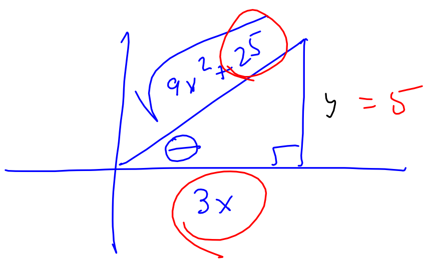
| Section 2.2 | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Video | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
| Textbook | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 26 | 12 | 27 | 31 | 37 | | 22 | 24 | | 41 | | 58 | 59 | 45 | 49 | 51 | 55 | 56 |

$$\tan\left(\arccos\left(\frac{3x}{\sqrt{9x^2+25}}\right)\right) = \tan\theta$$




$(3x)^2$

$x^2+y^2=r^2$

$a^2+b^2=c^2$

$3x^2+y^2=$

No!

$(3x)^2+y^2 = 9x^2+25$

$9x^2+y^2 = 9x^2+25$

$y^2 = 25$

$y = \pm\sqrt{25} = \pm 5$

TAKE THE "+"

in these type-problems

$(\quad)^2$

Do these in Q I

$$\Rightarrow \boxed{\tan\theta = \frac{5}{3x}}$$

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$\csc^2 x - 4\csc x + 4 = 0$

$u^2 - 4u + 4 = 0$

$(u-2)^2 = 0$

$a=1,\ b=-4,\ c=4$

Need parens or just write $4^2$

$\Rightarrow b^2 - 4ac = (-4)^2 - 4(1)(4)$

The $b^2$ is always $\geq 0$

$= 16 - 16 = 0$

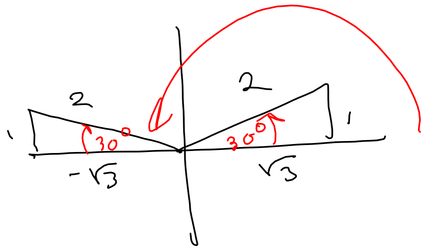
$\frac{\pi \text{ radians}}{180 \text{ degrees}} = 1$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{4 \pm 0}{2} = 2$$

$u = 2$

$\csc x = 2$

$\sin x = \frac{1}{2}$



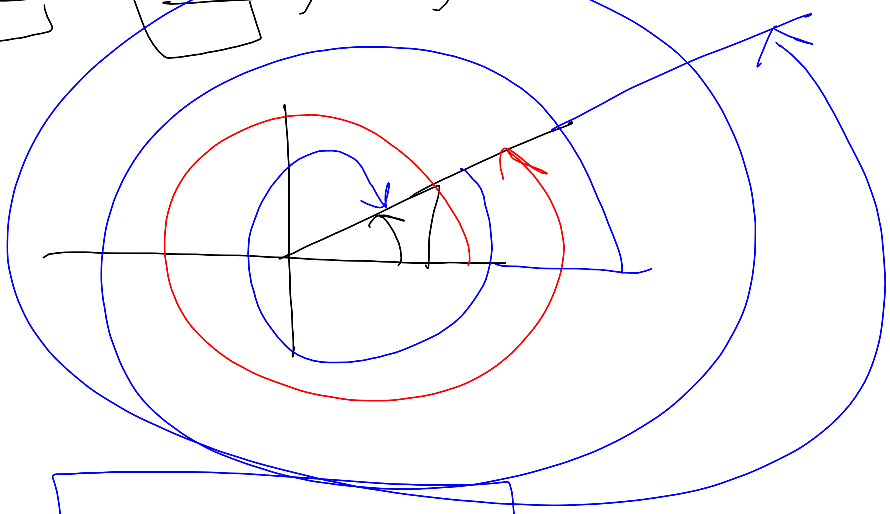

$x = 30°, 150°$

$$\boxed{= \frac{\pi}{6}, \frac{5\pi}{6}}$$

All sol'ns that live in $[0, 2\pi]$ or $[0°, 360°]$

$\exists$ $\infty$'ly many sol'ns.

