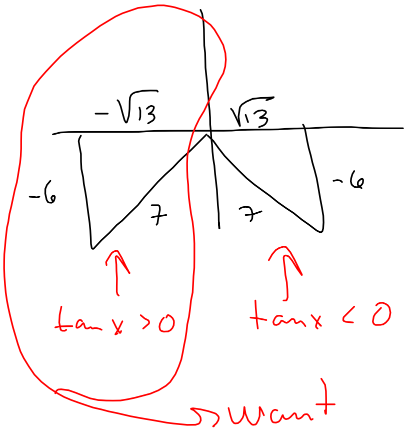
$\sin x = -\frac{6}{7}$

$\csc x = -\frac{7}{6}, \quad \tan x > 0$

§2.1

$49 - 36 = 13$



Double-Angle, Half-Angle §2.5

Find $\sin\left(\frac{19\pi}{12}\right)$ in 2 ways:

$240^\circ = \frac{4\pi}{3}$

$180^\circ + 60^\circ$

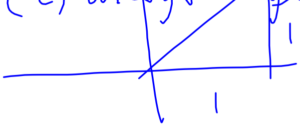
$19 = 1 + 18 = 2 + 17 = 3 + 16$ Ahhhh....

$\frac{3\pi}{3} + \frac{\pi}{3} = \pi + \frac{\pi}{3}$

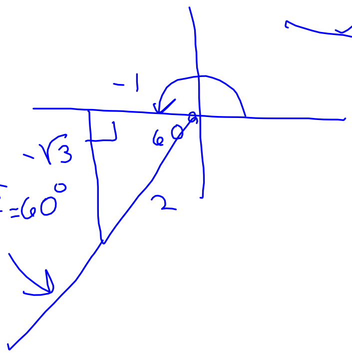
$\frac{19\pi}{12} = \frac{3\pi}{12} + \frac{16\pi}{12} = \frac{\pi}{4} + \frac{4\pi}{3}$

$\frac{\pi}{4} = 45^\circ$

(2) Using $\sqrt{2}$ angle sum identity



$\frac{\pi}{3} = 60^\circ$



$\sin(u+v) = \sin u \cos v + \sin v \cos u$

$= \sin\frac{\pi}{4}\cos\frac{4\pi}{3} + \sin\frac{4\pi}{3}\cos\frac{\pi}{4}$

$= \left(\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{2}\right) + \left(\frac{-\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) = \frac{-1-\sqrt{3}}{2\sqrt{2}}$ is OK by me,

unless I ask for simplified radical form

$\left(\frac{-1-\sqrt{3}}{2\sqrt{2}}\right)\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{-\sqrt{2}-\sqrt{3}\sqrt{2}}{2\cdot 2} = \boxed{\frac{-\sqrt{2}-\sqrt{6}}{4}}$

(b) By $\frac{1}{2}$-angle formula

$$\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos u}{2}}$$

From Power-Reduction Formula

$$\sin^2 u = \frac{1-\cos(2u)}{2} \qquad \cos^2 u = \frac{1+\cos(2u)}{2}$$

$$\sin^2\left(\frac{v}{2}\right) = \frac{1-\cos v}{2}$$

$$\sqrt{\sin^2\left(\frac{v}{2}\right)} = \sqrt{\frac{1-\cos v}{2}}$$

$$\left|\sin\left(\frac{v}{2}\right)\right| = \sqrt{\frac{1-\cos v}{2}}$$

$$\boxed{\sin\frac{v}{2} = \pm\sqrt{\frac{1-\cos v}{2}}}$$

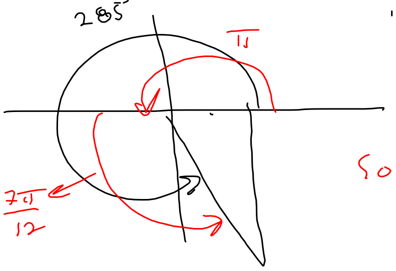
$$\frac{19\pi}{12} = \frac{1}{2}\left(\frac{19\pi}{6}\right)$$

Knowing which is HUGE.

