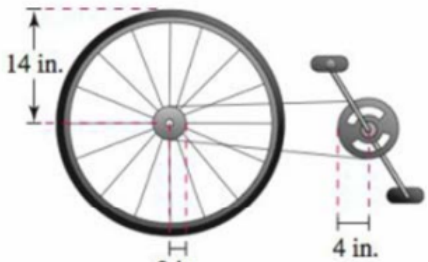
68. Speed of a Bicycle

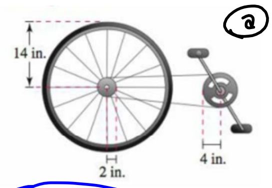
The radii of the pedal sprocket, the wheel sprocket, and the wheel of the bicycle in the figure are 4 inches, 2 inches, and 14 inches, respectively. A cyclist is pedaling at a rate of 1 revolution per second.



(a) Find the speed of the bicycle in feet per second and miles per hour.

(b) Use your result from part (a) to write a function for the distance d (in miles) a cyclist travels in terms of the number n of revolutions of the pedal sprocket.

(c) Write a function for the distance d (in miles) a cyclist travels in terms of the time t (in seconds). Compare this function with the function from part (b).



(a) FRONT RPM to Angular speed to feet per second.

Arc length (radians)
$s = r\theta$

$$\underbrace{\left(\frac{1\ \cancel{\text{rev front}}}{1\ \text{sec}}\right)\left(\frac{2\ \cancel{\text{rev back}}}{1\ \cancel{\text{rev front}}}\right)\left(\frac{2\pi\ \text{radians}}{1\ \cancel{\text{rev back}}}\right)}\cdot(14\ \text{in})$$

$\left(\frac{\theta}{\text{sec}}\right)(r)$ $\quad \frac{\text{radians}}{\text{sec}}$ = angular speed.

$= \frac{\text{arclength}}{\text{sec}}$

We're in inches/sec after multiplying by $14 = r$.

$$= \left((4\pi)(14)\frac{\text{in}}{\text{sec}}\right)\left(\frac{1\ \text{ft}}{12\ \text{in}}\right) = \frac{4\cdot 14\pi}{12}\ \frac{\text{ft}}{\text{sec}}$$

$$= \frac{14\pi}{3}\ \frac{\text{ft}}{\text{sec}} \approx 14.66207657\ \frac{\text{ft}}{\text{s}}$$

$$\approx \boxed{14.6621\ \frac{\text{ft}}{\text{s}}}$$

miles/hr:

$$\left(\frac{14\pi}{3}\ \frac{\text{ft}}{\text{s}}\right)\left(\frac{1\ \text{mi}}{5280\ \text{ft}}\right)\left(\frac{60\ \text{s}}{1\ \text{min}}\right)\left(\frac{60\ \text{min}}{1\ \text{hr}}\right)$$

Symbolically precise

$$\frac{88\ \text{ft}}{\text{s}} = \frac{60\ \text{miles}}{\text{hr}}$$

$$= \frac{(14\pi)(3600)}{3\cdot 5280}\quad 15 \rightarrow 5$$

$$\left(\frac{\overset{7}{\cancel{14}}\pi}{3}\ \frac{\text{ft}}{\text{s}}\right)\left(\frac{\frac{\overset{30}{\cancel{60}}\ \text{mi}}{\text{hr}}}{\frac{\underset{\underset{11}{\cancel{22}}}{\cancel{44}}\cancel{88}\ \text{ft}}{\text{s}}}\right) = \frac{7\cdot 5\pi}{11} = \frac{35\pi}{11}$$

$$\approx 19.99195325\ \frac{\text{mi}}{\text{hr}}$$

(b) $\boxed{D = rt \quad \text{Not what was asked!} \quad = \frac{35\pi}{11}t \text{ where } t \text{ is in hours.}}$

Want it in terms of the # of revolutions of the pedal. Let x = # of revs on front.

rate · time

$$\left(x\ \cancel{\text{revs}}\ \text{front}\right)\left(\frac{2\ \cancel{\text{revs rear}}}{\cancel{1\ \text{rev front}}}\right)\left(\frac{2\pi\ \text{radians}}{1\ \cancel{\text{rev rear}}}\right)\cdot(14\ \cancel{\text{in}})\left(\frac{1\ \text{ft}}{12\ \cancel{\text{in}}}\right)$$

Converting from angular speed to linear speed via $s = r\theta$ is tricky, because the units don't all cancel out.