There are infinitely many solutions



$\boxed{30° + 360° n, \ n \in \mathbb{Z}}$ = Integers $= \{0, \pm 1, \pm 2, \pm 3, \ldots\}$

$30°$
$30° + 360°$
$30° - 360°$
$30° + 2 \cdot 360°$
$30 - 2 \cdot 360°$
$\vdots$

$$x \in \{30° + 360° n \mid n \in \mathbb{Z}\}$$
$$= \left\{\frac{\pi}{6} + 2\pi n \mid n \in \mathbb{Z}\right\}$$

28. $\dfrac{\cot^3 t}{\csc t} = \cos t\,(\csc^2 t - 1)$

29. $\dfrac{1}{\tan\beta} + \tan\beta = \dfrac{\sec^2\beta}{\tan\beta}$

Answer

30. $\dfrac{\sec\theta - 1}{1-\cos\theta} = \sec\theta$

28

Pythagorus

$$\frac{\cot^{\cancel{3}} t}{\csc t} \stackrel{?}{=} \cos t\,(\csc^2 t - 1) = \cos t\,\cancel{(\cot^2 t)}$$

Just kind of manipulated both sides.

$$\frac{\cot t}{\csc t} \stackrel{?}{=} \cos t$$

$$\cot t = \cos t \csc t = (\cos t)\left(\frac{1}{\sin t}\right) = \frac{\cos t}{\sin t} = \cot t$$

$$\cot t = \cot t$$

$$\frac{\cot(t)}{\cot(t)} = \frac{\cot(t)}{\cot(t)}$$

$$1 = 1 \quad \text{IDENTITY}$$

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More formally: Reason from one to the other.

$$\frac{\cot^3(t)}{\csc(t)} = (\cos(t))(\csc^2(t) - 1)$$

$$\frac{\cot^3(t)}{\csc(t)} = \frac{\cot(t)\cot^2(t)}{\csc(t)} = \frac{\cot(t)}{\csc(t)}(\csc^2(t) - 1)$$

$$= \frac{\frac{\cos(t)}{\sin(t)}(\csc^2(t) - 1)}{\frac{1}{\sin(t)}} = \frac{\cos(t)}{\cancel{\sin(t)}}(\csc^2(t) - 1)\cdot\frac{\cancel{\sin(t)}}{1}$$

$$= \cos(t)(\csc^2(t) - 1) \quad \text{DONE}$$

28. $\dfrac{\cot^3 t}{\csc t} = \cos t\left(\csc^2 t - 1\right)$

29. $\dfrac{1}{\tan\beta} + \tan\beta = \dfrac{\sec^2\beta}{\tan\beta}$

Answer

30. $\dfrac{\sec\theta - 1}{1-\cos\theta} = \sec\theta$

29 $\quad \dfrac{1}{\tan\beta} + \tan\beta = \dfrac{\sec^2\beta}{\tan\beta}$

$$\frac{1}{\tan\beta} + \frac{\tan\beta}{1}\cdot\frac{\tan\beta}{\tan\beta} = \frac{1+\tan^2\beta}{\tan\beta} = \frac{\sec^2\beta}{\tan\beta}$$

30 $\quad \dfrac{\sec\theta - 1}{1-\cos\theta} = \sec\theta$

$$\boxed{a^2-b^2 = (a-b)(a+b)}$$

STANDARD TRICK

$$\frac{(\sec\theta - 1)}{(1-\cos\theta)}\cdot\frac{(1+\cos\theta)}{(1+\cos\theta)} = \frac{\sec\theta + \sec\theta\cos\theta - 1 - \cos\theta}{1-\cos^2\theta}$$

$$= \frac{\sec\theta + 1 - 1 - \cos\theta}{\sin^2\theta} = \frac{\sec\theta - \cos\theta}{\sin^2\theta}$$

