

B5 Find all solutions $x \in [0, 2\pi]$ (should be $[0, 2\pi]$)

of $2\sin^2(3x) - 1 = 0$

$$0 \leq x \leq 2\pi \Rightarrow$$

$$0 \leq 3x \leq 6\pi$$

$$2\sin^2(3x) = 1$$

$$\sin^2(3x) = \frac{1}{2}$$

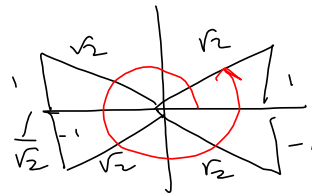
$$\sin(3x) = \pm \sqrt{\frac{1}{2}} = \pm \frac{1}{\sqrt{2}}$$

$$\sin(3x) = \pm \frac{1}{\sqrt{2}}$$

$$\Rightarrow 3x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4},$$

$$\frac{9\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4}, \frac{15\pi}{4}, \frac{17\pi}{4}$$

$$\frac{19\pi}{4}, \frac{21\pi}{4}, \frac{23\pi}{4}$$



$$3 \text{ TIMES : } 3 \cdot 2\pi = 6\pi = \frac{24\pi}{4}$$

\Rightarrow

$$x = \frac{\pi}{12}, \frac{3\pi}{12} = \frac{\pi}{4}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{9\pi}{12} = \frac{3\pi}{4}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{15\pi}{12} = \frac{5\pi}{4}, \frac{17\pi}{12}, \frac{19\pi}{12}$$

$$\frac{21\pi}{12} = \frac{7\pi}{4}, \frac{23\pi}{12}$$

$$2 \cdot \sin\left(\frac{3 \cdot 23 \cdot \pi}{12}\right)^2 - 1 = 0 \text{ is a random check and it checks.}$$

at ground level
B2 A gun with a muzzle velocity of 370 meters per second is fired, with an angle of 15° from the horizontal.

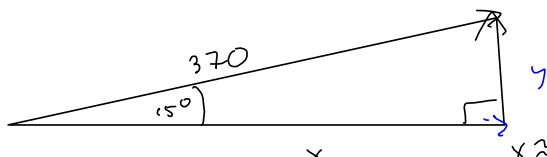
- (5 pts) Find the horizontal and vertical components of the bullet, as it leaves the muzzle, accurate to 4 decimal places.
- (5 pts) Use a half-angle formula to find the *exact* value for the answer to the previous.
- (5 pts) Using $-9.8 \frac{m}{s^2}$ for the acceleration due to gravity, and neglecting air friction, predict where and when the bullet will hit the ground, in the gun question.

IMO, IMHO

ISWYDT

FAIK

> FAIC



(a) $x = 370 \cos(15^\circ) \approx 357.3925557 \approx 357.3926 \frac{m}{s}$
 $y = 370 \sin(15^\circ) \approx 95.76304669$

(b) $x = 370 \sqrt{\frac{1 + \cos 30^\circ}{2}} = 370 \sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} = \frac{370 \sqrt{2 + \sqrt{3}}}{2} = 185 \sqrt{2 + \sqrt{3}} = x$
 $y = 370 \sqrt{\frac{1 - \cos 30^\circ}{2}} = 370 \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = \frac{370 \sqrt{2 - \sqrt{3}}}{2} = 185 \sqrt{2 - \sqrt{3}} = y$

(c) $y = \frac{1}{2}gt^2 + v_0t + y_0$ $v_0 = y\text{-component of the muzzle velocity.}$
 $= -4.9t^2 + v_0t + 0 \Rightarrow -t(4.9t - v_0) = 0$

$\Rightarrow t = \frac{185 \sqrt{2 - \sqrt{3}}}{4.9} \approx 19.54347893 \approx 19.5435 \text{ sec} \approx t$ OR $t = \frac{v_0}{4.9}$

How far downrange when it hits?

HORIZONTAL COMPONENT TIMES THE TIME

So $(185 \sqrt{2 + \sqrt{3}}) \left(\frac{185 \sqrt{2 - \sqrt{3}}}{4.9} \right) = x \approx 6984.693892 \approx 6984.6939 \text{ m}$

Calculations

$$\text{evalf}\left(370 \cdot \cos\left(\frac{15 \cdot \text{Pi}}{180}\right)\right)$$

$$u$$

$$357.3925557$$

$$\text{evalf}\left(370 \cdot \sin\left(\frac{15 \cdot \text{Pi}}{180}\right)\right)$$

$$95.76304669$$

$$185 \cdot \text{sqrt}(2 - \text{sqrt}(3))$$

$$\frac{185 \sqrt{6}}{2} - \frac{185 \sqrt{2}}{2}$$

$$\text{evalf}(\%)$$

$$95.7630467$$

$$\frac{185 \cdot \text{sqrt}(2 - \text{sqrt}(3))}{4.9}$$

$$18.87755102 \sqrt{6} - 18.87755102 \sqrt{2}$$

$$\text{evalf}(\%)$$

$$19.54347893$$

$$185 \cdot \text{sqrt}(2 - \text{sqrt}(3)) \cdot \frac{185 \cdot \text{sqrt}(2 + \text{sqrt}(3))}{4.9}$$

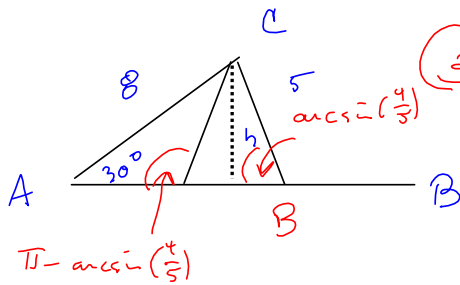
$$6984.693878 \left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right)$$

$$6984.693892$$

B7 The triangle described has 2 possible solutions:

Angle $A = 30^\circ$, side $b = 8$ and side $a = 5$.

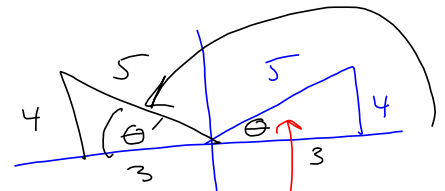
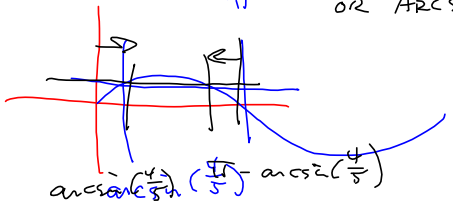
- a. (5 pts) Prove there are 2 possible triangles from this ambiguous information.
- b. (5 pts) Find both triangles.
- c. (5 pts) Use your work to find the area of both triangles. *Not a good investment of time.*



$\frac{h}{8} = \sin 30^\circ$
 $h = 8 \sin 30^\circ = 4$
 $5 > h = 4 \Rightarrow$ long enough for a sol'n.
 AND $5 < b = 8 \Rightarrow$ short enough for 2 sol'ns.

(b) $\frac{\sin 30^\circ}{5} = \frac{\sin B}{8}$
 $\Rightarrow \sin B = \frac{8 \sin 30^\circ}{5} = \frac{4}{5}$

$\sin^{-1}(\sin B) = \sin^{-1}(\frac{4}{5})$
 $\pi - \arcsin(\frac{4}{5})$ OR $\text{ARCSIN}(\frac{4}{5})$



$\pi - \arcsin(\frac{4}{5})$ calculator finds THIS one.
 OR $180^\circ - \arcsin(\frac{4}{5})$, $\text{ARCSIN}(\frac{4}{5})$