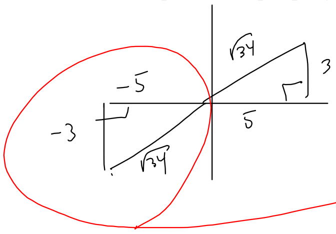
Test 2 Thursday. Questions from Test 2, Spring, 2018

1. (5 pts) Kindness Points. I'm so glad you followed all formatting preferences I've been talking about all semester, like margins, writing clearly and dark enough for me to read. Thanks for circling final answers, and not cramming too much stuff into a small space. Thanks for organizing your work so that it's easy to follow, from the top on down. Thanks for leaving a 1-inch margin in the top left of each page. I thank you and your classmates thank you.

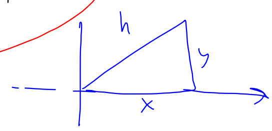
2. (10 pts) Find the values of all six trigonometric functions, given $\tan(u)=\frac{3}{5}$ and $\sin(u)<0$.

$\frac{y}{x}$ $\frac{y}{h}$

Test 1 question crops up, again. Know this.



$\sqrt{3^2+5^2} = \sqrt{9+25} = \sqrt{34}$



$\sin u = \frac{-3}{\sqrt{34}}$ $\csc u = -\frac{\sqrt{34}}{3}$

$\cos u = \frac{-5}{\sqrt{34}}$ $\sec u = -\frac{\sqrt{34}}{5}$

$\tan u = \frac{3}{5}$ $\cot u = \frac{5}{3}$

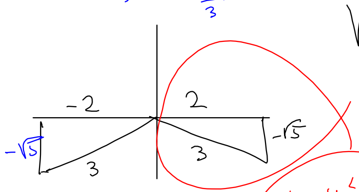
$= \frac{y}{h}$

3. (10 pts) Find $\sin\left(\frac{u}{2}\right)$, $\cos\left(\frac{u}{2}\right)$, and $\tan\left(\frac{u}{2}\right)$, given that $\sin(u)=-\frac{\sqrt{5}}{3}$ and $\tan(u)<0$.

Assume $0<u<2\pi$. Give final answers in simplified radical form.

$\sin u = -\frac{\sqrt{5}}{3}$

$\sqrt{3^2-\sqrt{5}^2} = \sqrt{9-5} = \sqrt{4} = 2$



$\cos(u) = \frac{2}{3}$

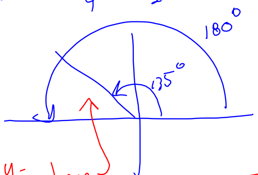
$\tan u < 0$

The tough analysis is finding the quadrant in which the half-angle lives.

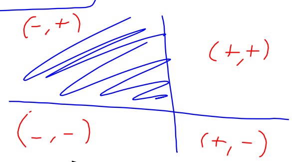
$u \in$ QIV

$\frac{3\pi}{2} < u < 2\pi$

$135^\circ = \frac{3\pi}{4} < \frac{u}{2} < \pi = 180^\circ \rightarrow \boxed{\frac{u}{2} \in \text{QII}}$

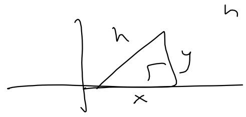


$\frac{u}{2}$ in here



sine: y's $\frac{y}{h}$

cosine: x's $\frac{x}{h}$



$\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos(u)}{2}}$

$= +\sqrt{\frac{1-\cos(u)}{2}} = \sqrt{\frac{1-\frac{2}{3}}{2}}$

QII!

$= \sqrt{\frac{\frac{1}{3}}{2}} = \boxed{\sqrt{\frac{1}{6}} = \sin\left(\frac{u}{2}\right) \text{ Fine.}}$ $= \frac{1}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{6}}{6}$

is simplified.

$\cos\left(\frac{u}{2}\right) = -\sqrt{\frac{1+\cos(u)}{2}} = -\sqrt{\frac{1+\frac{2}{3}}{2}} = -\sqrt{\frac{\frac{5}{3}}{2}} = \boxed{-\sqrt{\frac{5}{6}} = \cos\left(\frac{u}{2}\right)}$

$\cos\left(\frac{u}{2}\right) < 0$, from QII info.

4. Consider the equation $3\tan^3(x) - 3\tan^2(x) - \tan(x) + 1 = 0$.

   a. (10 pts) Find all solutions x, in radians, to the equation, above, in the interval $[0, 2\pi)$. Give *exact* answers, here. (Hint: Factor by grouping.)

   b. (10 pts) Find *all* real solutions x, in radians.

Factoring skills... Know your algebra. This is a "nice" cubic. I won't give you a tough one. But I'll ask any quadratic, so know your quadratic equations.