$$= \frac{\frac{1}{\cos\theta} - \frac{\cos\theta}{1}\cdot\frac{\cos\theta}{\cos\theta}}{\sin^2\theta} = \frac{\frac{1-\cos^2\theta}{\cos\theta}}{\sin^2\theta} = \frac{\frac{\sin^2\theta}{\cos\theta}}{\frac{\sin^2\theta}{1}}$$

$$= \frac{\sin^2\theta}{\cos\theta}\cdot\frac{1}{\sin^2\theta} = \frac{1}{\cos\theta} = \sec\theta.$$

$$\frac{\sec\theta - 1}{1-\cos\theta} = \frac{\frac{1}{\cos\theta} - \frac{1}{1}\cdot\frac{\cos\theta}{\cos\theta}}{1-\cos\theta} = \frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{1-\cos\theta}{1}}$$

$$= \frac{1-\cos\theta}{\cos\theta}\cdot\frac{1}{1-\cos\theta} = \frac{1}{\cos\theta} = \sec\theta$$

Tortoise wins the race. Be systematic, deliberate, leaving plenty of breadcrumbs for yourself, for when things go vershizzle,

*WHICH THEY WILL!!!*

$$\left(\sin^{\frac{1}{2}} x\right)\left(\cos x\right) - \left(\sin^{\frac{5}{2}} x\right)\left(\cos x\right) =$$

Factor out GCF $= \sin^{\frac{1}{2}} x \cos x$

$\frac{5}{2} - \frac{1}{2} = \frac{4}{2} = 2$

$$\left(\sin^{\frac{1}{2}}(x)\right)\left(\cos(x)\right)\left[1 - \sin^2 x\right] = \sin^{\frac{1}{2}}(x)\cos(x)\left[\cos^2(x)\right]$$

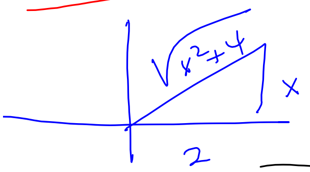
$$= \sin^{\frac{1}{2}}(x)\cos^3(x) = \sqrt{\sin(x)}\ \cos^3(x) = \cos^3 x\sqrt{\sin x}$$

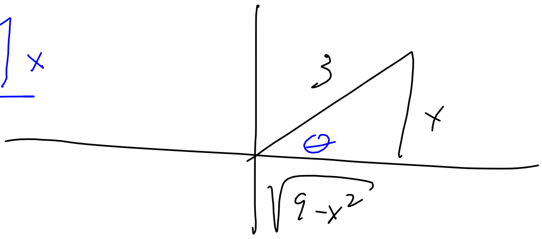
$$\left(\sin^{\frac{1}{2}} x \cos x\right)\left[\frac{\sin^{\frac{1}{2}} x \cos x}{\sin^{\frac{1}{2}} x \cos x} - \frac{\sin^{\frac{5}{2}} x \cos x}{\sin^{\frac{1}{2}} x \cos x}\right]$$ See?

$$6x + 9 = 3\left(\frac{6x}{3} + \frac{9}{3}\right) = 3(2x+3)$$

$$\boxed{\int \frac{dx}{\sqrt{x^2+4}}}$$ Calc. II

$$\sqrt{9-x^2} = \sqrt{9-(3\sin\theta)^2} = \sqrt{9-9\sin^2\theta}$$
$$= 3\sqrt{1-\sin^2\theta} = 3\sqrt{\cos^2\theta} = 3\left|\cos\theta\right|$$




$\frac{x}{3} = \sin\theta$

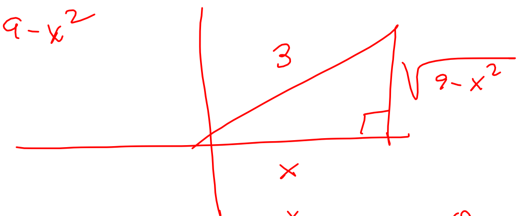
$x = 3\sin\theta$

$\sqrt{x^2} = |x|$

$\frac{x}{2} = \tan\theta$

$x = 2\tan\theta$

$\sqrt{9-x^2}$




$\frac{x}{3} = \cos\theta$

$x = 3\cos\theta$