Questions from Test 3, Fall, 2016

$f(x) = 3x^3 - 8x^2 + 10x - 4$

(2) $f(2)$:

$$\begin{array}{r|rrrr} 2 & 3 & -8 & 10 & -4 \\ & & 6 & -4 & 12 \\ \hline & 3 & -2 & 6 & \boxed{8 = f(2)} \end{array}$$

This work says

$f(x) = (x-2)(3x^2 - 2x + 6) + 8$

Dividend = Divisor · Quotient + Remainder

$\frac{29}{3} = 9 + \frac{2}{3}$

$\Rightarrow 29 = 3 \cdot 9 + 2$

We just divided $f(x)$ by $x-2$ to find $f(2)$

$$\text{Divisor}\overline{\left)\text{Dividend}\right.}^{\,\text{Quotient}}$$

$$\begin{array}{r|rrrr} 1+i & 3 & -8 & 10 & -4 \\ & & 3+3i & -8-2i & 4 \\ \hline & 3 & -5+3i & 2-2i & 0 \end{array}$$

See? $1+i$ is a zero of $f(x)$

$(a-b)(a+b) = a^2 - b^2$

$(a+bi)(a-bi) = a^2 + b^2$

$(-5+3i)(1+i)$
$= -5 - 5i + 3i + 3i^2$
$= -5 - 2i - 3$
$= -8 - 2i$

NOTE $2 - 2i = 2\boxed{(1-i)}$

$2(1+i)(1-i)$

$= 2(1^2 + 1^2) = 2(2) = 4$

$1^2 - i^2 = 1^2 + 1^2$

$b^2 i^2 = -b^2$

$-b^2 i^2 = b^2$

Conjugate pairs theorem:

$f(a+bi) = 0$

$\Rightarrow f(a-bi) = 0$

The 1st Synthetic Division Says:

$f(x) = (x-(1+i))\underbrace{(3x^2 + (-5+3i)x + (2-2i))}_{\text{Depressed Polynomial}}$

$$\begin{array}{r|rrrr} 1+i & 3 & -8 & 10 & -4 \\ & & 3+3i & -8-2i & 4 \\ \hline 1-i & 3 & -5+3i & 2-2i & 0 \\ & & 3-3i & -2+2i & \\ \hline & \boxed{3} & \boxed{-2} & 0 & \end{array}$$

sweet!

Sweet!

$3x - 2$

$$\boxed{f(x) = (3x-2)(x-(1+i))(x-(1-i))}$$

5. Let $f(x)=3x^3-8x^2+10x-4$.
   a. (5 pts) Use synthetic division to find $f(2)$.
   b. (5 pts) Use synthetic division to show that $x=1+i$ is a solution of the equation $f(x)=0$.
   c. (5 pts) Find the linear factorization of f that is promised to us in the Fundamental Theorem of Algebra.

How I'd write it up for the test:

$f(x) = 3x^3 - 8x^2 + 10x - 4$

(a) $f(2)$:

$$\begin{array}{r|rrrr} 2 & 3 & -8 & 10 & -4 \\ & & 6 & -4 & 12 \\ \hline & 3 & -2 & 6 & \boxed{8 = f(2)} \end{array}$$

(b)

$$\begin{array}{r|rrrr} 1+i & 3 & -8 & 10 & -4 \\ & & 3+3i & -8-2i & 4 \\ \hline 1-i & 3 & -5+3i & 2-2i & \boxed{0} \\ & & 3-3i & -2+2i & \\ \hline & 3 & -2 & 0 & \end{array}$$

Nice!

(b) See? $f(1+i)=0$!

(c)

So $\boxed{f(x) = (x-(1+i))(x-(1-i))(3x-2)}$

is the linear factorization.

(All factors are linear (x^1-power))

$(-5+3i)(1+i)$
$= -5-2i+3i^2$
$= -8-2i$

$(1+i)(2-2i)$
$= 2(1+i)(1-i)$
$= 2(1^2+1^2)$
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