$3u^3 - 3u^2 - u + 1 = 3u^2(u-1) - 1(u-1)$

$= (u-1)(3u^2-1) = (u-1)\boxed{(\sqrt{3}u-1)(\sqrt{3}u+1)}$

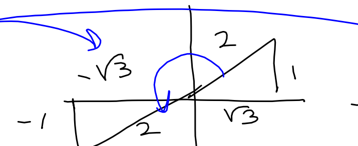
If you can broaden your vision of a^2-b^2

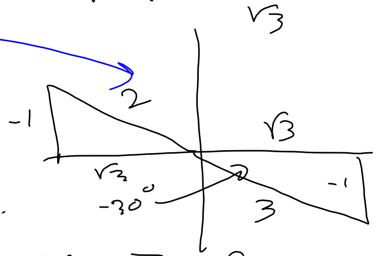
$\Rightarrow u = 1$ $\qquad \sqrt{3}u - 1 = 0$ $\qquad \sqrt{3}u + 1 = 0$

$\tan x = 1$ $\qquad \sqrt{3}u = 1$ $\qquad \sqrt{3}\tan x = -1$

$\tan x = u = \frac{1}{\sqrt{3}}$ $\qquad \tan x = -\frac{1}{\sqrt{3}}$

Answers π apart.





$\frac{\pi}{4}$ or 45° ref. angle. $\qquad \frac{\pi}{6} = 30^\circ$ ref. angle.

45° OR $180^\circ + 45^\circ$

$\frac{\pi}{4}$ $\qquad = 225^\circ = \frac{5\pi}{4}$

$30^\circ = \frac{\pi}{6}$

$180^\circ + 30^\circ = 210^\circ$

$= \frac{7\pi}{6}$

Also $\frac{\pi}{6}$ reference angle.

$180^\circ - 30^\circ = 150^\circ = \frac{5\pi}{6}$

$360^\circ - 30^\circ = 330^\circ = \frac{11\pi}{6}$

conversions:

$(225^\circ)\left(\frac{\pi}{180^\circ}\right)$

$\frac{\cancel{225}}{\cancel{180}} = \frac{5\pi}{4}$

$$\boxed{x \in \left\{\frac{\pi}{4}, \frac{5\pi}{4}, \frac{\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}\right\}}$$

$$x \in \left\{\frac{\pi}{4} + 2n\pi, \frac{5\pi}{4} + 2n\pi, \frac{\pi}{6} + 2n\pi, \frac{7\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi, \frac{11\pi}{6} + 2n\pi \;\middle|\; n \in \mathbb{Z}\right\}$$

$n = 0, \pm 1, \pm 2, \pm 3, \ldots$

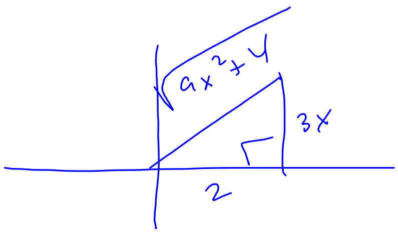
$$x \in \left\{ w + 2n\pi \;\middle|\; w = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}, n \in \mathbb{Z}\right\}$$

$$x \in \left\{ w + n\pi \;\middle|\; w = \frac{\pi}{4}, \frac{\pi}{6}, \frac{5\pi}{6}, n \in \mathbb{Z}\right\}$$

captures all of 'em, when you notice the pattern of sol'ns being $\pi = 180^\circ$ apart in each picture.

5. (10 pts) Re-write $\boxed{\sin\left(\arctan\left(\frac{3x}{2}\right)\right)}$ as an algebraic expression. $\Rightarrow \sin\theta$

Relates to Calc II and trigonometric Substitution.



$$\sqrt{(3x)^2+(2)^2} = \sqrt{9x^2+4}$$

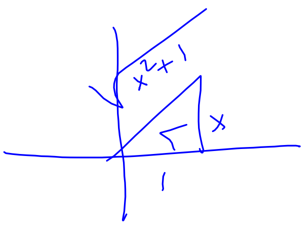
$$\Rightarrow \sin\theta = \frac{3x}{\sqrt{9x^2+4}}$$

7. (10 pts) Re-write $\cos[\arctan(x) - \arcsin(x)]$ as an algebraic expression. You will have a radical expression in the denominator. Leave it that way.

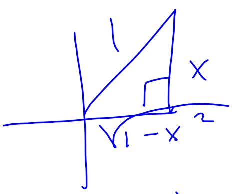
Similar to #5, but a little deeper down the rabbit-hole.

