§2.3 #26. 68,

26) $2\sin^2 x + 3\sin x + 1 = 0$

$(2\sin x + 1)(\sin x + 1) = 0$

Let $u = \sin x$

Then $2u^2 + 3u + 1 = 0$

$(2u+1)(u+1) = 0$

$a = 2,\ b = 3,\ c = 1$

$b^2 - 4ac = 3^2 - 4(2)(1)$

$= 9 - 8 = 1 = 1^2$

$u = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-3 \pm \sqrt{1}}{2(2)} = \frac{-3 \pm 1}{4}$

$\nearrow \frac{-3+1}{4} = \frac{-2}{4} = -\frac{1}{2} = u$

$\searrow \frac{-3-1}{4} = \frac{-4}{4} = -1 = u$

How to cheat a factoring problem

$u = -\frac{1}{2}, -1 \implies$

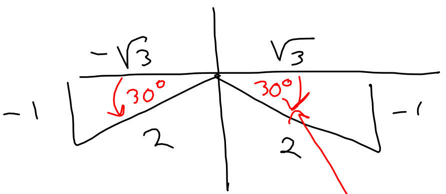
$2\left(u - \left(-\frac{1}{2}\right)\right)\left(u - (-1)\right) = 2\left(u + \frac{1}{2}\right)(u+1)$

$= (2u+1)(u+1) = 0$!

This is the factoring sledgehammer, reasoning backwards from the roots to the factored form

$u = -\frac{1}{2},\ u = -1$

$\sin x = -\frac{1}{2}$

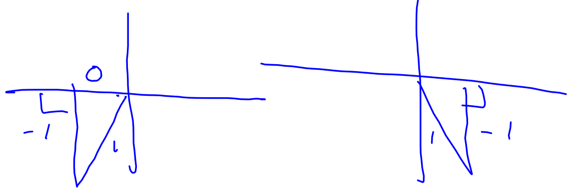


$\sin^{-1}\left(-\frac{1}{2}\right) = -30°$

what if we only look for answers in $[0, 360°]$ or $[0, 2\pi]$

$\sin x = -1$

The hypotenuse is ALWAYS positive.



$\implies x = 270° = \frac{3\pi}{2}$

$180^\circ + 30^\circ = 210^\circ = \frac{7\pi}{6}$

$360^\circ - 30^\circ = 330^\circ = \frac{11\pi}{6}$

So, $\frac{7\pi}{6}$, $\frac{11\pi}{6}$ for radians answers on $[0, 2\pi]$. But when they say "Solve" without specifying an interval, you should be thinking ALL solutions:

I will specify any interval over which to seek solutions, like $[0, 2\pi]$ or $[0, 360^\circ]$

If I don't specify, then I want 'em all:

for all, in

$$\boxed{\frac{7\pi}{6} + 2n\pi,\ \frac{11\pi}{6} + 2n\pi,\ \forall n \in \mathbb{Z}}$$

$\frac{7\pi}{6}, \frac{11\pi}{6}, \frac{3\pi}{2}$ Book → Sol'ns in $[0, 2\pi]$

OR $\boxed{\frac{3\pi}{2} + 2n\pi, n \in \mathbb{Z}}$

ALL sol'ns

Quiz Tuesday over Test 1 #3-type problem, only better-posed than the idiot teacher gave you the first time.

#68 $\sec^2 x + 2\sec x - 8 = 0$

$(\sec x + 4)(\sec x - 2) = 0$

$u^2 + 2u - 8 = 0$

$(u+4)(u-2) = 0$

$a=1, b=2, c=-8$

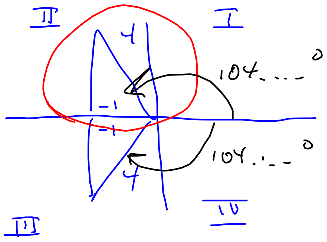
$b^2-4ac = 2^2 - 4(1)(-8)$

$= 4+32 = 36 \rightsquigarrow \sqrt{36} = 6$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-2 \pm 6}{2} = \frac{-1 \pm 3}{1}$ ↗ 2, ↘ −4

$\Rightarrow u = \sec x = -4$ or $u = \sec x = 2$

Flip it: $\cos x = -\frac{1}{4}$ $\cos x = \frac{1}{2}$



$\arccos(-\frac{1}{4})$ or $-\arccos(-\frac{1}{4})$

ALL: $\arccos(-\frac{1}{4}) + 2n\pi$,

$\arccos(-\frac{1}{4}) + 2n\pi$, $n \in \mathbb{Z}$

in $[0, 2\pi]$ we have

$\arccos(-\frac{1}{4})$ or $\operatorname{arcsec}(-4) =$

≈ 104.4775122

How to capture the other one in the 3rd quadrant that our picture says is also a sol'n?

The other piece:

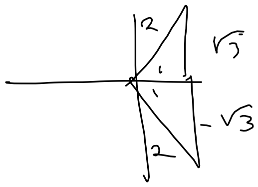
~~$\operatorname{arcsec}(x) = 2$~~

No $\sec(x) = 2$ $\boxed{\cos(x) = \frac{1}{2}}$

$\operatorname{arcsec}(\sec(x)) = \operatorname{arcsec}(2)$

$x = \operatorname{arcsec}(2)$

$= \arccos(\frac{1}{2})$



$60°, 300°$

$\frac{\pi}{3}, \frac{5\pi}{3}$

$\frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi, \forall n \in \mathbb{Z}$

Angle Sum formulas

I care more about $\sin(u+v)$ & $\cos(u+v)$ than anything else.

