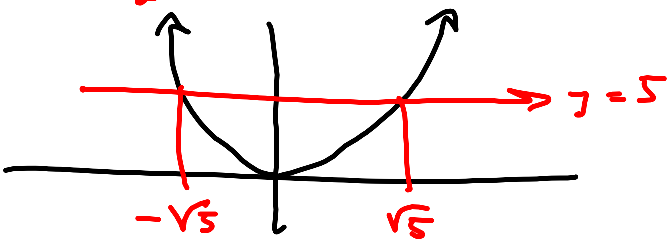
$x^2 = 5$

$\sqrt{x^2} = \sqrt{5}$

$x = \pm\sqrt{5}$

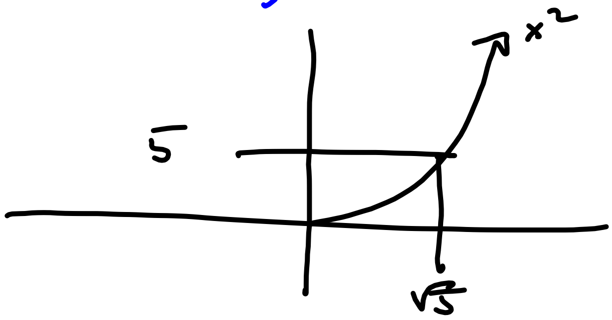
2-valued output

$\sqrt{x}$ & x^2 are inverses



2 values means this inverse is not a FUNCTION.

But we can MAKE it a function by restricting the domain of x^2 to $\{x \mid x \geq 0\}$



Thus if $f(x) = x^2$, then $f^{-1}(x) = \sqrt{x}$ is a function.

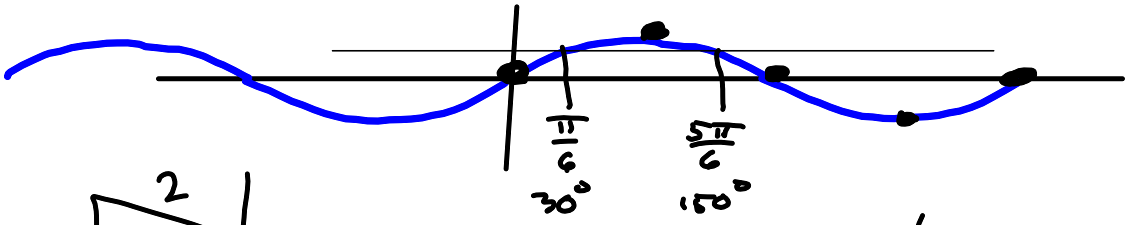
want to solve

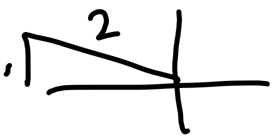
$f(x) = \sin x = \frac{1}{2}$

$f^{-1}(x) = \sin^{-1}(x) = \arcsin(x)$

Not $\frac{1}{\sin(x)}$, but the angle whose sine is x.

$\sin(x)$

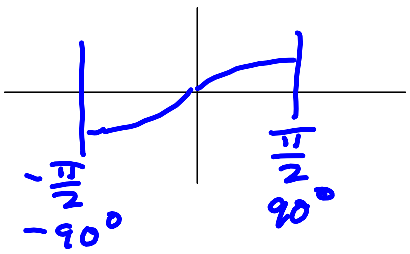




2 values that satisfy $\sin(x) = \frac{1}{2}$

$\sin(x)$ & x^2 lack an important property.

Guesses? NOT 1-to-1.



Restrict Sine to make it 1-to-1.

