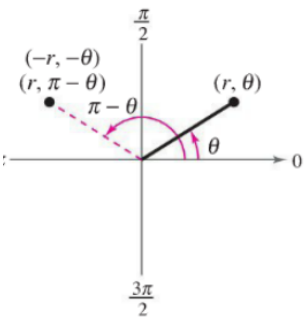
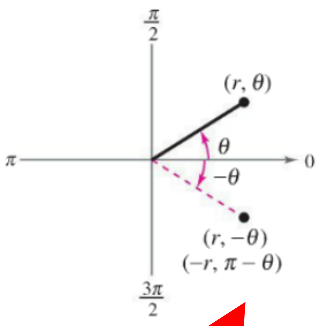
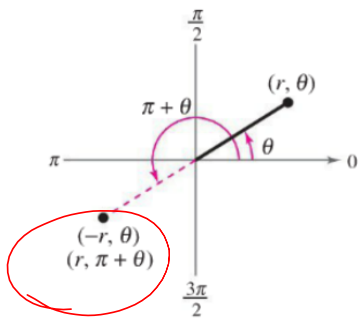


Symmetry with Respect to the Line $\theta = \frac{\pi}{2}$



Symmetry with Respect to the Polar Axis



Symmetry with Respect to the Pole

Tests for Symmetry in Polar Coordinates

The graph of a polar equation is symmetric with respect to the following when the given substitution yields an equivalent equation.

1. The line $\theta = \pi/2$: Replace (r, θ) by $(r, \pi - \theta)$ or $(-r, -\theta)$.
2. The polar axis: Replace (r, θ) by $(r, -\theta)$ or $(-r, \pi - \theta)$.
3. The pole: Replace (r, θ) by $(r, \pi + \theta)$ or $(-r, \theta)$.

Quick Tests for Symmetry in Polar Coordinates

1. The graph of $r = f(\sin \theta)$ is symmetric with respect to the line $\theta = \frac{\pi}{2}$.
2. The graph of $r = g(\cos \theta)$ is symmetric with respect to the polar axis.

Nov 29-8:16 AM

$r = 1 + \sin\theta = f(\sin\theta)$

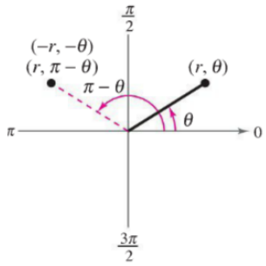
$\Rightarrow$ symmetric wrt $\theta = \frac{\pi}{2}$ (y-axis)

Here's the test:

$r = 1 + \sin(\pi - \theta)$

$\sin(\theta + \phi)$

$= \sin\theta\cos\phi + \sin\phi\cos\theta$

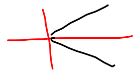


$= 1 + \sin\pi\cos(-\theta) + \sin(-\theta)\cos\pi$

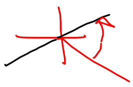
$= 1 + 0 + -\sin\theta(-1) = 1 + \sin\theta$!

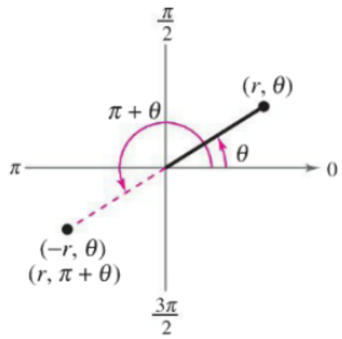
Yes!

Check wrt the polar axis



Check wrt the pole





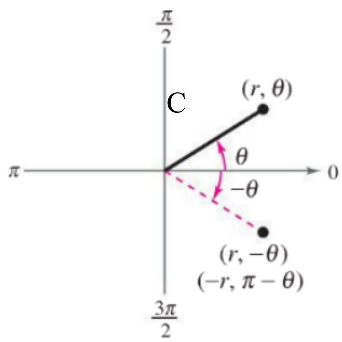
Symmetry with Respect to the Pole

$-r = 1 + \sin\theta$ No

$r = 1 + \sin(\pi + \theta)$?

$= 1 + \sin\pi\cos\theta + \sin\theta\cos\pi$

$= 1 - \sin\theta$ No



Symmetry with Respect to the Polar Axis

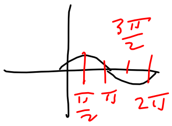
$r = 1 + \sin\theta$

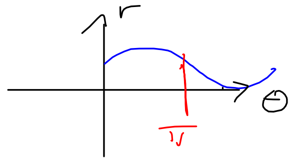
$r = 1 + \sin(-\theta) = 1 - \sin\theta$ No

