1. (10 pts) Find two angles, between -2π and 2π (i.e., 0^0 and 360^0) that are coterminal with $\frac{55\pi}{4}$. Give exact answers in degrees and radians.

2. Arc Length and Area of Sector. Suppose we have a circle of radius $r = 6$ cm.
   a. (5 pts) Find the arc length on the circle, that is intercepted by an angle of 1724^0. Round to 3 decimal places.
   b. (5 pts) Find the *exact* area of the sector that is intercepted (swept through) by an angle of $\theta = \frac{11\pi}{6}$

3. Basic concept: Draw the doggone pictures!

   a. (5 points) Sketch two triangles that satisfy $\cos(\theta)=\frac{-2}{\sqrt{13}}$.

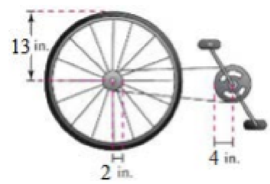
b. (5 pts) Assume the terminal side of the angle θ lies in the 2nd quadrant. Find the other five trigonometric functions of θ.

c. (5 pts) Again, assuming θ's terminal side lies in Q II, and $0 \leq \theta < 2\pi$, find θ, in radians *and* degrees, rounded to 3 decimal places.

d. (5 pts) Give *all* solutions to the equation $\cos(\theta) = \frac{-2}{\sqrt{13}}$, in degrees *and* radians, rounded to three (3) decimal places. (Good one to skip and come back to, if time permits).

4. (10 pts) Sketch one period of the graphs of $y = \sin(x)$ and $y = \csc(x)$ on the same set of coordinate axes.

5. (10 pts) The radii of the pedal sprocket, the wheel sprocket, and the wheel of the bicycle in the figure are 4 inches, 2 inches and 13 inches, respectively. A cyclist is pedaling at a rate of 2 revolutions per second. Find the speed of the bicycle in feet per second. Then convert that to miles per hour. Round final answers to 1 decimal place.



6. (10 pts) Sketch the graph of $f(x)=11\sin\left(\frac{\pi}{8}x-\frac{7\pi}{8}\right)-9$.

7. (10 pts) Write the cosine function that achieves its maximum height of $y = 25$ centimeters at time $t = 4$ seconds and its minimum height of $y = -5$ centimeters at $t = 48$ seconds.

8. (5 pts) Solve the triangle on the right. That means, find all lengths and angles. Exact answers required.

Another way

$\frac{14}{c} = \cos 45^\circ$

$\boxed{\frac{14}{\cos 45^\circ} = C} = \frac{14}{\left(\frac{1}{\sqrt{2}}\right)} = 14\left(\frac{\sqrt{2}}{1}\right)$

$\boxed{= 14\sqrt{2} = C}$

$\frac{a}{14} = \tan 45^\circ \implies$

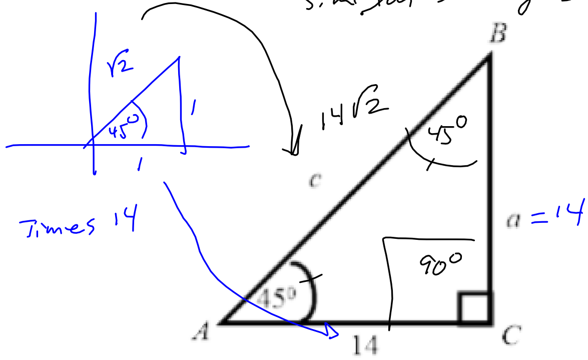
$a = 14 \tan 45^\circ = 14 \cdot 1 \boxed{= 14 = a}$

$B = 45^\circ$

$C = 90^\circ$

$(A = 45^\circ)$

similar triangles way



9. Find the exact value of…

a. … (5 pts) $\sin\left(\arctan\left(\frac{13}{7}\right)\right)$.

b. … (5 pts) $\arcsin\left(\sin\left(\frac{7\pi}{4}\right)\right)$

10. (5 pts) Draw the sketch and use it to find an algebraic expression that is equivalent to $\tan\left(\arccos\left(\frac{3x}{\sqrt{9x^2+5}}\right)\right)$. Assume that everything is taking place in the 1st quadrant.

