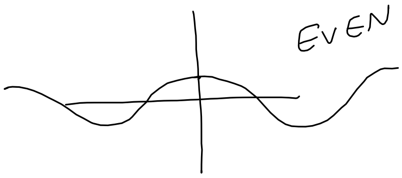
" — "



ODD

$\cos(-x) = \cos(x)$

EVEN



$$\frac{(-5)(10)(\frac{1}{3})(-6)}{(-4)(4)} \rightsquigarrow -$$

$\sin(x)\cos(x)$ ODD

$(-)(+) = -$

$$\frac{x^2 \sin(x) \cos(x)}{(+)(-)(+) = -}$$ ODD

$\tan(x)$ is $\frac{\sin x}{\cos x} = \frac{-}{+} = -$ ODD

Special

Even + Even = Even

odd + odd = ODD

odd + Even = Neither!

$x^2 + x^3 + 1$ is neither

$$\boxed{\sin(u+v) = \sin u \cos v + \sin v \cos u}$$

$$\sin(u-v) = \sin(u + (-v))$$

$$= \sin u \cos(-v) + \sin(-v)\cos(u)$$

$$= \sin u \cos v + -\sin(v)\cos u$$

$$= \sin u \cos v - \sin v \cos u$$

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u-v) = \cos u \cos(-v) - \sin u \sin(-v)$$

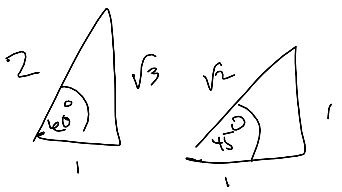
$$= \cos u \cos v + \sin u \sin v$$

$\tan(u+v) = \dfrac{\sin(u+v)}{\cos(u+v)}$ so you already know it!

$\sin\left(\frac{7\pi}{12}\right)$ I want it <u>exact</u>.

$$\frac{7\pi}{12} = \frac{6\pi}{12} + \frac{\pi}{12}$$
$$= \frac{5\pi}{12} + \frac{2\pi}{12}$$
$$= \frac{4\pi}{12} + \frac{3\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$$

$$\sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$$
$$= \sin\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\frac{\pi}{3}$$



$\frac{\pi}{3}$ $\frac{\pi}{4}$

$$= \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\cdot\frac{1}{2}$$
$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

If I ask for "simplified radical form."

$$= \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$
$$= \frac{\sqrt{3}\sqrt{2}+\sqrt{2}}{2\cdot 2} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$$

$$\cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right) = \cos\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{3}\sin\frac{\pi}{4}$$
$$= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}} = \frac{1-\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{2}-\sqrt{6}}{4}$$

$$\tan\left(\frac{7\pi}{12}\right) = \frac{\sin\left(\frac{7\pi}{12}\right)}{\cos\left(\frac{7\pi}{12}\right)} = \frac{\frac{\sqrt{6}+\sqrt{2}}{4}}{\frac{\sqrt{2}-\sqrt{6}}{4}} = \frac{\sqrt{6}+\sqrt{2}}{4}\cdot\frac{4}{\sqrt{2}-\sqrt{6}}$$

$$= \frac{\sqrt{6}+\sqrt{2}}{\sqrt{2}-\sqrt{6}}$$ is fine.

Simplified radical form.

$$\left(\frac{\sqrt{6}+\sqrt{2}}{\sqrt{2}-\sqrt{6}}\right)\left(\frac{\sqrt{6}+\sqrt{2}}{\sqrt{2}+\sqrt{6}}\right) = \frac{(\sqrt{6}+\sqrt{2})^2}{-4} =$$

Nooooooo!

$$\frac{6+2}{-4} = \frac{8}{-4} = -2$$

"conjugate" idea.

$$(a-b)(a+b) = a^2-b^2$$

$$(\sqrt{2}-\sqrt{6})(\sqrt{2}+\sqrt{6}) = 2-6 = -4 \,!$$

$$(a+b)^2 = a^2+2ab+b^2 = (a+b)(a+b)$$

$$(a-b)^2 = a^2-2ab+b^2 = (a-b)(a-b)$$

$$\frac{(\sqrt{6}+\sqrt{2})^2}{-4} = \frac{\sqrt{6}^2+2\sqrt{6}\sqrt{2}+\sqrt{2}^2}{-4}$$

$$= \frac{6+2\sqrt{2}\sqrt{3}\sqrt{2}+2}{-4} = \frac{6+4\sqrt{3}+2}{-4}$$

$$= \frac{8+4\sqrt{3}}{-4} = \frac{4(2+\sqrt{3})}{-4} = -(2+\sqrt{3}) \,!$$

same!?

$$\frac{\sqrt{2}+\sqrt{6}}{\sqrt{2}-\sqrt{6}}$$ is fine.

