$f(x) = \sin(x)$ parameter is 'x.'

& $\{(x, f(x)) \mid x \in \text{some domain}\}$

= Graph of f = a curve

§6.6 see Let $x = t$, $y = \sin(t)$

$f(t) = x = x(t)$, $y = y(t) = g(t)$

$(x(t), y(t)) = (f(t), g(t))$

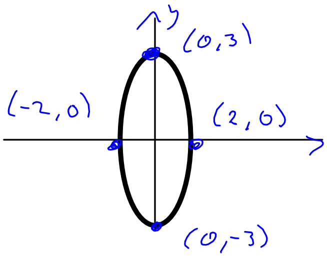
Review

$\frac{x^2}{4} + \frac{y^2}{9} = 1$

Ellipse

$x^2 + y^2 = 9$

$\frac{x^2}{9} + \frac{y^2}{9} = 1$



$x = 9\cos\theta$, $y = 4\sin\theta$

$\frac{x}{9} = \cos\theta$, $\frac{y}{4} = \sin\theta$

This is called eliminating the parameter.

$\left(\frac{x}{9}\right)^2 + \left(\frac{y}{4}\right)^2 = \cos^2\theta + \sin^2\theta = 1$

$\frac{x^2}{81} + \frac{y^2}{16} = 1$

26. $x = \cos\theta$

$y = 2\sin 2\theta = 4\sin\theta\cos\theta$

$\sin(2\theta) = 2\sin\theta\cos\theta$

$x^2 + y^2 = \cos^2\theta + 4\sin^2\theta\cos^2\theta$

$= \cos^2\theta(1 + 4\sin^2\theta)$

$\frac{y}{2} = \sin 2\theta = 2\sin\theta\cos\theta$

$\frac{y}{4} = \sin\theta\cos\theta$

$x^2 - \left(\frac{y}{4}\right)^2 = \cos^2\theta - \sin^2\theta\cos^2\theta$

$= \cos^2\theta(1 - \sin^2\theta)$

$= \cos^2\theta(\cos^2\theta)$

$= x^4$

$x^4 - x^2 = -\frac{y^2}{16}$

$16x^4 - 16x^2 = -y^2$

$y^2 = 16x^2 - 16x^4$

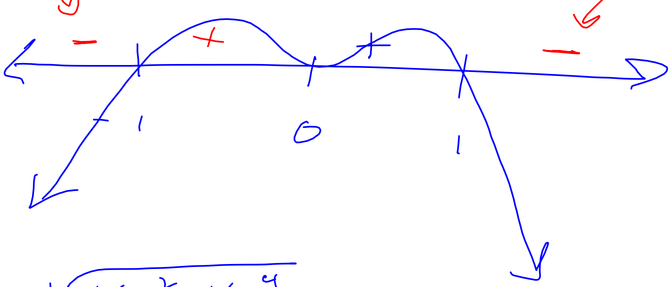
$y = \pm\sqrt{16x^2 - 16x^4} = \pm 4|x|\sqrt{1-x^2}$

$\sqrt{x^2} = |x|$

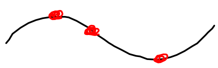
$16x^2 - 16x^4 = 16x^2(1-x^2) = 16x^2(1-x)(1+x)$

$-16x^4$

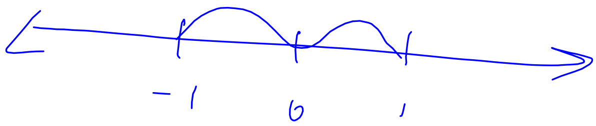
End behavior



$\sqrt{16x^2 - 16x^4}$

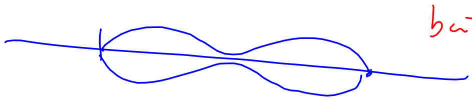


$= 16x^4$



$= \sqrt{16x^2 - 16x^4}$

This is not ideal, but it's good analysis



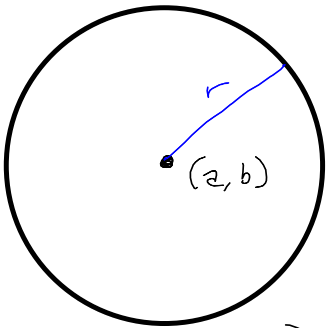
#50 $x = a + r\cos\theta$ $\qquad y = b + r\sin\theta$