$\sin(u\pm v) = \sin u \cos v \pm \sin v \cos u$

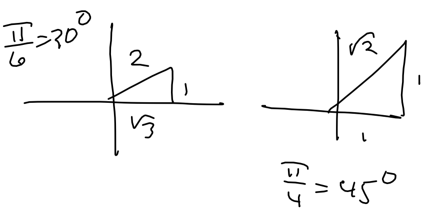
$\cos(u+v) = \cos u \cos v - \sin u \sin v$

$\frac{5\pi}{12}$

Evaluate $\sin\left(\frac{5\pi}{12}\right)$ & I want it EXACT.

$\frac{5}{12} = \frac{1}{12} + \frac{4}{12} = \frac{2}{12} + \frac{3}{12} = \frac{1}{6} + \frac{1}{4} \rightsquigarrow \frac{\pi}{6} + \frac{\pi}{4}$

So, $\sin\left(\frac{5\pi}{12}\right) = \sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right) = \sin\frac{\pi}{6}\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\frac{\pi}{6}$



$= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{1+\sqrt{3}}{2\sqrt{2}}$ is fine by me.

Bonus: Simplified radical form:

$\left(\frac{1+\sqrt{3}}{2\sqrt{2}}\right)\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{\sqrt{2} + \sqrt{3}\sqrt{2}}{2\cdot 2}$

$= \frac{\sqrt{2}+\sqrt{6}}{4}$

$\sqrt{3}\sqrt{2} = \sqrt{3\cdot 2} = \sqrt{6}$

$\tan(u+v) = \frac{\sin(u+v)}{\cos(u+v)}$ is my approach.

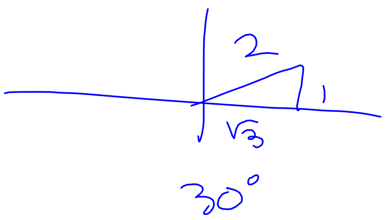
#s 11-26 get you to play the game

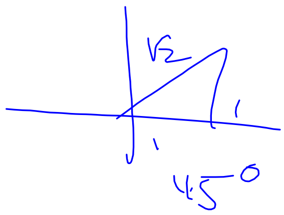
#s 11-18 spoon-fed

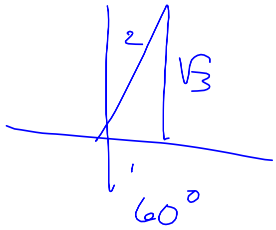
#s 19-26 are good.

$$\frac{\pi}{12} = \frac{2\pi}{12} - \frac{\pi}{12} = \frac{3\pi}{12} - \frac{2\pi}{12} = \frac{\pi}{4} - \frac{\pi}{6}$$

Looking for divisors of 2, 4, 6 from the 12-pt unit circle.







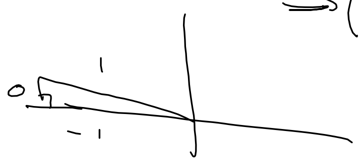
69 $\sin(x+\pi) - \sin x + 1 = 0$

SWIDT

$\sin x \cos\pi + \sin\pi \cos x - \sin x + 1 = 0$

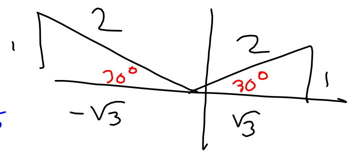
π

$\Rightarrow (\sin(x))(-1) + 0 - \sin x + 1 = 0$



$-2\sin x + 1 = 0$

$\sin x = \frac{1}{2}$



Book specifies
$x \in [0, 2\pi]$

$30^\circ, \ 180^\circ - 30^\circ = 150^\circ$

$\frac{\pi}{6}, \ \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$x \in \left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$.

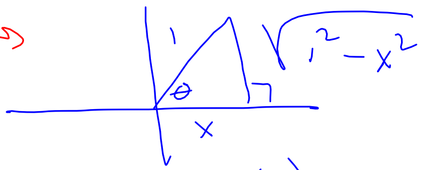
#s 53-56 $\boxed{\text{Assume these are in QI}}$ → Mainly for the $\arcsin(\sin x)$ & $\arccos(\cos(x))$ only spit out x, if x is in ~~the~~ a right quadrant

$$\sin(\underbrace{\arcsin(x)}_{u} + \underbrace{\arccos(x)}_{v}) = \sin\theta$$

$$= \sin(u+v) = \sin u \cos v + \sin v \cos u$$

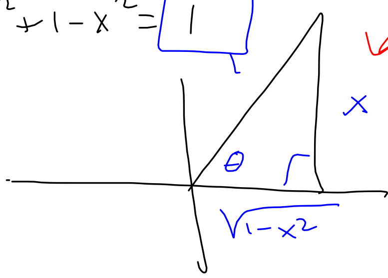
$$= \sin(\arcsin(x))\cos(\arccos(x))$$
$$+ \boxed{\sin(\arccos(x))}\boxed{\cos(\arcsin(x))}$$

$$= x \cdot x + \sqrt{1-x^2}\sqrt{1-x^2} = x^2 + (1-x^2)$$
$$= x^2 + 1 - x^2 = \boxed{1}$$



$\theta = \arccos(x)$

$\Rightarrow \sin\theta = \dfrac{\sqrt{1-x^2}}{1}$



$\theta = \arcsin(x)$

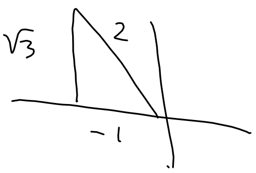
$\cos\theta = \sqrt{1-x^2}$

$\boxed{\#s\ 1\text{-}8}$

Not spitting out the $\frac{2\pi}{3}$

$$\arcsin\left(\sin\left(\frac{2\pi}{3}\right)\right) = \arcsin\left(\frac{\sqrt{3}}{2}\right) = 60^\circ = \frac{\pi}{3}$$

$\frac{2\pi}{3}$



So be careful.