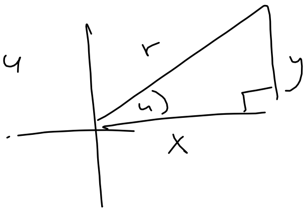
$$\frac{\sin u}{\cos u} = \frac{\frac{y}{r}}{\frac{x}{r}} = \frac{y}{r}\cdot\frac{r}{x} = \frac{y}{x} = \tan u$$



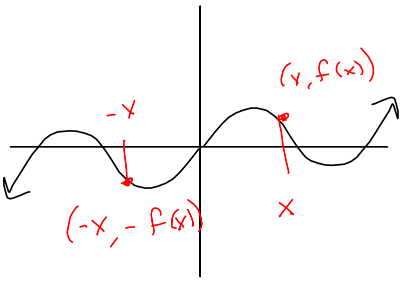
§ 2.1 #s 1–6

ODD FUNCS

$f(-x) = -f(x)$

$\sin x$, $\tan x$, $\csc x$

x^3, x^5, x^{2n+1}, ...

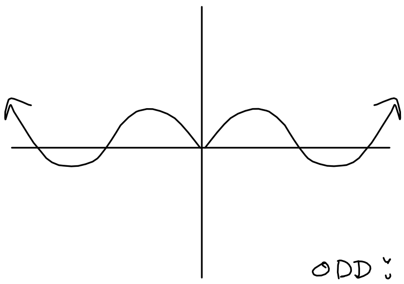


Even Functions

$f(-x) = f(x)$

$\cos x$, $\sec x$

x^2, x^4, x^{2n}, ...



ODD: $-$

EVEN: $+$

$$\frac{\tan x \sec x}{\csc x + \sin x} = \frac{\frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}}{\frac{1}{\sin x} + \sin x}$$

$(-)(-) = +$

$(-) + (-) = -$

$(+) + (-) = $ Neither.

$(+)(-) = -$

$$= \frac{\frac{(-)}{(+)}\cdot(+)}{(-) + (-)} = \frac{(-)}{(-)} = + \text{ EVEN}$$

$$f(x) = \frac{\tan x \sec x}{\csc x + \sin x}$$

$$\boxed{f(-x)} = \frac{\tan(-x)\sec(-x)}{\csc(-x) + \sin(-x)} = \frac{-\tan x \sec x}{-\csc(x) - \sin(x)} = \frac{-\tan x \sec x}{-(\csc x + \sin x)}$$

$$= \frac{\tan x \sec x}{\csc x + \sin x} \boxed{= f(x)}$$ means EVEN

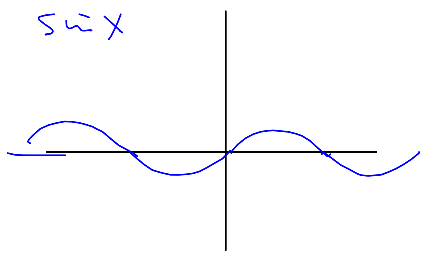
Cofunction jazz

f & cof $\qquad f\left(\frac{\pi}{2}-x\right) = \text{cof}(x)$

sine & cosine $\qquad \sin\left(\frac{\pi}{2}-x\right) = \cos(x)$

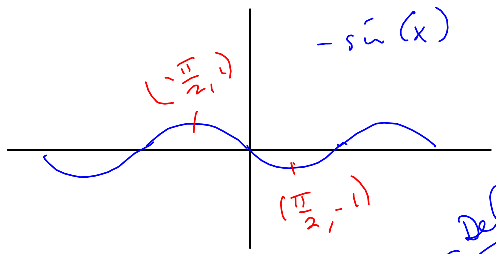
secant & cosecant

tangent & cotangent



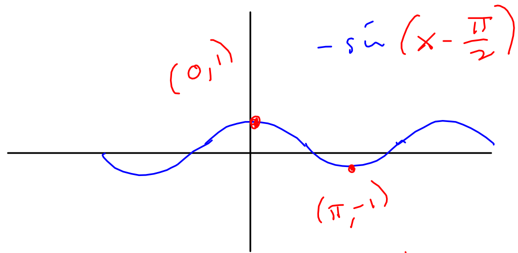
$\sin\left(\frac{\pi}{2}-x\right) = \sin\left(-\left(x-\frac{\pi}{2}\right)\right)$

$= -\sin\left(x-\frac{\pi}{2}\right)$



So, indeed, $\sin\left(\frac{\pi}{2}-x\right)$
$= \cos(x)$

Delay by $\pi/2$: shift right $\pi/2$

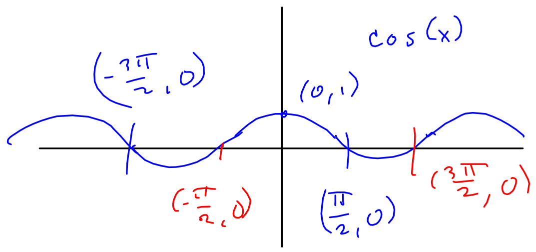


is cosine!