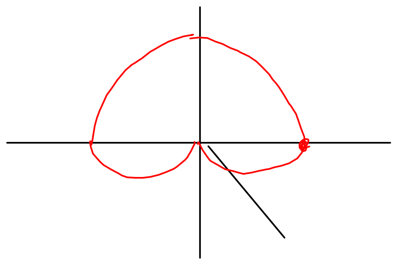
$-r = 1 + \sin(\pi - \theta)$

$-r = 1 + \sin\theta$ No

$r = 1 + \sin\theta$







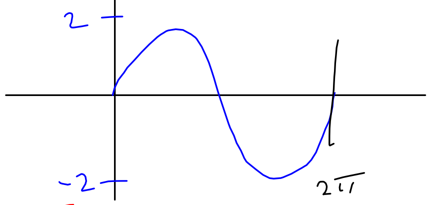
Cardioid

$r = a + b\cos\theta$ $a > b,\ a < b,\ a = b$

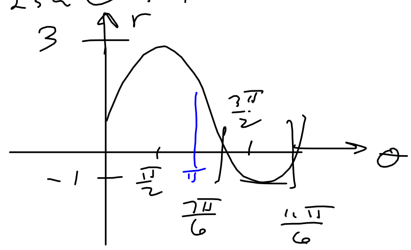
$r = a + b\sin\theta$ I just did $a = b = 1$

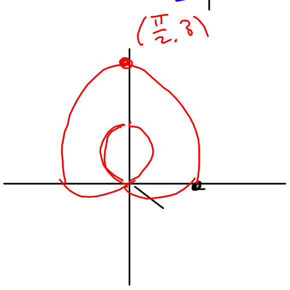
what about $1 + 2\sin\theta$?

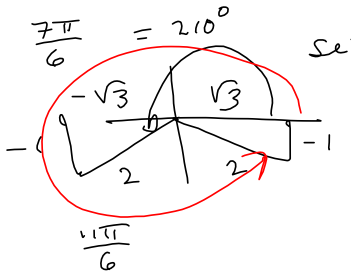
$2\sin\theta$



$2\sin\theta + 1$



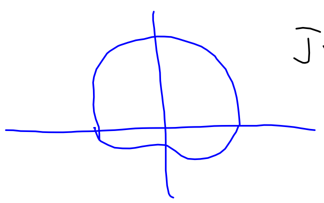




set $2\sin\theta + 1 = 0$

$\sin\theta = -\frac{1}{2}$

$r = 3 + \sin\theta$



Just a circle

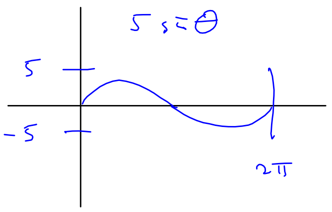
$r = 3\csc\theta = \frac{3}{\sin\theta} = \frac{3}{\frac{y}{r}} = \frac{3}{\frac{y}{\sqrt{x^2+y^2}}} = \sqrt{x^2+y^2}$

$\Rightarrow \frac{3}{\left(\frac{y}{\sqrt{x^2+y^2}}\right)} = \frac{3\sqrt{x^2+y^2}}{y} = \sqrt{x^2+y^2} = \frac{3}{y} = 1$

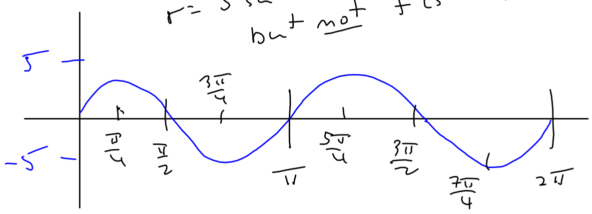
$\Rightarrow y = 3$!? Yes, grasshoppah.

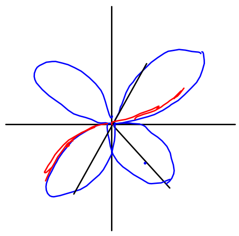
$r = 5\sin(2\theta)$

$5\sin\theta$



$r = 5\sin(2\theta) = f(\sin 2\theta)$ but not $f(\sin\theta)$





4-petal rose.

Missed symmetry wrt polar axis!

$(-r, \pi-\theta)$ test

$r = 5\sin(2(-\theta))$

$r = 5\sin(2(\pi-\theta))$

$= 5\sin(2\pi - 2\theta)$

$= 5(\sin(2\pi)\cos(-2\theta)$

$+ \sin(-2\theta)\cos(2\pi))$

$= -5\sin 2\theta$ Nope.

$-r = 5\sin(2\pi - 2\theta)$

$r = 5\sin(2\theta)$

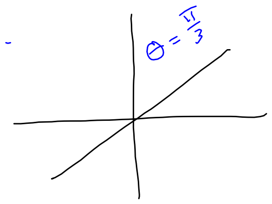
see

$-r \overset{?}{=} 5\sin(-2\theta)$

$-r = -5\sin(2\theta)$

$r = 5\sin(2\theta)$! Yes

$\theta = \frac{\pi}{2}$ Yes.



$r = \sin(2(\pi+\theta))$

$= \sin 2\pi\cos 2\theta$

$+ \sin 2\theta\cos 2\pi$

$= \sin 2\theta$ Yes!

Polar!

$r = \frac{\pi}{3}$

deception!