11. (5 pts) Sketch the pictures corresponding to:

a. $\cos(x)=0$

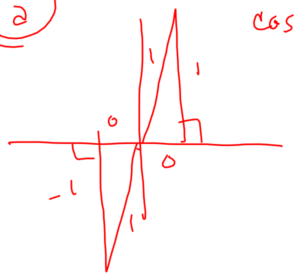
b. $\sin(x)=1$

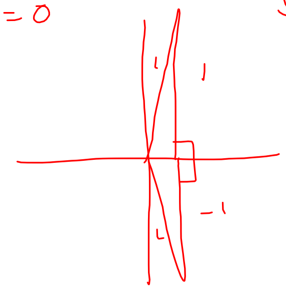
c. $\sin(x)=\frac{-\sqrt{3}}{2}$

d. $\tan(x)=\sqrt{3}$

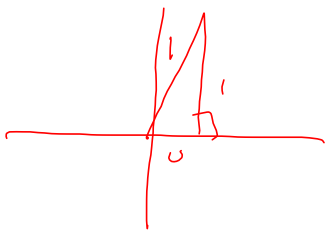
e. $\csc(x)=0$

a $\cos(x)=0$

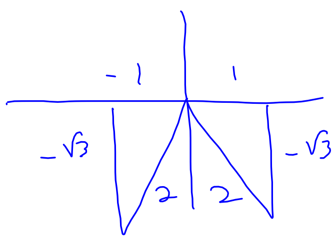




b $\sin(x)=1$

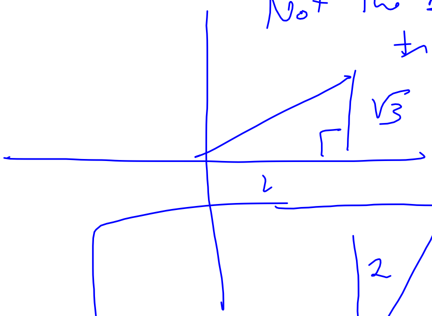


c $\sin(x)=-\frac{\sqrt{3}}{2}$

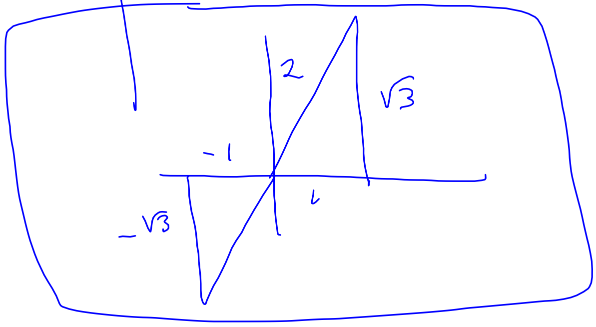


d $\tan(x)=\sqrt{3}$

Not the best triangle



Better



e $\csc(x)=0$

$$\frac{1}{\sin(x)}=0$$

$$1=\frac{1}{\sin(x)}\cdot\sin(x)=0\cdot\sin(x)=0$$

$1=0$ impossible.

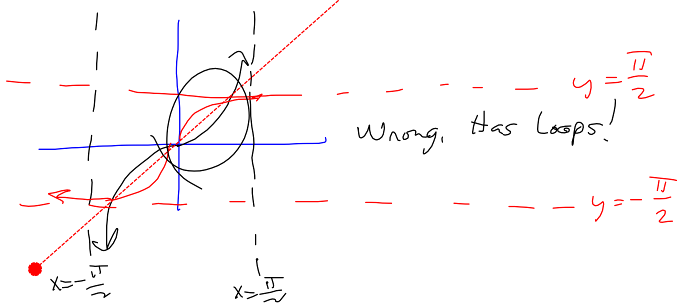
This says $\sin x=\frac{1}{\csc(x)}=\frac{1}{0}$ Never happen.

12. (5 pts) Sketch the graph of one period of $y=\tan(x)$ (restricted to make it 1-to-1) *and* $y=\arctan(x)$ on the same set of coordinate axes. I want to see the function and its inverse in the same picture. Label key points as ordered pairs (ALWAYS). State the domain and range of the restricted tangent function and its inverse.