$$\sin\frac{19\pi}{12} = \sin\left(\frac{1}{2}\left(\frac{19\pi}{6}\right)\right) = \pm\sqrt{\frac{1-\cos\frac{19\pi}{6}}{2}}$$

$$\frac{19\pi}{12} = \frac{12\pi}{12} + \frac{7\pi}{12} = \pi + \frac{7\pi}{12}$$

$$180^\circ + 105^\circ = 285^\circ$$



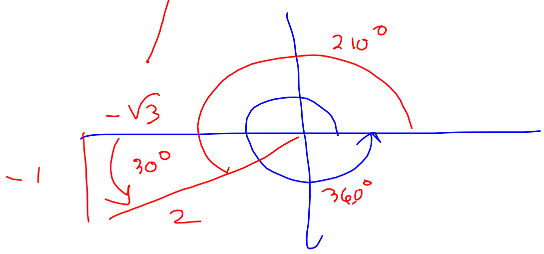
So $\sin\frac{19\pi}{12} = -$

$$= -\sqrt{\frac{1-\cos\frac{19\pi}{6}}{2}} = -\sqrt{\frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{2}}$$

$\frac{8}{10}$ out of simplified below

$$\left(\frac{19\pi}{6}\right)\left(\frac{180}{\pi}\right) = 19\cdot 30 = 570^\circ$$

$$570 - 360 = 210$$



$$-\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = -\sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}} = -\sqrt{\frac{2+\sqrt{3}}{4}} = \boxed{-\frac{\sqrt{2+\sqrt{3}}}{2}}$$

$$= \boxed{\frac{-\sqrt{2}-\sqrt{6}}{4}}$$

Same?!

$$\frac{A}{B} = A\cdot\frac{1}{B} \qquad \frac{\frac{2+\sqrt{3}}{2}}{2} = \frac{2+\sqrt{3}}{2}\cdot\frac{1}{2}$$

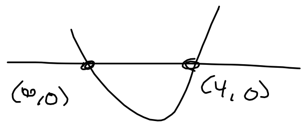
$$= \frac{2+\sqrt{3}}{4}$$

2.3 #23

Solve $3\tan^3 x = \tan x$

$x^2 - 4x$

$x(x-4)$



$\Rightarrow \quad 3\tan^3 x - \tan x = 0$

$\tan(x)\left(3\tan^2 x - 1\right) = 0$

$\tan(x)\left(\sqrt{3}\tan x - 1\right)\left(\sqrt{3}\tan x + 1\right) = 0$

$3\tan^2 x - 1 = 0$

$a = 3,\ b = 0,\ c = 1$

$b^2 - 4ac = 0^2 - 4(3)(-1) = +12$

$2\,\underline{|\,12}$
$2\,\underline{|\,6}$
3

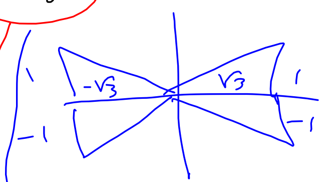
$\sqrt{12} = 2\sqrt{3}$

$3u^2 - 1 = 0$

$u = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{0 \pm 2\sqrt{3}}{2(3)} = \pm \dfrac{\sqrt{3}}{3}$

$3u^2 = 1$

$u^2 = \frac{1}{3} \Rightarrow u = \pm\sqrt{\frac{1}{3}} = \pm \frac{1}{\sqrt{3}} = \tan x$



$\pm \dfrac{\sqrt{3}}{3} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{3}{3\sqrt{3}}$

$= \dfrac{1}{\sqrt{3}}$

$x^2 - 9 = (x-3)(x+3)$

$x^2 - 7 = (x-\sqrt{7})(x+\sqrt{7})$

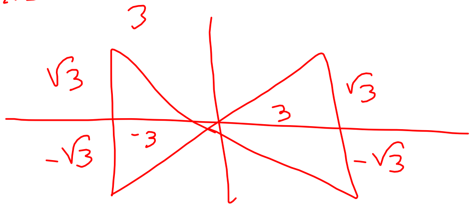
$x^2 + 7 = x^2 - (\sqrt{7}\,i)^2 = (x - \sqrt{7}\,i)(x + \sqrt{7}\,i)$