$x - a = r\cos\theta \qquad y - b = r\sin\theta$

$\frac{x-a}{r} = \cos\theta \qquad \frac{y-b}{r} = \sin\theta$

$$\left(\frac{x-a}{r}\right)^2 + \left(\frac{y-b}{r}\right)^2 = \cos^2\theta + \sin^2\theta = 1$$

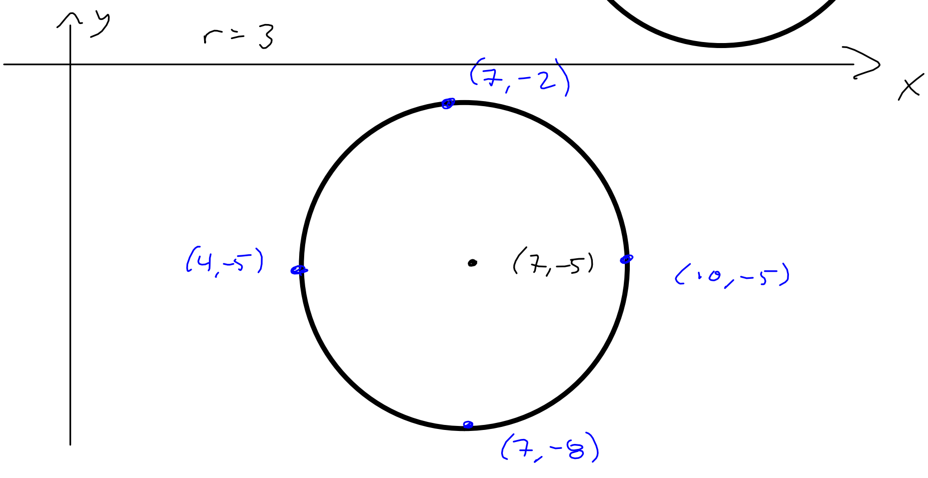
$$\frac{(x-a)^2}{r^2} + \frac{(y-b)^2}{r^2} = 1$$



$(x-7)^2 + (y+5)^2 = 9$

$(h, k) = \text{center} = (7, -5)$

$r = 3$



§6.7 Polar Coords

$(x, y) \longmapsto (r, \theta)$

r = distance from origin

θ = angle measured from the positive x-axis
↳ polar axis

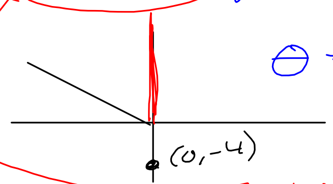
Polar coordinates are handy for describing "round things."

The circle of radius 5, in polar coordinates:

$\boxed{r = 5}$

Positive y-axis

$\theta = \frac{\pi}{2}$!



The WHOLE y-axis if you allow r to be negative

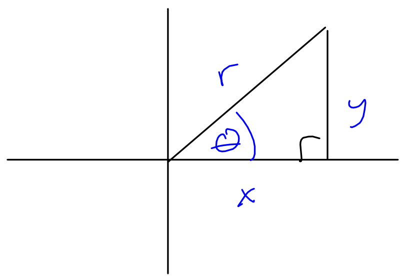
$(x,y) = (0,-4)$

$\longmapsto (r,\theta) = (-4, \frac{\pi}{2})$

$= (4, \frac{3\pi}{2})$

These representations are NOT UNIQUE!

i.e., There's more than one way to express a given point



$r^2 = x^2 + y^2$

$x = r\cos\theta$ $\qquad \frac{x}{r} = \cos\theta$

$y = r\sin\theta$

$\frac{y}{x} = \tan\theta \longrightarrow$

(-4

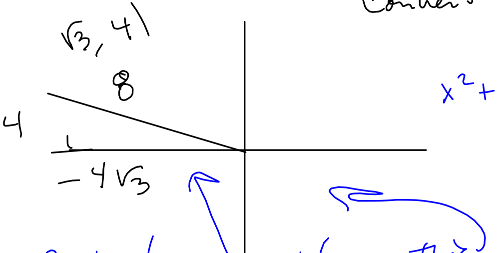
Range of arctangent is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Anything outside of QI & QIV requires analysis.