$\cos\left(\frac{\pi}{2}-x\right)$

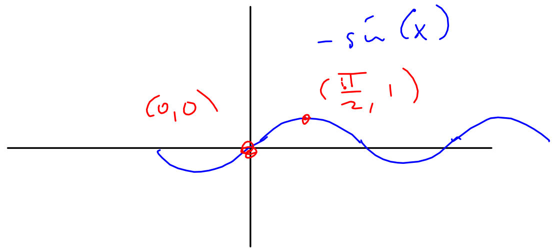
$= \cos\left(-\left(x-\frac{\pi}{2}\right)\right)$ is even

$= \cos\left(x-\frac{\pi}{2}\right)$ Right $\frac{\pi}{2}$.



$\cos(-\text{Man}) = \cos(\text{man})$

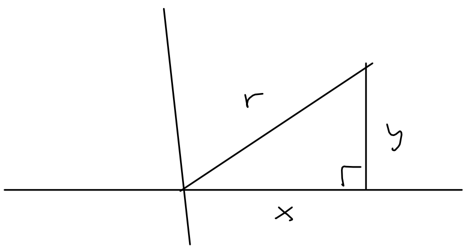
$\cos\left(x-\frac{\pi}{2}\right)$



Pythagorus (BASIC)

what if $r=1$?

we're on the unit circle



$x^2+y^2=r^2$

if $r=1$, then

$$\boxed{\cos^2\theta+\sin^2\theta=1}$$

KNOW IT.

$\frac{x}{r}=\cos\theta$

$x=r\cos\theta$

& if $r=1$,

then $x=\cos\theta$

$$\boxed{\tan^2\theta+1=\sec^2\theta}$$

CHEAT SHEET MATERIAL

$$\tan^2\theta+1=\frac{\sin^2\theta}{\cos^2\theta}+\frac{1}{1}\cdot\frac{\cos^2\theta}{\cos^2\theta}$$

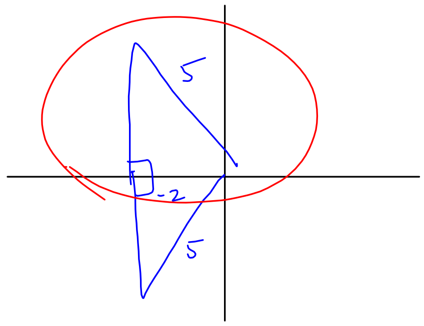
$$=\frac{\sin^2\theta+\cos^2\theta}{\cos^2\theta}=\frac{1}{\cos^2\theta}=\sec^2\theta$$

Reciprocal Identities,

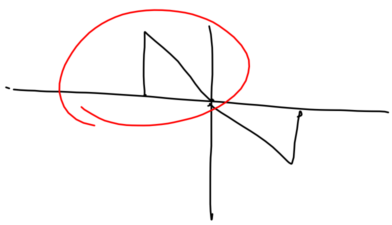
like $\csc\theta=\frac{1}{\sin\theta}$, I expect you to know.

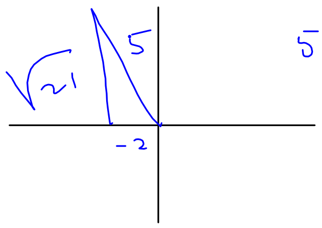
$\sec x = -\frac{5}{2}$ $\tan x < 0$

$\Rightarrow \cos x = -\frac{2}{5}$



$\tan x < 0$





$5^2 - (-2)^2 = 25 - 4 = 21$

$\rightsquigarrow \sqrt{21}$

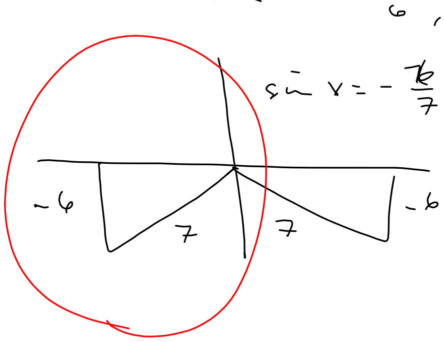
$\sin\theta = \frac{\sqrt{21}}{5}$ $\csc\theta = \frac{5}{\sqrt{21}}$

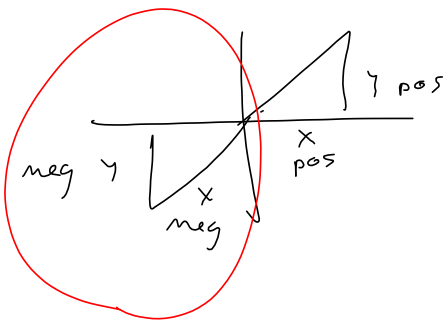
$\cos\theta = \frac{-2}{5}$ $\sec\theta = -\frac{5}{2}$