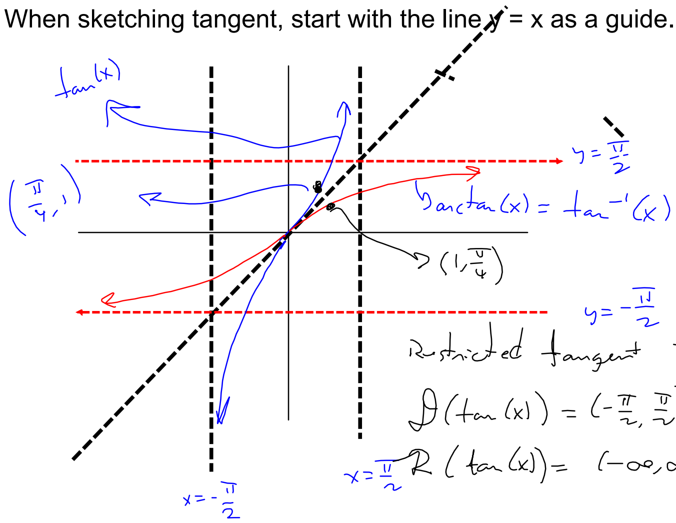
Remember: Slope of sine and slope of tangent are 1 at their x-intercepts.

The tendency is to make them steeper. This will create loops in your picture for #12 that shouldn't be there.

less steep for tangent



When sketching tangent, start with the line y = x as a guide.



Restricted tangent to keep it 1-to-1.

$\mathcal{D}(\tan(x)) = (-\frac{\pi}{2},\frac{\pi}{2}) = \mathcal{R}(\arctan(x))$

$\mathcal{R}(\tan(x)) = (-\infty,\infty) = \mathcal{D}(\arctan(x))$

13. (5 pts) Find all solutions θ of the trigonometric polynomial $4\cos^2(\theta)-3=0$ in the interval $[0,2\pi)$

Let $u=\cos\theta$

I didn't specify degrees or radians/ I probably will ask specifically for both.

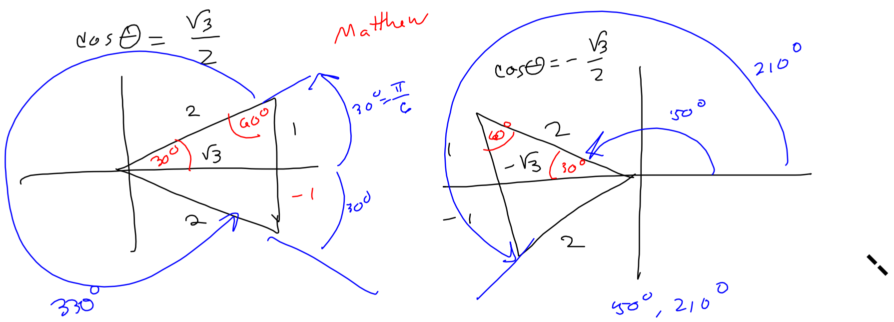
$4u^2-3=0$

$4u^2=3$

$u^2=\frac{3}{4}$

$u=\pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{\sqrt{4}} = \pm\frac{\sqrt{3}}{2} = \cos\theta$

$\cos\theta=\frac{\sqrt{3}}{2}$ Matthew



$30^\circ, 330^\circ$ $\qquad 150^\circ, 210^\circ$

$x\in\{30^\circ, 150^\circ, 210^\circ, 330^\circ\} = \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\}$

$(150^\circ)(\frac{\pi}{180^\circ}) = \frac{5\pi}{6}$

.

14. (5 pts) Super-duper bonus Find all solutions θ of the trigonometric polynomial $4\cos^3(\theta)+4\cos^2(\theta)-3\cos\theta-3=0$ in the interval $[0,2\pi)$.

Let $\cos\theta = u$

$4u^3+4u^2-3u-3$ Factors by grouping. (Contined)

$4u^2(u+1)-3(u+1)$

$=(u+1)(4u^2-3) = 0$

$u+1=0$

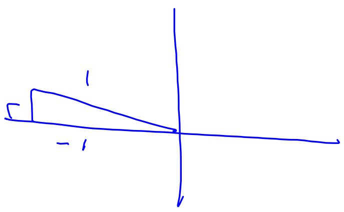
$\cos\theta = -1$

$4u^2-3=0$

$4\cos^2\theta - 3 = 0$

$x \in \{30^\circ, 150^\circ, 210^\circ, 330^\circ\} = \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\} = A$