Double-Angle Formulas

$\sin(2x) = \sin(x+x) = \sin x\cos x + \sin x\cos x$

Huge in Calculus. $= 2\sin x\cos x$

$\cos(2x) = \cos(x+x) = \cos x\cos x - \sin x\sin x$

$= \cos^2 x - \sin^2 x$ is ONE

$= 1-\sin^2 x - \sin^2 x = 1 - 2\sin^2 x$ TWO

$= \cos^2 x - (1-\cos^2 x) = 2\cos^2 x - 1$ Three

Depends on what you need.

$\tan(2x) = \dfrac{\sin(2x)}{\cos(2x)}$ so we don't need it.

$\sin(2x) = 2\sin x\cos x$

$\boxed{\cos(2x) = 1-2\sin^2 x = 1-2\cos^2 x}$

For when you know $\sin x$ & $\cos x$ but don't know $\sin(2x)$ or $\cos(2x)$.

Half-Angle: when you know $\sin(2x)$ or $\cos(2x)$, but don't know $\sin x$ or $\cos x$

$\sin(x)$ from the above formula:

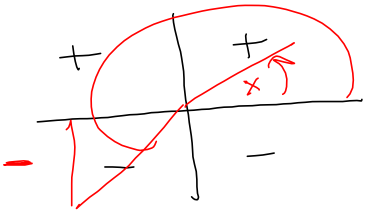
$1 - 2\sin^2 x = \cos(2x)$ solve for $\sin(x)$

$-2\sin^2 x = \cos(2x) - 1$

$\sin^2 x = \dfrac{\cos(2x)-1}{-2} = \dfrac{1-\cos(2x)}{2}$

$\sin x = \pm\sqrt{\dfrac{1-\cos(2x)}{2}}$

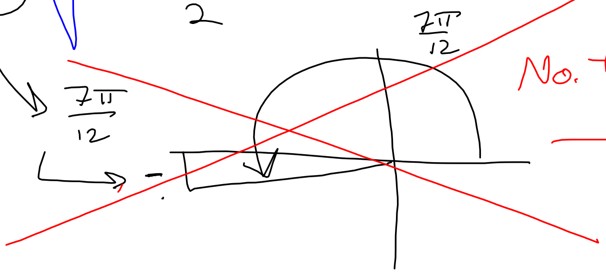
whether it's "+" or "−" depends on where x lives.

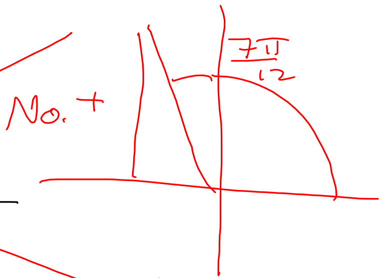


$\frac{1}{2}$- angle ! Find $\sin\left(\frac{7\pi}{12}\right)$

$$x = \frac{7\pi}{12} = \frac{1}{2}\left(\frac{7\pi}{6}\right)$$

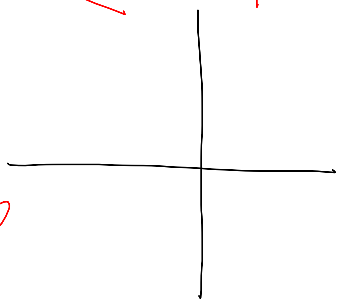
$$\Rightarrow \sin\frac{7\pi}{12} = \pm\sqrt{\frac{1-\cos\left(\frac{7\pi}{6}\right)}{2}}$$