Then $\sin^{-1}(x) = \arcsin(x)$

NOT $\csc(x) = \frac{1}{\sin(x)}$

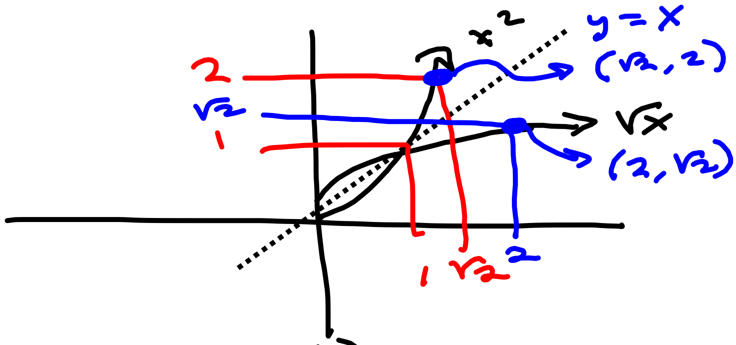
Student needs to know when $\sin^n(x) = (\sin(x))^n$ & be careful about $\sin^{-1}(x)$.

$\sin^2(x) = (\sin(x))^2$

Restricting sine to $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ MAKES it 1-to-1.

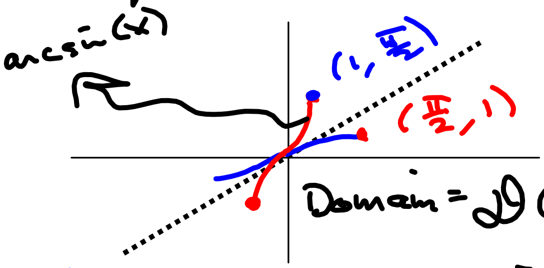
This MAKES $\sin^{-1}(x) = \arcsin(x)$ a function.

Facts about inverses



$f(x)$ & $f^{-1}(x)$ are reflections about $y=x$ of one another.

Make the slope $m=1$ @ $x=0$



Restricted

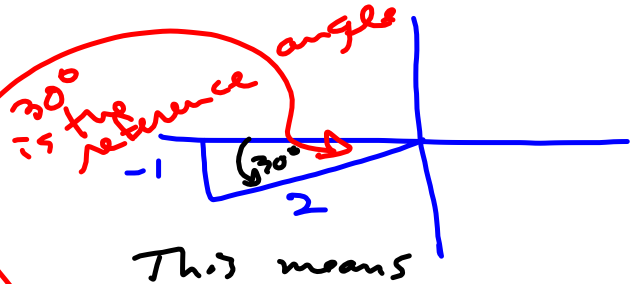
Domain $= \mathcal{D}(\text{sine}) = [-\frac{\pi}{2}, \frac{\pi}{2}]$
$= \mathcal{R}(\text{arcsine})$
$=$ Range of inverse

$\mathcal{R}(\text{sine}) = [-1, 1]$
$= \mathcal{D}(\text{arcsine})$

In the sequel, we'll use calculators to find arcsine, but will need to use some pictures & logic, too.

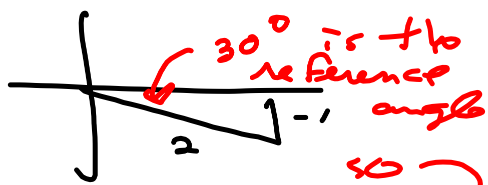
Suppose we know this picture:

$\sin x = -\frac{1}{2}$ &
$x \in Q\ \mathrm{III}$



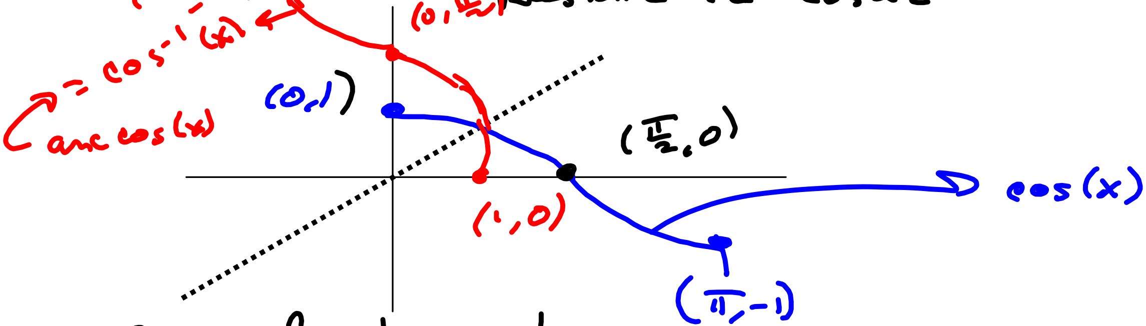
This means
$x = 180^\circ + 30^\circ = 210^\circ = x$