$$= \cos(u+v) = \cos u\cos v + \sin u \sin v$$

$$= \cos(\arctan(x))\cos(\arcsin(x)) + \sin(\arctan(x))\sin(\arcsin(x))$$



arctan(x)



arcsin(x)

$$= \frac{1}{\sqrt{x^2+1}}\cdot\frac{\sqrt{1-x^2}}{1} + \frac{x}{\sqrt{x^2+1}}\cdot x = \frac{\sqrt{1-x^2}+x^2}{\sqrt{x^2+1}}$$

$$u - v = u + (-v)$$

$$\cos(u-v) =$$

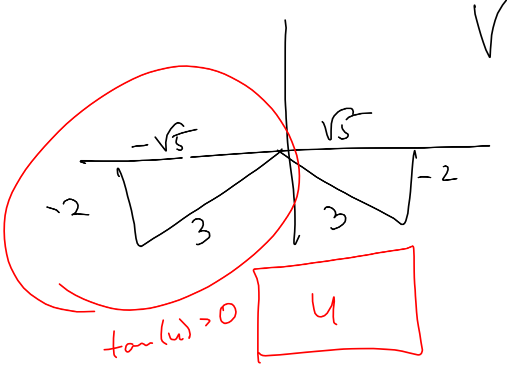
$$= \cos(u)\cos(-v) - \sin(u)\sin(-v) = \cos u \cos v - \sin u(-1)\sin v$$

$$= \cos u \cos v + \sin u \sin v$$

8. (10 pts) Find $\sin(2u)$, $\cos(2u)$ and $\tan(2u)$, given that $\sin(u) = -\frac{2}{3}$ and $\tan(u) > 0$. Give *exact* answers, in simplified radical form.

Analysis part: Find the quadrant in which u resides.

$\sqrt{3^2 - 2^2} = \sqrt{9-4} = \sqrt{5}$, silly.

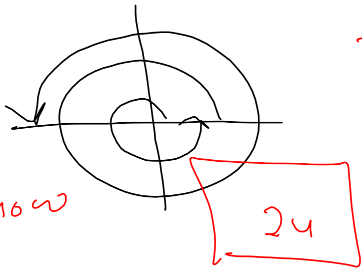


$u \in$ Q III

$180^\circ < u < 270^\circ$

$360^\circ < 2u < 540^\circ$

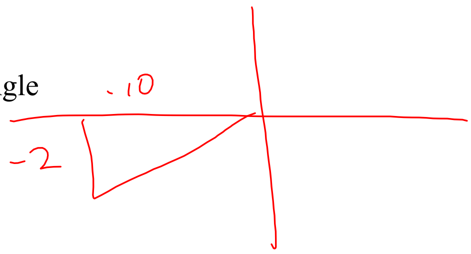
$540^\circ - 360^\circ = 180^\circ$

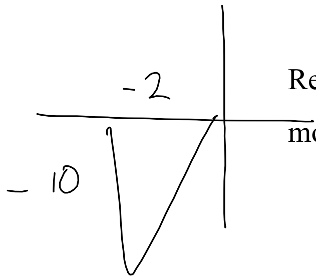


This puts 2u in QI OR QII

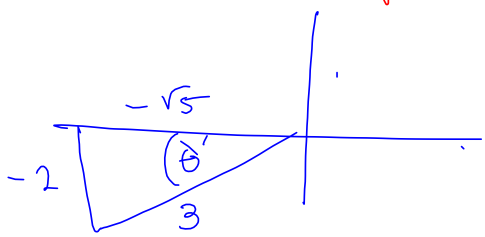
We need to narrow that down!

Reference angle less than 45^0





Reference angle more than 45^0



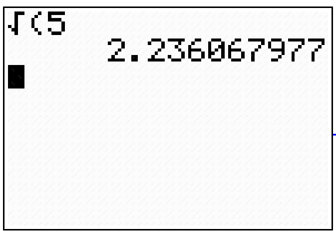
$\theta' < 45^\circ$, so

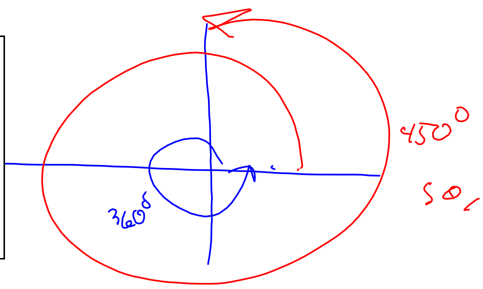
(because $\sqrt{5} > 2$)

$180^\circ < u < 225^\circ$

$360^\circ < 2u < 450^\circ$

$450^\circ - 360^\circ = 90^\circ$





so, $2u \in$ Q I

Bonus: Answer *up to three* (3) for up to 15 extra points:

1. A wheel of diameter $d = 20$ cm rolls 300 m. To the nearest full revolution, how many revolutions of the wheel were there? (BEWARE CONFLICTING UNITS!)

2. Build a cosine function that achieves its maximum height of $y = 200$ m at time $x = 5$ seconds and its minimum height of $y = -2$ at $x = 53$ seconds.

3. What is the area of the sector intercepted by an arc of 290^0 in a circle of radius 60 cm? Give an *exact* answer!

4. Sketch the graph of $100\sin\left(\frac{\pi}{16}x + \frac{11\pi}{8}\right) + 20$.

Any question from any previous test is fair game. Re-hash your tests, always. Know how to work the whole thing or ask me questions. Due diligence...