$\tan\theta = -\frac{\sqrt{21}}{2}$ $\cot\theta = -\frac{2}{\sqrt{21}}$

$\sin x = -\frac{6}{7}$

$\csc x = -\frac{7}{6}$, $\tan\theta > 0$ $\tan\theta > 0$





$$\frac{\cos^2\left(\frac{\pi}{2}-x\right)}{\cos x} = \frac{\sin^2(x)}{\cos(x)} = \frac{\sin x}{\cos x}\cdot\frac{\sin x}{1} = \tan x \sin x$$

Show that

$$\frac{\cos^2\left(\frac{\pi}{2}-x\right)}{\cos x} = \sin x \tan x$$

is standard question

53 $\left(\frac{\tan x + 1}{\sec x + \csc x}\right)\left(\frac{\tan x - 1}{\tan x - 1}\right)$ Based on intuition & trickiness.

when in doubt, break it down. Goal is one of the 6 trig functions.

$$\frac{\frac{\sin x}{\cos x} + \frac{\cos x}{\cos x}}{\frac{1}{\cos x}\cdot\frac{\sin x}{\sin x} + \frac{1}{\sin x}\cdot\frac{\cos x}{\cos x}} = \frac{\frac{\sin x + \cos x}{\cos x}}{\frac{\sin x + \cos x}{\sin x \cos x}}$$

$$= \frac{\cancel{(\sin x + \cos x)}}{\cancel{\cos x}}\cdot\frac{\sin x \cancel{\cos x}}{\cancel{(\sin x + \cos x)}} = \sin x.$$

Always be mindful of the $(a+b)(a-b)$ trick.

$= a^2 - b^2$

even though I didn't use it, here.

FACTOR

$6\cos^2\theta + 5\cos\theta - 6$ Let $u = \cos\theta$.

Then $6u^2 - 5u - 6 = 6u^2 - 9u + 4u - 6$

$= 3u(2u-3) + 2(2u-3)$

$= 3u ☺ + 2 ☺$

$= ☺(3u+2) = (2u-3)(3u+2)$

$= (3\cos\theta + 2)(2\cos\theta - 3)$

This is build-up to these probs:

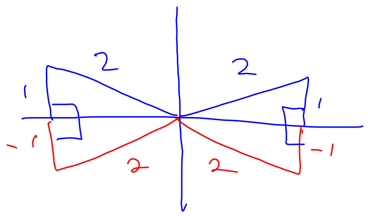
Solve:

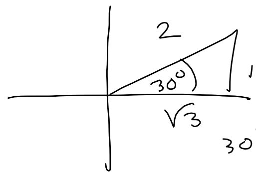
$4\sin^2\theta - 1 = 0$ Find all sol'ns in $[0, 2\pi]$

$(2\sin\theta - 1)(2\sin\theta + 1) = 0$

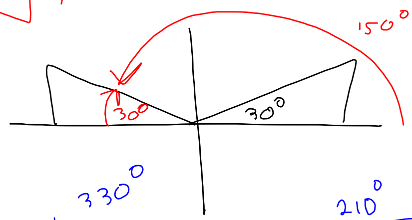
$\Rightarrow 2\sin\theta = 1$ $\quad 2\sin\theta = -1$

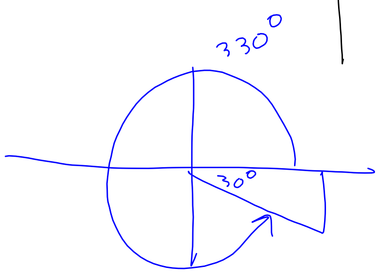
$\sin\theta = \frac{1}{2}$ $\quad \sin\theta = -\frac{1}{2}$

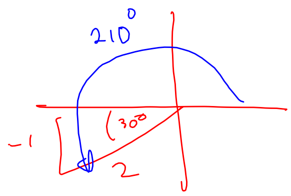




$30°$ OR $\frac{\pi}{6}$







§2.1

#s 1-6, 23-28

So, $\theta = 30°, 150°, 210°, 330°$

$4\sin^2\theta - 1 = 0$

$4u^2 - 1 = 0$

$4u^2 + 0u - 1 = 0$ $\quad a = 4,\ b = 0,\ c = -1$

$b^2 - 4ac = 0^2 - 4(4)(-1) = 16 \rightsquigarrow \sqrt{16} = 4$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{0 \pm \sqrt{16}}{2(4)} = \frac{\pm 4}{8} = \pm\frac{1}{2}$