$\sin x = -\frac{1}{2}$
$\arcsin(-\frac{1}{2}) = -30^\circ$

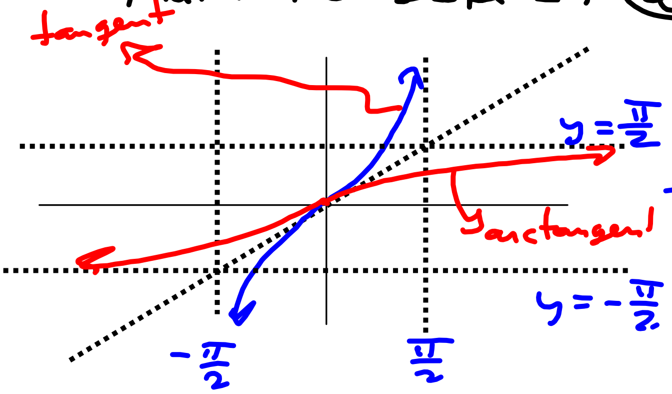


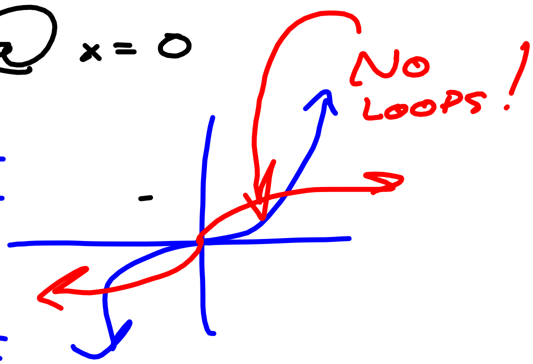
so

Restricted cosine



Same for tangent
Make the slope = 1 @ x = 0





$\mathcal{D}$(restricted tangent)
$= \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$= \mathcal{R}$(arctangent)

$\mathcal{R}$(tangent) $= (-\infty, \infty)$
$= \mathcal{D}$(arctangent)

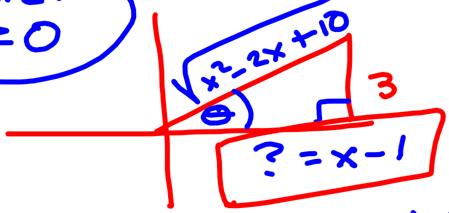
But what about
$\cot(x) = \frac{5}{3}$?

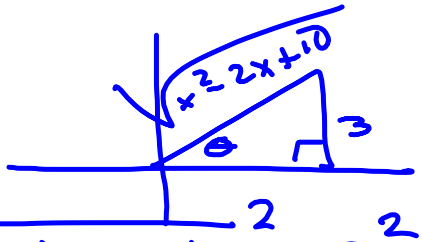
$\Longleftrightarrow$ $\tan(x) = \frac{3}{5}$

Like #s 75–78

$\theta = \arcsin\left(\frac{3}{\sqrt{x^2-2x+10}}\right) = \arccos$ ______ . ?