$\arctan\left(\frac{y}{x}\right) = \arctan(\tan\theta)$

$= \theta$, almost.

arctangent is "true" in quadrants I & IV

Convert $(x,y) = (-4\sqrt{3}, 4)$



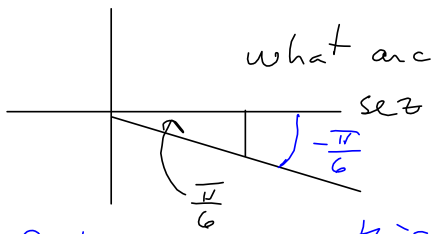
$x^2+y^2 = (-4\sqrt{3})^2 + 4^2 = 16\cdot 3 + 16 = 64 = r^2$

$\Rightarrow r = \pm\sqrt{64} = \pm 8$. Pick the positive, here. $\boxed{r=8}$

Pretend you can't see this

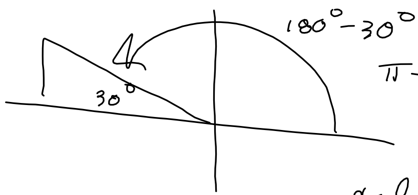
$\arctan\left(\frac{y}{x}\right) = \arctan\left(\frac{-4}{4\sqrt{3}}\right)$

$= \arctan\left(-\frac{1}{\sqrt{3}}\right) = -30^\circ = -\frac{\pi}{6}$



what arctangent sez

But we know this



$180^\circ - 30^\circ$ or $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$, even though calculator says "$-\frac{\pi}{6}$" (-30°)

So $(r,\theta) = \left(8, \frac{5\pi}{6}\right)$ from $(x,y) = (-4\sqrt{3}, 4)$

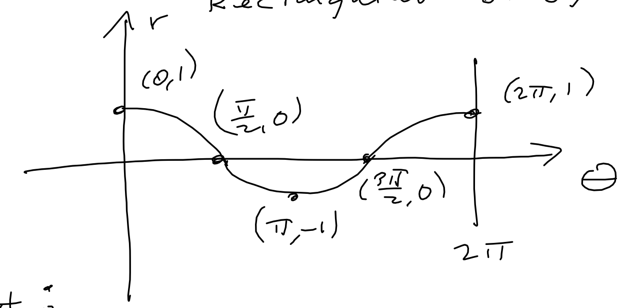
Graphing in Polar Coordinates:

Use rectangular graph of r-vs-θ

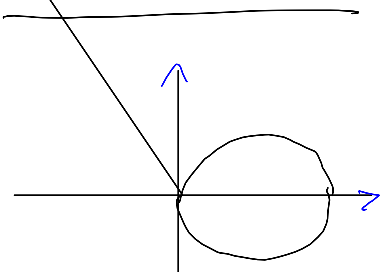
$r = f(\theta)$

$r = \cos\theta$

Rectangular "bridge."

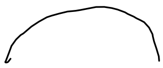


POLAR GRAPH:



θ	$\cos\theta$
0	1
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$

It turns out, this is a circle!



$r = \cos\theta$

$$\cos\theta = \frac{x}{r} = \frac{x}{\sqrt{x^2+y^2}}$$

$$\sqrt{x^2+y^2} = \frac{x}{\sqrt{x^2+y^2}} \Rightarrow$$

$$x = x^2+y^2$$

$$\Rightarrow x^2 - x + y^2 = 0$$

$$x^2 - x + \left(\tfrac{1}{2}\right)^2 + y^2 = 0 + \tfrac{1}{4}$$

$$\left(x-\tfrac{1}{2}\right)^2 + y^2 = \tfrac{1}{4}$$

Completing the square

Circle, centered @ $(\frac{1}{2}, 0)$ of radius $\frac{1}{2}$.