$x^2 = -7$

$x = \pm\sqrt{-7} = \pm i\sqrt{7} = x$

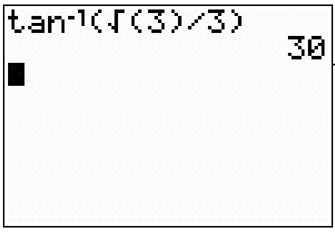
$(x - i\sqrt{7})(x - (-i\sqrt{7}))$

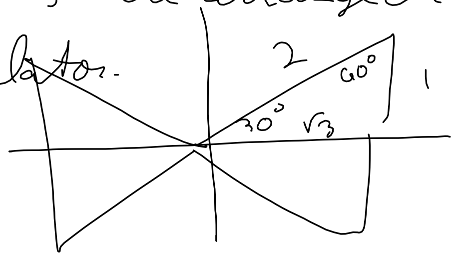
$\tan x = \pm \dfrac{\sqrt{3}}{3}$



Tougher to see this as a 1-2-√3 triangle.

If you can't see it's a $1-2-\sqrt{3}$ or how the $1, 2, \& \sqrt{3}$ are arranged. Go to Degrees calculator.





Find all sol'ns in $[0, 2\pi)$

$30°, 150°, 210°, 330°$

$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Highest Level of Difficulty will be the $f(\text{trig}(2x)) = 0$ situations

Double angle.

Find $\cos\left(\frac{11\pi}{12}\right)$ using a half-angle

$\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos(u)}{2}}$ $\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos u}{2}}$

Next time: Double-angle applications.

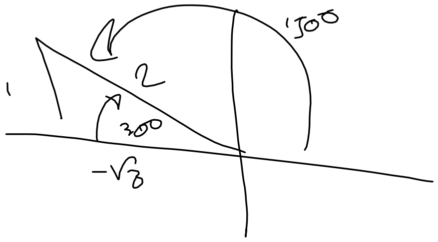
S25 #35 is like this
& idiot didn't assign it.

Find $\sin 75^\circ$, $\cos 75^\circ$, but don't fool with $\tan 75^\circ$; rather, $\tan 75^\circ = \frac{\sin 75^\circ}{\cos 75^\circ}$

Avoid $\tan\left(\frac{u}{2}\right)$, entirely, by doing sine & cosine!
↳ FORMULA.

No Double Jeopardy.

$$\sin 75^\circ = \sin\left(\frac{150^\circ}{2}\right) = + \sqrt{\frac{1-\cos 150^\circ}{2}} = \sqrt{\frac{1+\sqrt{3}/2}{2}}$$



$$= \sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}}$$

$$= \boxed{\frac{\sqrt{2+\sqrt{3}}}{2}} = \sin 75^\circ$$

$$\cos(150^\circ) = \sqrt{\frac{1+\left(-\frac{\sqrt{3}}{2}\right)}{2}}$$

$$= \ldots = \boxed{\frac{\sqrt{2-\sqrt{3}}}{2}} = \cos 75^\circ$$

$$\tan 75^\circ = \frac{\sin 75^\circ}{\cos 75^\circ} = \frac{\sqrt{2+\sqrt{3}}}{2} \cdot \frac{2}{\sqrt{2-\sqrt{3}}} = \boxed{\frac{\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}} = \tan 75^\circ$$

Follow-up: Rationalize the denominator in your result:

$$\sqrt{\left(\frac{2+\sqrt{3}}{2-\sqrt{3}}\right)\left(\frac{2+\sqrt{3}}{2+\sqrt{3}}\right)}$$

$$= \sqrt{\frac{4+4\sqrt{3}+3}{4-3}} = \sqrt{\frac{7+4\sqrt{3}}{1}} = \sqrt{7+4\sqrt{3}}$$

Idiot teacher: You were going to give a 10-minute quiz on the poorly-posed question on Test 1. You must do it on Thursday of this week, or never lose the label "idiot."