by previous



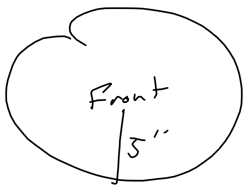
$\theta = \pi$

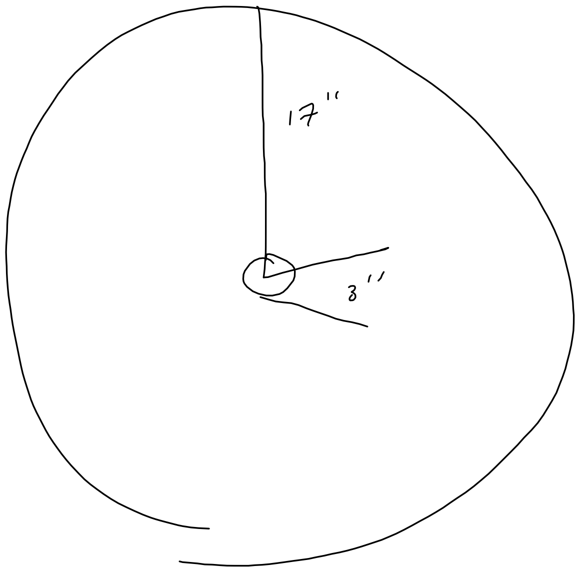
$\theta \in \{\pi\} = B$

$\boxed{x \in A \cup B}$

One like #5

Pedaling @ $\boxed{7 \text{ revs/sec.}}$





$S = r\theta$

$$\underbrace{\frac{S}{\text{unit time}}}_{\text{WANT}} = \underbrace{\frac{r\theta}{\text{unit time}}}_{\text{HAVE}}$$

$$\underbrace{\left(\frac{7 \text{ revs front}}{1 \text{ sec}}\right)\left(\frac{5 \text{ revs rear}}{3 \text{ revs front}}\right)\left(\frac{2\pi}{1 \text{ rev rear}}\right)}_{\frac{\theta}{\text{sec}}}\underbrace{\left(17 \text{ inch radius}\right)}_{r}\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)$$

$$= \frac{85\pi}{18}\frac{\text{ft}}{\text{s}} = \left(\frac{85\pi}{18}\frac{\text{ft}}{\text{sec}}\right)\left(\frac{60 \frac{\text{mi}}{\text{hr}}}{88 \frac{\text{ft}}{\text{sec}}}\right) \approx$$

70.80483444

2. Arc Length and Area of Sector. Suppose we have a circle of radius $r = 6$ cm.
   a. (5 pts) Find the arc length on the circle, that is intercepted by an angle of 1724^0. Round to 3 decimal places. $s = r\theta$ $C = 2\pi r$
   b. (5 pts) Find the *exact* area of the sector that is intercepted (swept through) by an angle of $\theta = \frac{11\pi}{6}$

$\theta \rightsquigarrow r\theta$

$\frac{1}{2}r^2\theta$

$\theta = 2\pi$

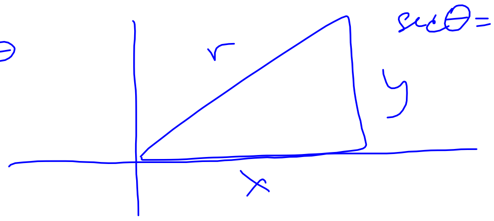
$A = \pi r^2 = \frac{1}{2}(2\pi)r^2 = \boxed{\frac{1}{2}\theta r^2}$

$\text{Area} = \frac{1}{2}r^2\theta$

$\text{Arc Length} = r\theta$

$\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, $\tan\theta = \frac{y}{x}$

$\csc\theta = \frac{1}{\sin\theta}$, $\sec\theta = \frac{1}{\cos\theta}$, $\cot\theta = \frac{x}{y}$



Pythagorous $x^2 + y^2 = r^2$

when $r = 1$:

$x^2 + y^2 = 1$

$$\boxed{\cos^2\theta + \sin^2\theta = 1}$$

$$\left.\begin{aligned} \tan^2\theta + 1 &= \sec^2\theta \\ \cot^2\theta + 1 &= \csc^2\theta \end{aligned}\right\} \text{ Cheatsheet material, later.}$$

$$\tan^2\theta + 1 = \frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta}$$

$$\tan\theta = \frac{y}{x} = \frac{\frac{y}{r}}{\frac{x}{r}} = \frac{\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos^2\theta} = \sec^2\theta$$