Restrict $\theta \geq 0$





$\sqrt{(x^2-2x+10)}^{\,2} - 3^2$

$= x^2-2x+10-9$

$= x^2-2x+1 = ?^2$

$\Longrightarrow \sqrt{x^2-2x+1} = \sqrt{?^2}$

$? = \pm\sqrt{x^2-2x+1}$

CRAP EXAMPLE, MILLS

& $\theta \geq 0 \Longrightarrow$

$? = \sqrt{x^2-2x+1} =$

$= \sqrt{(x-1)^2}$

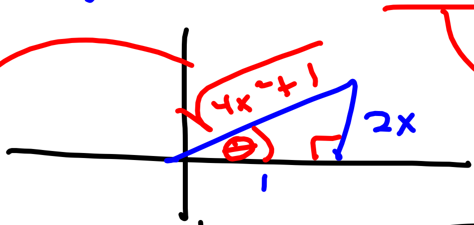
$= |x-1| = x-1$
with these restrictions

Now, for a question more like I want to ask:

Simplify:

$$f(x) = \sin(\arctan(2x)) = \frac{2x}{\sqrt{4x^2+1}}$$

angle whose tangent is $2x$ → θ



Relates to trig. substitution in CALC II

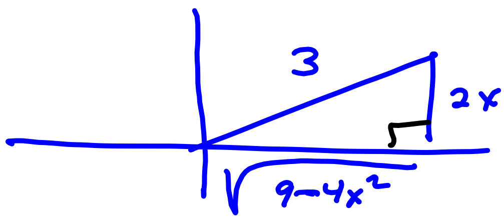
$$\sqrt{(2x)^2+1^2} = \sqrt{4x^2+1}$$

Cool!

See # 73 in S1.7

Write $\tan(\arcsin(\frac{2x}{3}))$ as an algebraic expression.

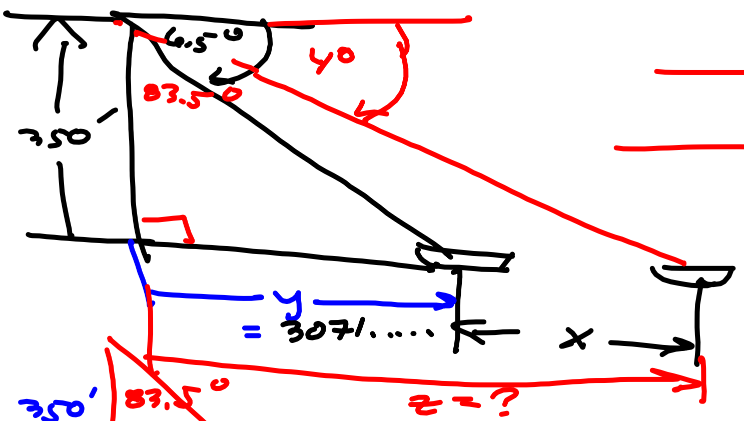
$(2x)^2 - 3^2$

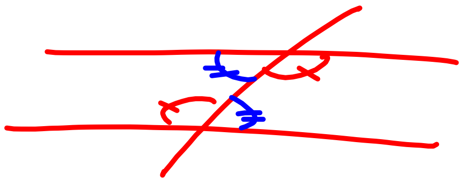


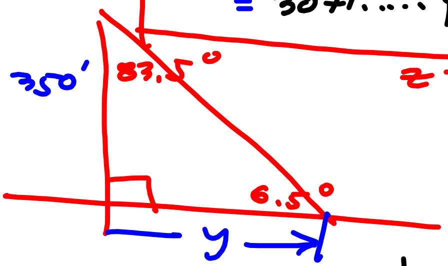
$$\sqrt{3^2-(2x)^2} = \sqrt{9-4x^2}$$

$$\tan(\arcsin(\tfrac{2x}{3})) = \frac{2x}{\sqrt{9-4x^2}}$$

S 1.8 #22







$$\frac{y}{350} = \tan 83.5°$$

$$y = 350 \tan 83.5°$$

$$= 3071.9105775 \text{ ft}$$

Good catch, Alex & Toni-Lynn

Get z the same as y.

$350 \tan 4° = z$

$\boxed{x = z - y}$ to finish