~~So~~

~~$$\sin\frac{7\pi}{12} = -\sqrt{\frac{1-\cos\frac{7\pi}{6}}{2}}$$~~



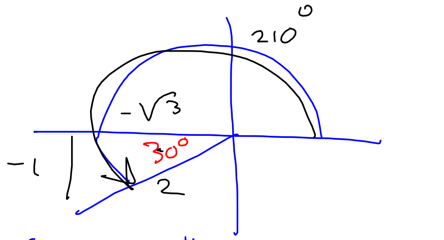
No. $\frac{7\pi}{12} = \frac{6\pi}{12} + \frac{\pi}{12} = \frac{\pi}{2} + \text{small}$

QII

So it's

$$+\sqrt{\frac{1-\cos\frac{7\pi}{6}}{2}}$$

$$= \sqrt{\frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{2}}$$



$$\left(\frac{7\pi}{6}\right)\left(\frac{180^\circ}{\pi}\right) = 7\cdot 30 = 210^\circ$$

Scratch: $\dfrac{1+\frac{\sqrt{3}}{2}}{2} = \dfrac{\frac{2}{2}+\frac{\sqrt{3}}{2}}{2} = \dfrac{\frac{2+\sqrt{3}}{2}}{2} = \dfrac{2+\sqrt{3}}{4}$

$\dfrac{\frac{2+\sqrt{3}}{2}}{\frac{2}{1}}$ $\dfrac{2+\sqrt{3}}{2}\cdot\dfrac{1}{2} = \dfrac{2+\sqrt{3}}{4}$

$= \sqrt{\dfrac{\sqrt{3}+2}{4}}$

$\sqrt{\dfrac{\sqrt{3}+2}{2}}$ is Bad.

Test (

$\sqrt{4-9x^2} = 2-3x$

Noooooo!

$\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$ GooD

$\sqrt{ab} = \sqrt{a}\sqrt{b}$

$\boxed{\sqrt{a+b} = \sqrt{a}+\sqrt{b}}$ BAD

No!

$5 \neq 7$

$5 = \sqrt{25} = \sqrt{16+9} = \sqrt{16}+\sqrt{9} = 4+3 \neq 7$

Nooooooo!

$= \sqrt{\dfrac{\sqrt{3}+2}{4}} = \dfrac{\sqrt{\sqrt{3}+2}}{\sqrt{4}} = \boxed{\dfrac{\sqrt{\sqrt{3}+2}}{2}} = \sin\left(\frac{7\pi}{12}\right)$

$\cos(x) =$

$\cos(2x) = 1 - 2\sin^2 x = 2\cos^2 x - 1$

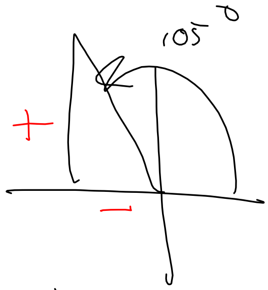
$\Longrightarrow \quad 2\cos^2 x = \cos(2x) + 1$

$\cos^2 x = \dfrac{\cos(2x)+1}{2}$

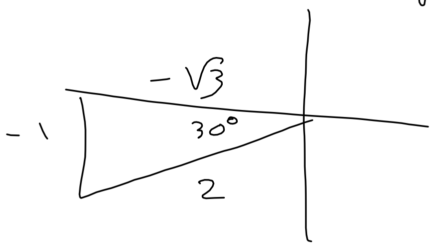
$\cos x = \pm\sqrt{\dfrac{\cos(2x)+1}{2}} = \pm\sqrt{\dfrac{1+\cos(2x)}{2}}$

$\cos\left(\frac{7\pi}{12}\right) = \cos\left(\frac{1}{2}\left(\frac{7\pi}{6}\right)\right)$

$\left(\frac{7\pi}{12}\right)\left(\frac{180^\circ}{\pi}\right) = 7\cdot(15)^\circ = 105^\circ$



$\cos\left(\frac{7\pi}{12}\right) = -\sqrt{\dfrac{1+\cos\left(\frac{7\pi}{6}\right)}{2}}$



$= -\sqrt{\dfrac{1-\frac{\sqrt{3}}{2}}{2}} = -\sqrt{\dfrac{\frac{2-\sqrt{3}}{2}}{2}}$

$= -\sqrt{\dfrac{2-\sqrt{3}}{4}} = \boxed{-\dfrac{\sqrt{2-\sqrt{3}}}{2} = \cos\left(\frac{7\pi}{12}\right)}$