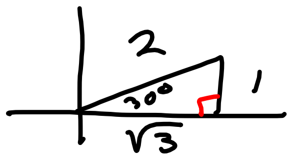
technically

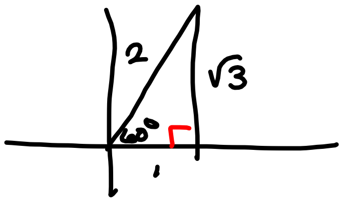
"radian · ft units" because of the relationship between angle, in radians & arc length.

DO NOT MEMORIZE THE UNIT CIRCLE
TRIG VALUES LIKE THEY WANT YOU TO.

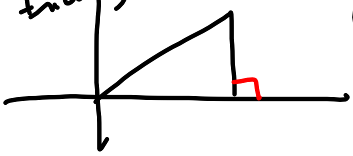
All the triangles you need:

$30^\circ - 60^\circ$ right triangle





$45^\circ - 45^\circ$ right triangle.

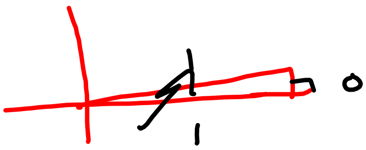


Degenerate triangles

QUADRANT OR
QUADRANTAL ANGLES

$\sin 0^\circ = \frac{0}{1} = 0$

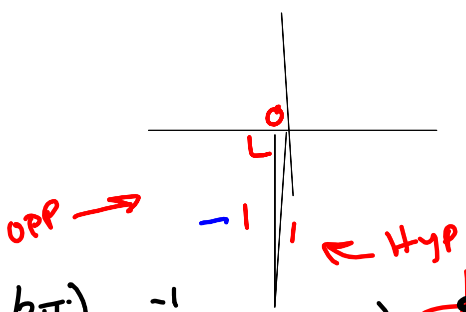
0°

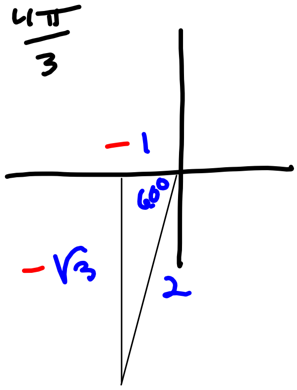


90°

$\left(\frac{3\pi}{2} \cdot \frac{180^\circ}{\pi}\right)$
$= 270^\circ$

$240^\circ = \frac{4\pi}{3}$

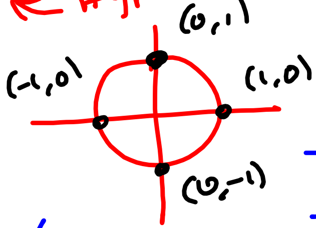




$\sin\left(\frac{3\pi}{2}\right) = \frac{-1}{1} = -1$

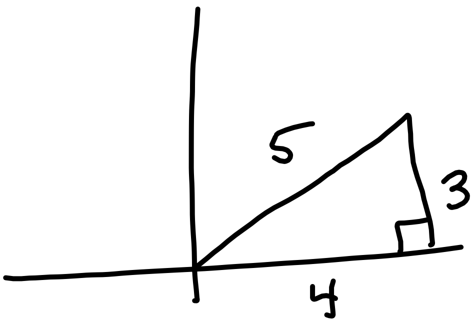
$\cos\left(\frac{3\pi}{2}\right) = \frac{0}{1} = 0$

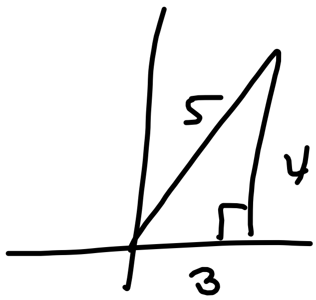
$\tan\left(\frac{3\pi}{2}\right) = \frac{-1}{0}$ $\nexists$

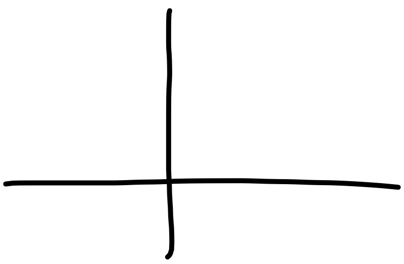


$\exists$ - there is, there exists

DNE







sin